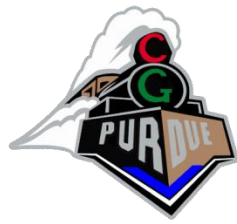




# Light Transport

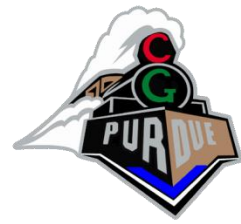
CS434

Daniel G. Aliaga  
Department of Computer Science  
Purdue University



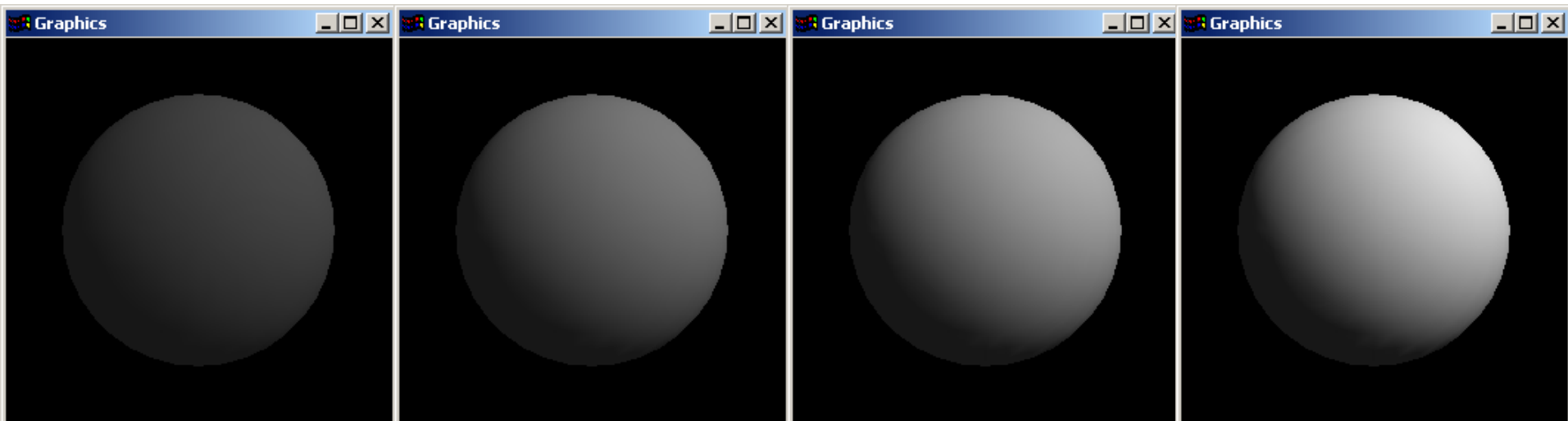
# Topics

- Local and Global Illumination Models
- Helmholtz Reciprocity
- Dual Photography/Light Transport (in Real-World)



# Diffuse Lighting

- A.k.a. Lambertian illumination
- A fraction of light is radiated in every direction
- Intensity varies with cosine of the angle with normal



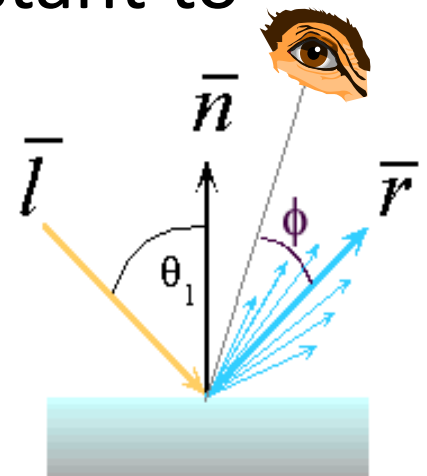


# Specular Lighting

- The most common lighting model was suggested by Phong

$$I_{spec} = \rho_{spec} I_{Light} (\cos \phi)^{n_{shiny}}$$

- The  $n_{shiny}$  term is an empirical constant to model the rate of falloff
- The model has no exact physical basis, but it “sort of works”



