Rasterization Parameter Interpolation

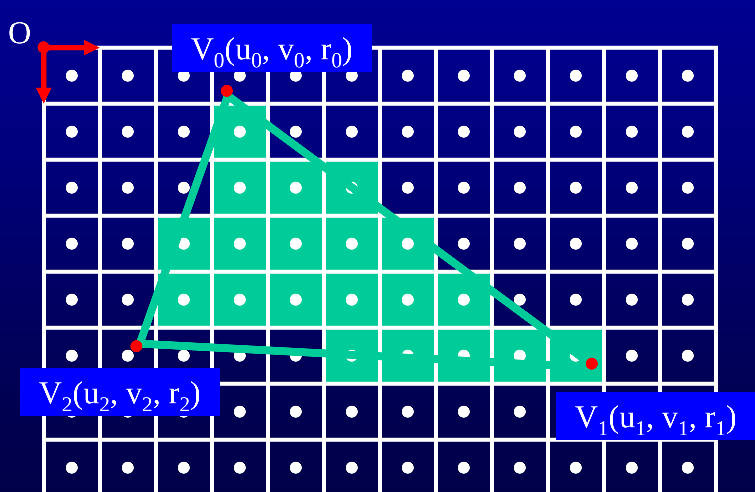
Overview

- screen-space interpolation
- perspectively-correct interpolation

Screen-space interpolation

- Given
  - image plane coordinates of 3 vertices
    $(u_0, v_0), (u_1, v_1), (u_2, v_2)$
  - rasterization parameter value at the 3 vertices
    (r_0, r_1, r_2)
- Find
  - coefficients of linear expression
    $au + bv + c = r$

Screen-space interpolation



$$au + bv + c = r$$

$$\begin{cases} au_0 + bv_0 + c = r_0 \\ au_1 + bv_1 + c = r_1 \\ au_2 + bv_2 + c = r_2 \end{cases}$$

$$\begin{bmatrix} u_0 & v_0 & 1 \\ u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix}$$

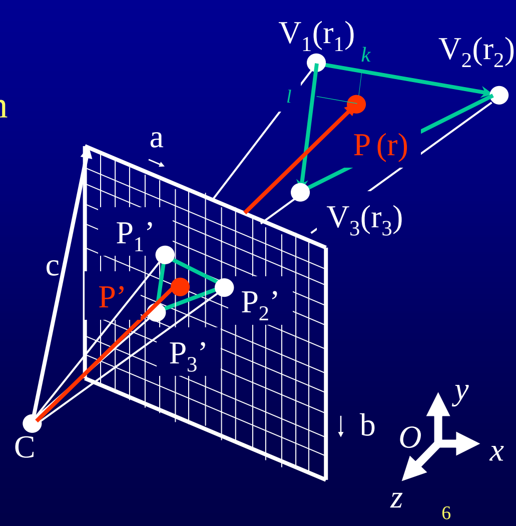
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} u_0 & v_0 & 1 \\ u_1 & v_1 & 1 \\ u_2 & v_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix}$$

Screen-space interpolation

- using barycentric interpolation
- r is any rasterization parameter (red, green, blue, 1/z, z, s, t, n_x, n_y, n_z)
- 1/z is the only parameter linear in screen space, for others it is an approximation

Persp. corr. interpolation

- Linear interpolation in triangle plane rather than in image plane
  - pick convenient 2D coordinate system
  - choose axes V_2V_1 and V_3V_1
  - P has coordinates (k, l)
  - $P=V_1+(V_2-V_1)k+(V_3-V_1)l$
  - $r = r_1+(r_2-r_1)k+(r_3-r_1)l$



Perspectively correct interpolation of parameter "r"

$$\dot{P} = \dot{V}_1 + (\dot{V}_2 - \dot{V}_1)k + (\dot{V}_3 - \dot{V}_1)l$$

$$\dot{P} = \dot{C} + (\bar{c} + u\bar{a} + v\bar{b})w$$

$$\begin{bmatrix} \dot{V}_1 - \dot{C} & \dot{V}_2 - \dot{C} & \dot{V}_3 - \dot{C} \end{bmatrix} \begin{bmatrix} 1-k-l \\ k \\ l \end{bmatrix} = \begin{bmatrix} \bar{a} & \bar{b} & \bar{c} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} w$$

$$\begin{bmatrix} 1-k-l \\ k \\ l \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} w$$

$$w = \frac{1}{(q_{11}+q_{21}+q_{31})u + (q_{12}+q_{22}+q_{32})v + q_{13}+q_{23}+q_{33}}$$

$$k = \frac{q_{21}u + q_{22}v + q_{23}}{(q_{11}+q_{21}+q_{31})u + (q_{12}+q_{22}+q_{32})v + q_{13}+q_{23}+q_{33}}$$

$$l = \frac{q_{31}u + q_{32}v + q_{33}}{(q_{11}+q_{21}+q_{31})u + (q_{12}+q_{22}+q_{32})v + q_{13}+q_{23}+q_{33}}$$

$$1-k-l = \frac{q_{11}u + q_{12}v + q_{13}}{(q_{11}+q_{21}+q_{31})u + (q_{12}+q_{22}+q_{32})v + q_{13}+q_{23}+q_{33}}$$

$$r = r_1 + (r_2 - r_1)k + (r_3 - r_1)l = r_1(1-k-l) + r_2 k + r_3 l$$

$$r = \frac{(q_{11}r_1 + q_{21}r_2 + q_{31}r_3)u + (q_{12}r_1 + q_{22}r_2 + q_{32}r_3)v + q_{13}r_1 + q_{23}r_2 + q_{33}r_3}{(q_{11}+q_{21}+q_{31})u + (q_{12}+q_{22}+q_{32})v + q_{13}+q_{23}+q_{33}}$$