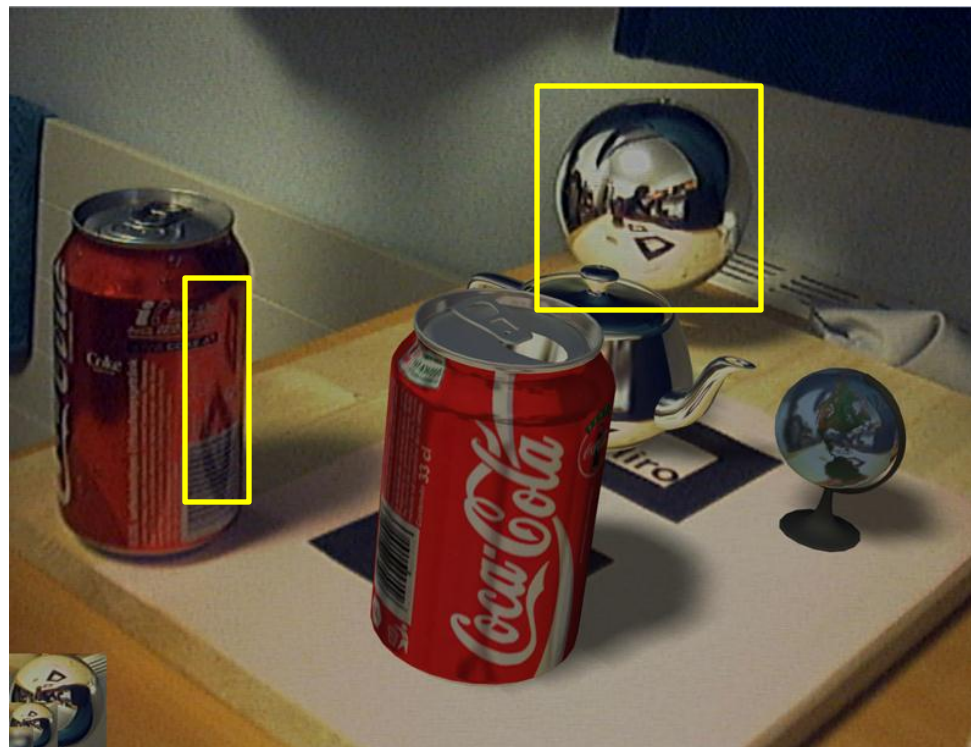


# Example





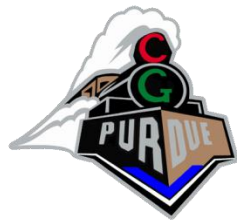
# Inter-reflections



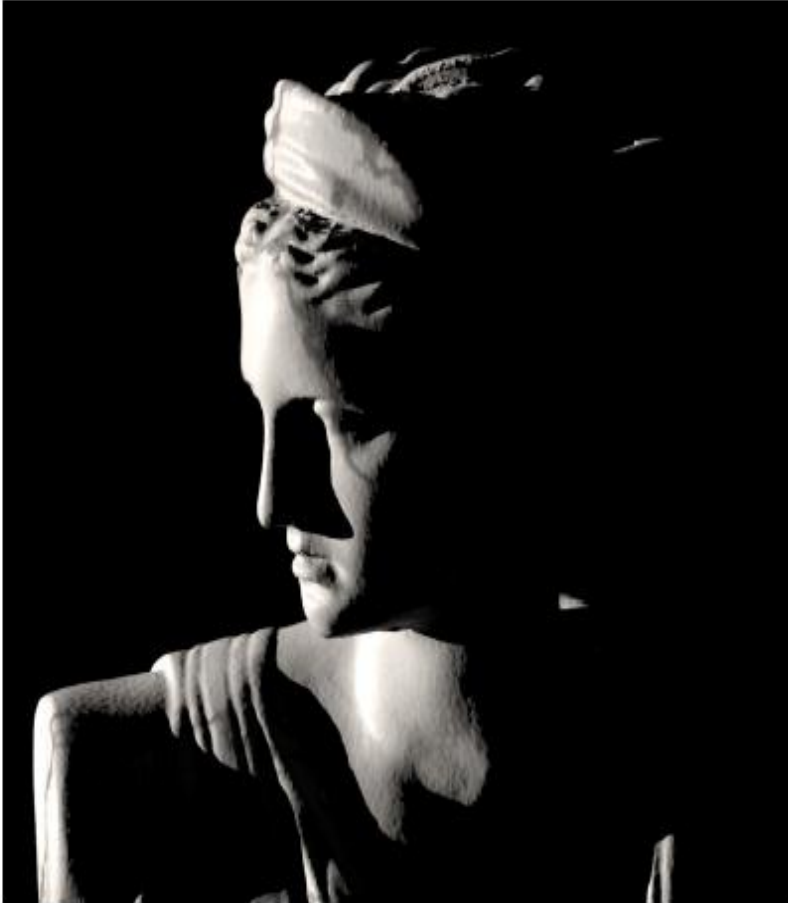


# Scattering





# Scattering



Without (subsurface) scattering

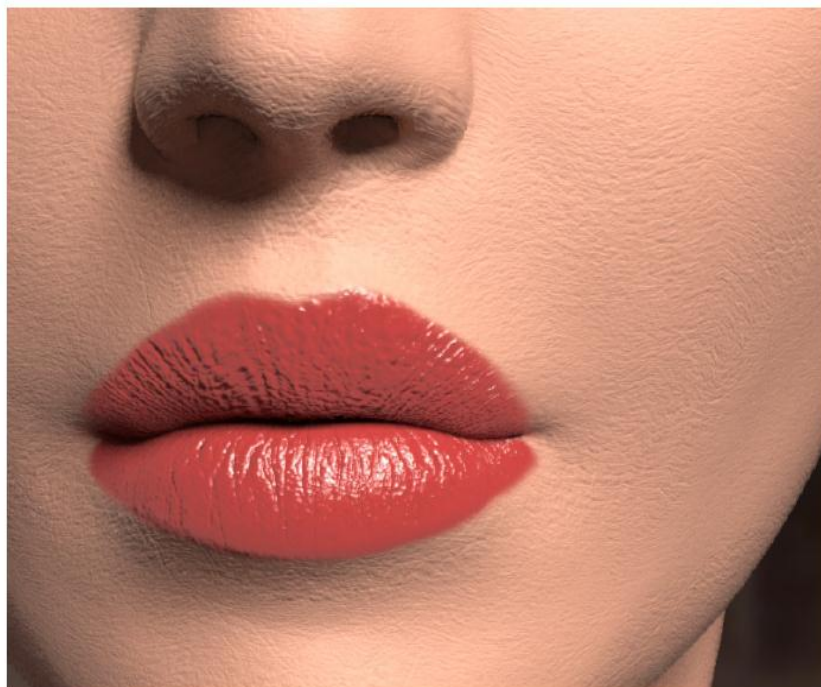


With (subsurface) scattering





# Scattering



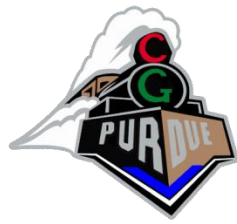
BRDF

Without (subsurface) scattering

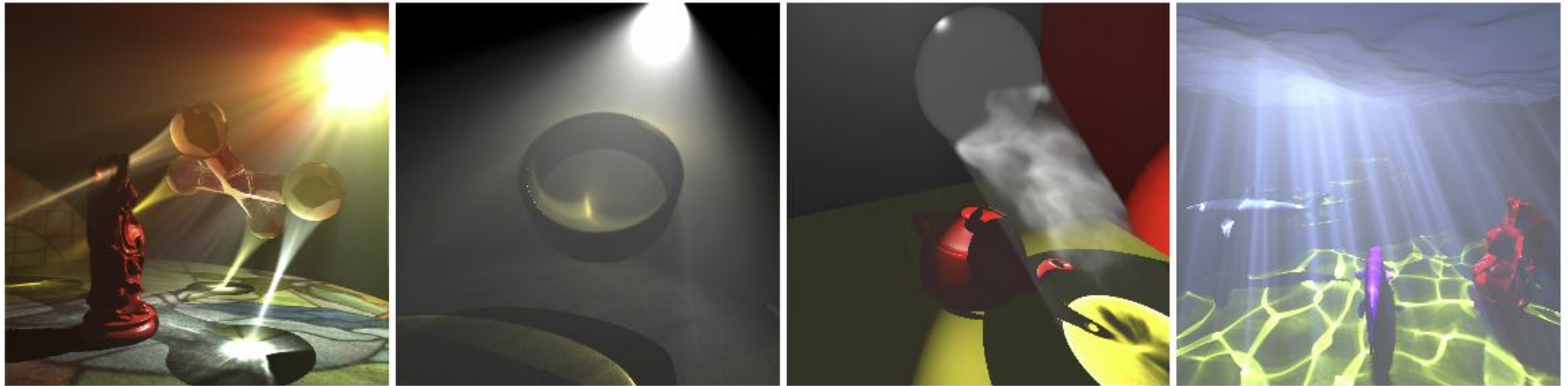


BSSRDF

With (subsurface) scattering



# Scattering



Hu et al. 2010

Scattering through participating media with volume caustics...



# Rendering Equation (also known as the light-transport equation)

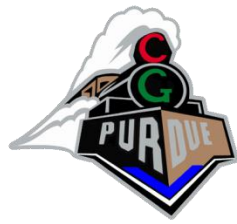
- Illumination can be generalized to

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
-----------------------------------	----------	--------------------	------	-----------------------------

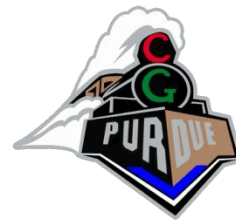
(note: equation is recursive)

**...but it does not model all illumination effects!**



# Conclusion

- Modeling physical illumination is hard
- “Undoing” physically-observed illumination in order to discover the underlying geometry is even harder
- Insight: let’s sample it and “re-apply” it!



## Recall the Linear Operator Equation...

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light  
(Output Image)

Emission

Reflected  
Light

BRDF

Cosine of  
Incident angle



$$L = E + KL$$

where  $K$  can be thought of as the “light transport matrix”; i.e., it transports light from the previous surface (=light) to the next surface



# Dual Photography

- Compute a light transport matrix  $T$  that “transports light” from an illumination vector  $P$  to a camera image vector  $C$
- Thus rendering equation is now...



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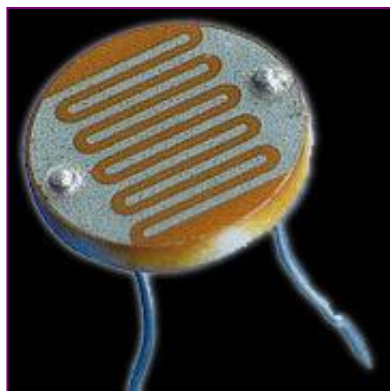
$$C = TP$$





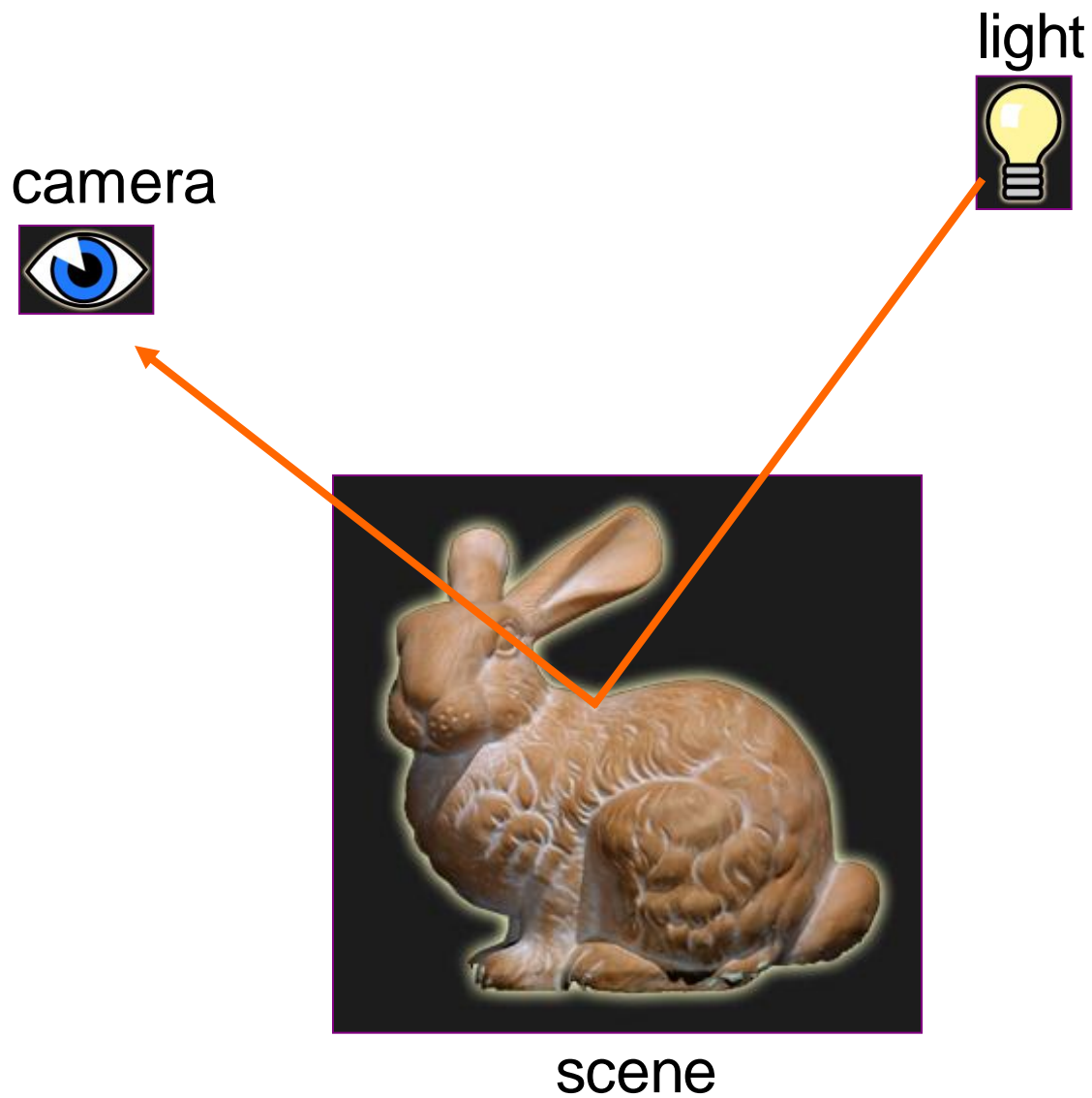
# Dual Photography

- Compute a light transport matrix  $T$  that “transports light” from an illumination vector  $P$  to a camera image vector  $C$

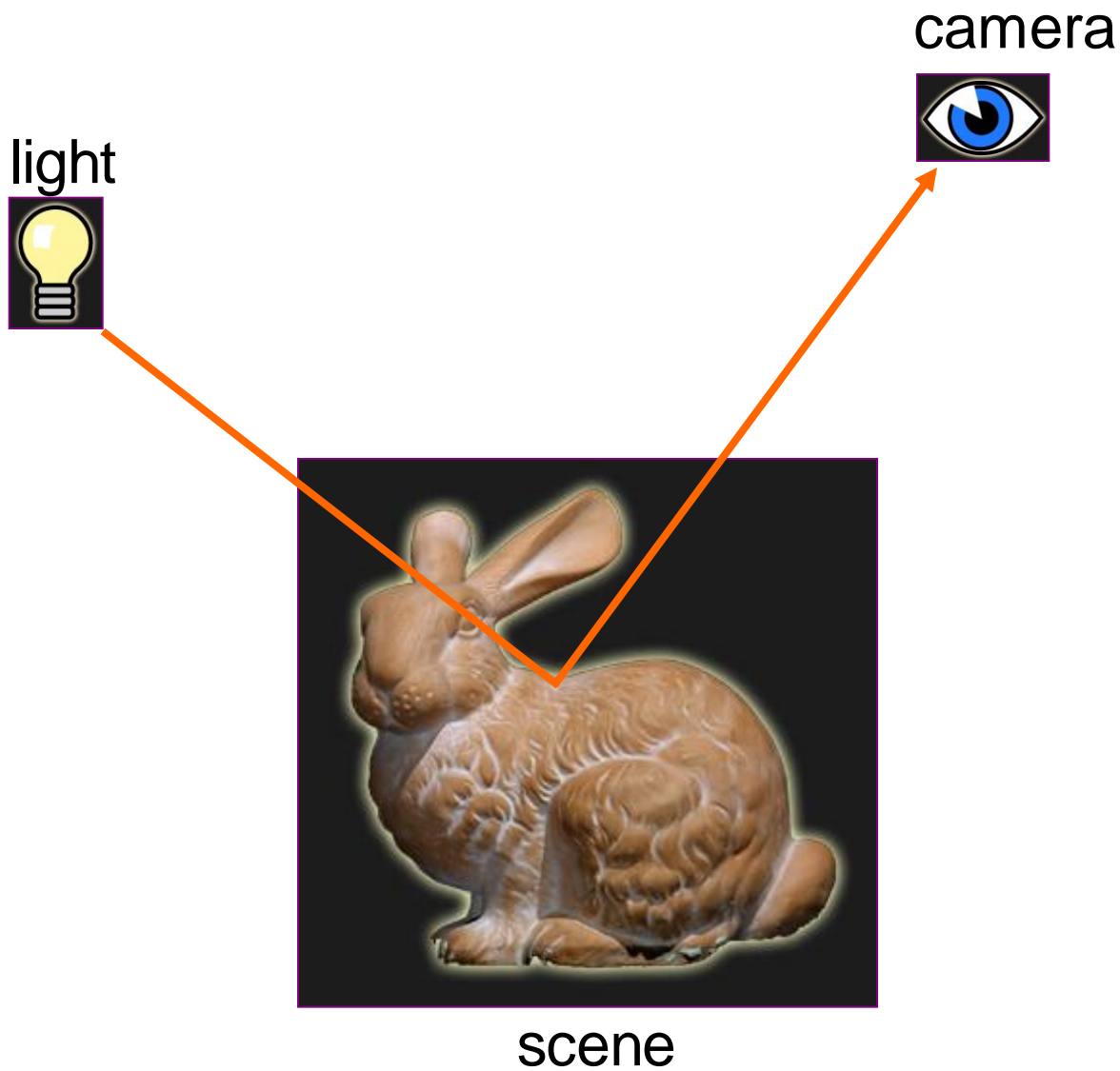


[Sen et al., SIGGRAPH 2005] (slides based on those from the paper)

# Helmholtz Reciprocity



# Helmholtz Reciprocity



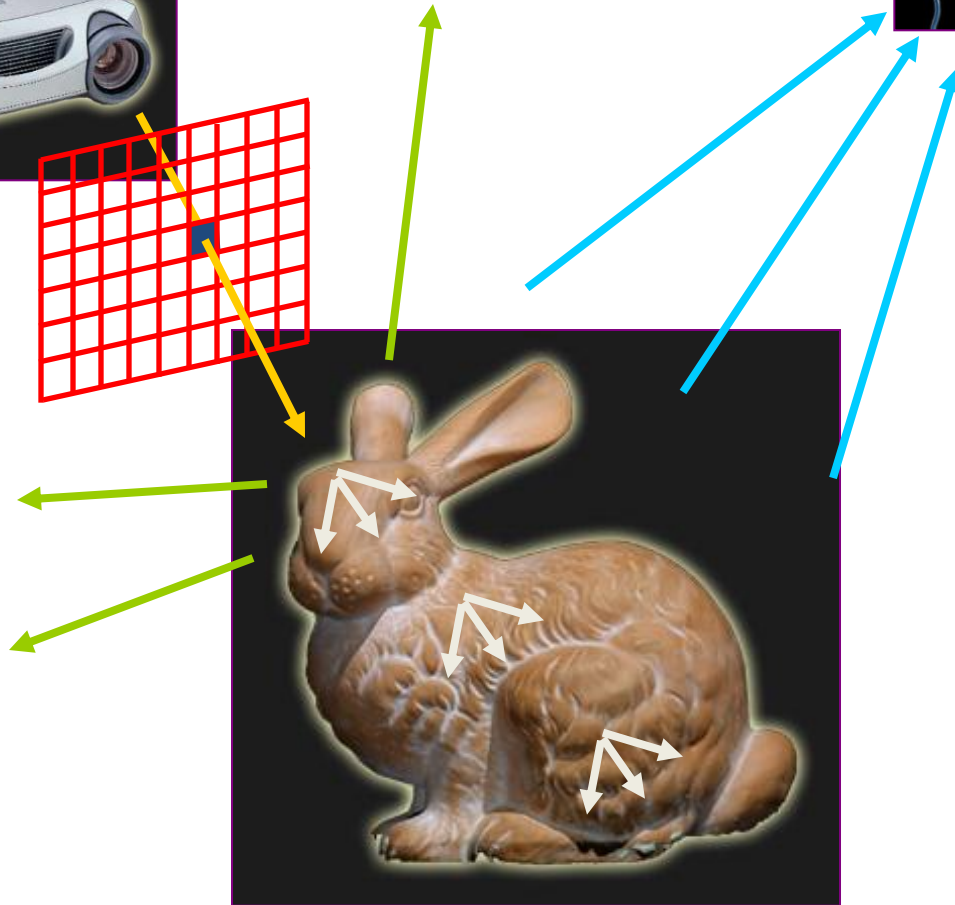
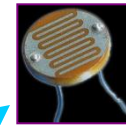
# Measuring transport along a set of paths



projector



photocell

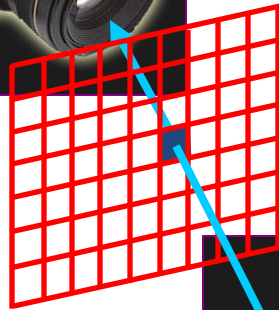


scene

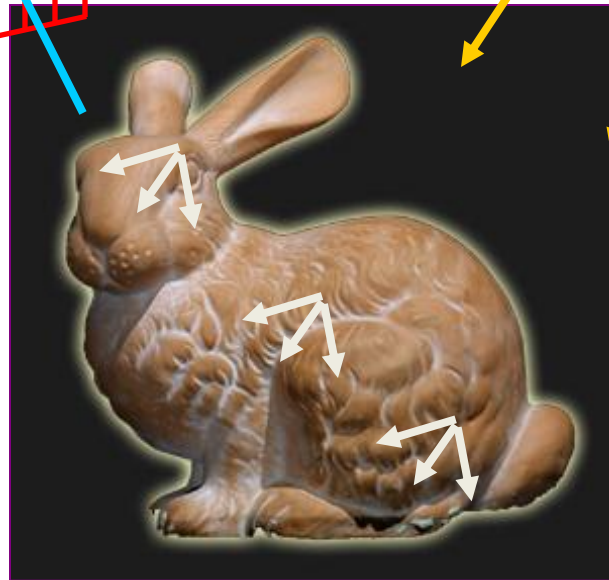
# Reversing the paths



camera



point light

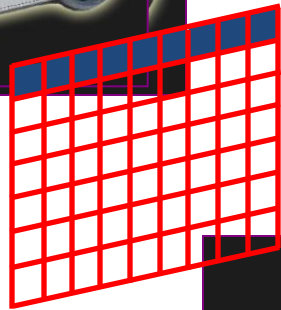


scene

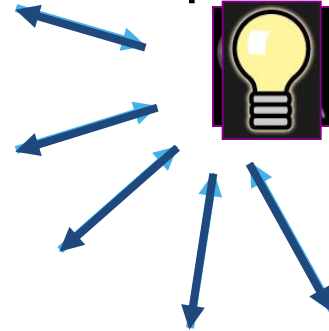
# Forming a dual photograph



“dual” camera  
projector



“dual” light  
protocol



scene





# Forming a dual photograph

“dual” camera

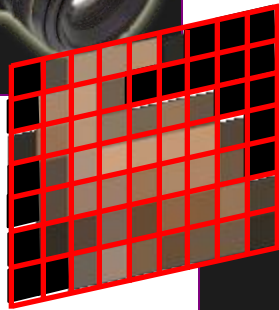
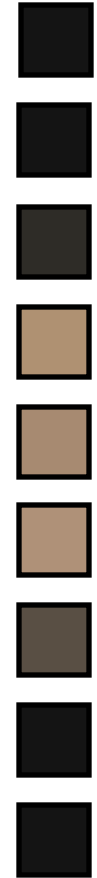
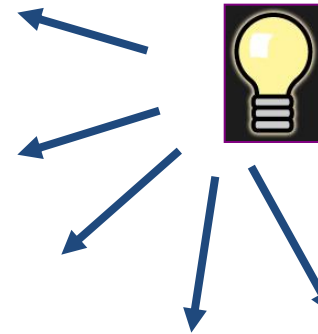


image of scene



scene

“dual” light



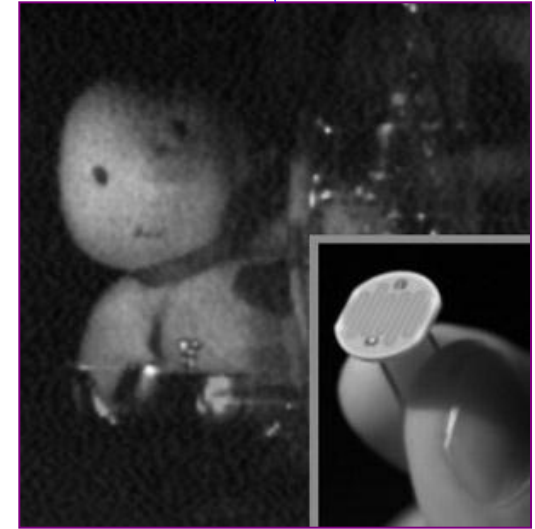


# Physical demonstration

- light replaced with projector
- camera replaced with photocell
- projector scanned across the scene



conventional photograph,  
with light coming from right



dual photograph,  
as seen from projector's position  
and as illuminated from photocell's position





# Related imaging methods

- time-of-flight scanner
  - if they return reflectance as well as range
  - but their light source and sensor are typically coaxial
- scanning electron microscope



Velcro® at 35x magnification,  
Museum of Science, Boston

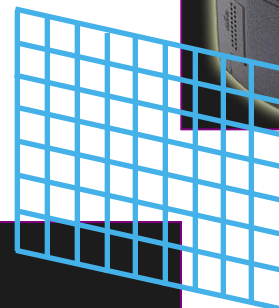
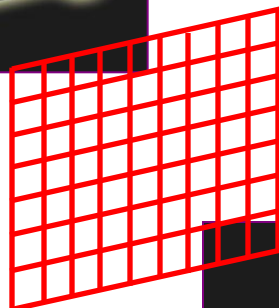
# The 4D transport matrix



projector



photocell

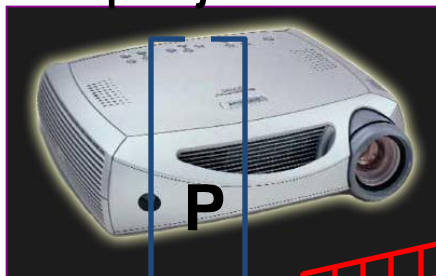


scene

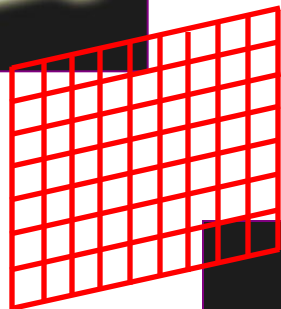
# The 4D transport matrix



projector



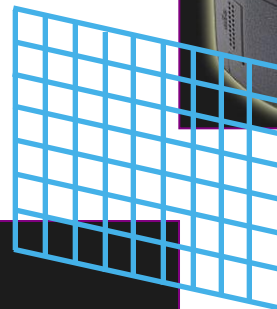
$pq \times 1$



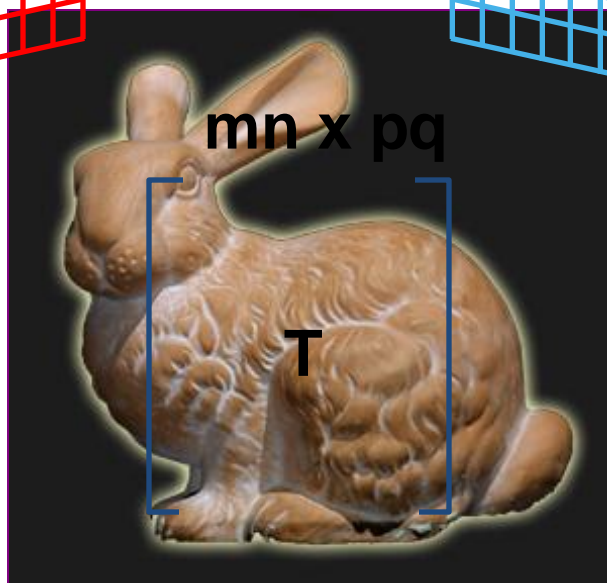
camera



$mn \times 1$



$mn \times pq$



scene

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P} \end{array} \right] \\ pq \times 1 \end{array}$$

# The 4D transport matrix



$$\begin{matrix} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{matrix} = \begin{matrix} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \end{array} \right] \end{matrix} \begin{matrix} \left[ \begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \\ pq \times 1 \end{matrix}$$

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right] \\ \text{mn} \times \mathbf{1} \end{array} = \begin{array}{c} \text{mn} \times \text{pq} \\ \left[ \begin{array}{|c|c|} \hline \text{light orange bar} & \text{orange bar} \\ \hline \end{array} \right] \\ \mathbf{T} \end{array} \begin{array}{c} \left[ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right] \\ \text{pq} \times \mathbf{1} \end{array}$$

The diagram shows a matrix equation. On the left, a vertical column of 10 empty square brackets represents a vector of size  $mn \times 1$ , labeled 'C'. This is followed by an equals sign. In the middle, a matrix of size  $mn \times pq$  is shown, labeled 'T'. This matrix is represented by two vertical bars: a light orange one on the left and a darker orange one on the right. On the right, another vertical column of 10 empty square brackets represents a vector of size  $pq \times 1$ . This vector contains the values 0, 1, 0, 0, 0, 0, 0, 0, 0, and 0 from top to bottom.

# The 4D transport matrix



$$\begin{matrix} \left[ \begin{array}{c} \mathbf{C} \end{array} \right] \\ mn \times 1 \end{matrix} = \begin{matrix} mn \times pq \\ \left[ \begin{array}{c} \text{orange bars} \end{array} \right] \mathbf{T} \end{matrix} \begin{matrix} \left[ \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} \right] \\ pq \times 1 \end{matrix}$$

# The 4D transport matrix



$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C} \\ \hline \end{array} \right] \\ mn \times 1 \end{array} = \begin{array}{c} mn \times pq \\ \left[ \begin{array}{c} \mathbf{T} \\ \hline \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P} \\ \hline \end{array} \right] \\ pq \times 1 \end{array}$$



# The 4D transport matrix



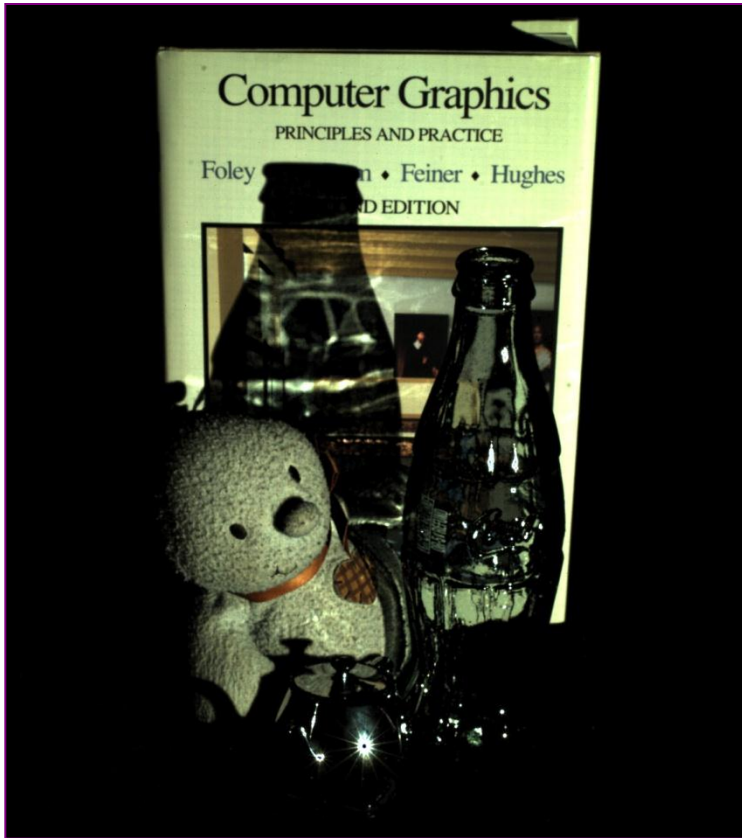
$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C} \\ \mathbf{C} \end{array} \right] \\ \text{mn} \times \mathbf{1} \end{array} = \begin{array}{c} \text{mn} \times \text{pq} \\ \left[ \begin{array}{c} \mathbf{T} \\ \mathbf{T} \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P} \\ \mathbf{P} \end{array} \right] \\ \text{pq} \times \mathbf{1} \end{array}$$

applying Helmholtz reciprocity...

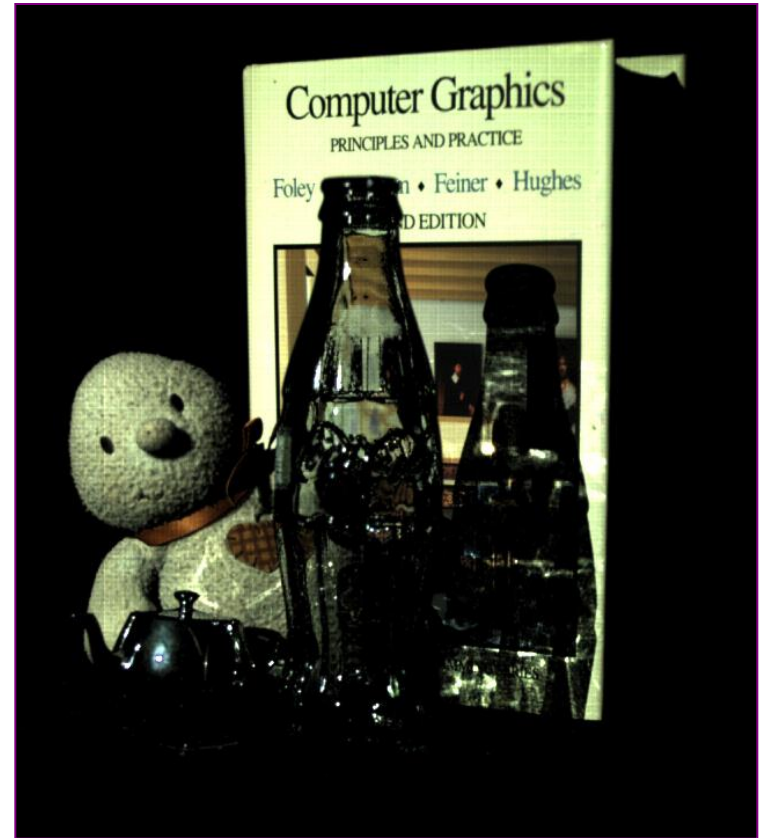
$$\begin{array}{c} \left[ \begin{array}{c} \mathbf{C}' \\ \mathbf{C}' \end{array} \right] \\ \text{pq} \times \mathbf{1} \end{array} = \begin{array}{c} \text{pq} \times \text{mn} \\ \left[ \begin{array}{c} \mathbf{T}^T \\ \mathbf{T}^T \end{array} \right] \end{array} \begin{array}{c} \left[ \begin{array}{c} \mathbf{P}' \\ \mathbf{P}' \end{array} \right] \\ \text{mn} \times \mathbf{1} \end{array}$$



# Example



conventional photograph  
with light coming from right

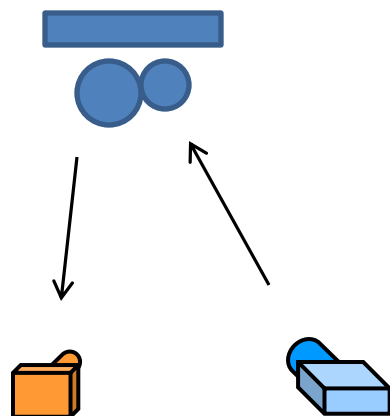


dual photograph  
as seen from projector's position



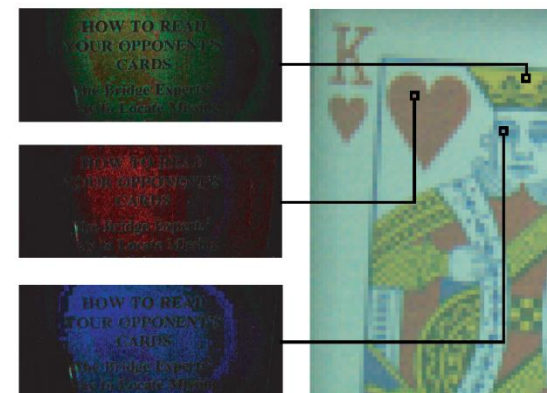
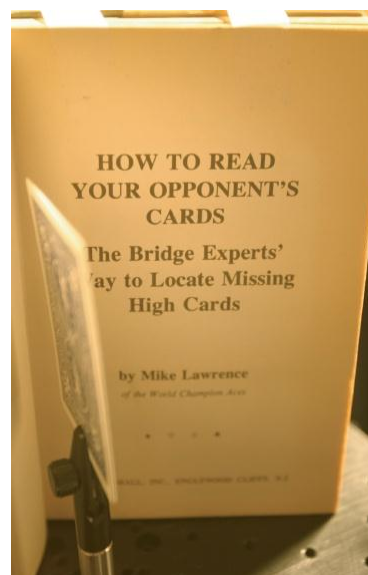
# Example

- Can encode light (or projector) to camera “transport” in a large matrix  $T$



Camera  $c$

Projector  $p$

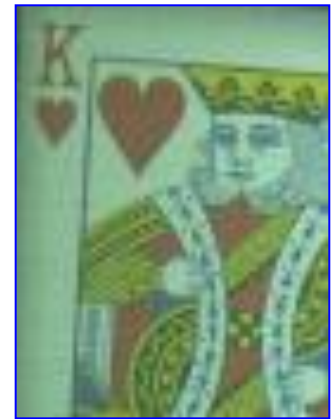
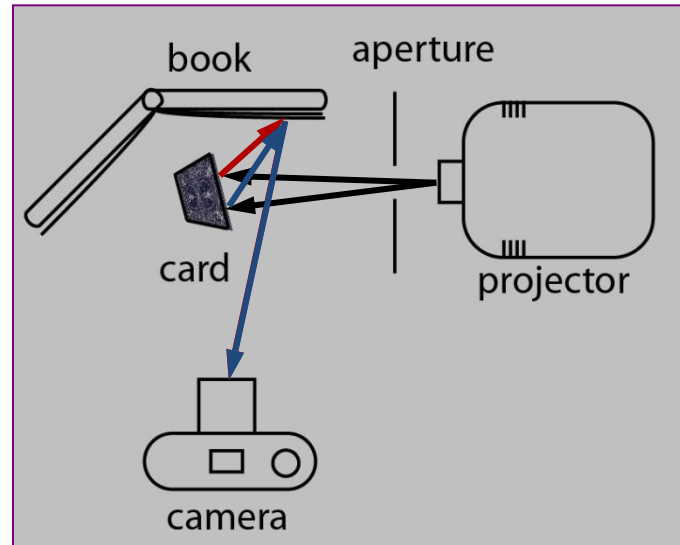
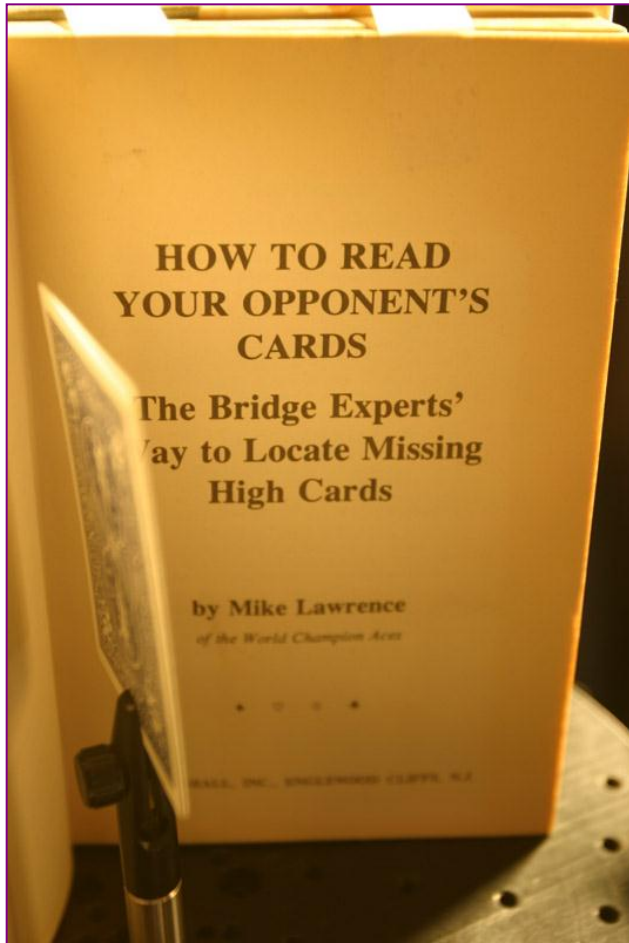


$$\begin{bmatrix} c \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} p \end{bmatrix}$$

$$\begin{bmatrix} p \end{bmatrix} = \begin{bmatrix} T^t \end{bmatrix} \begin{bmatrix} c \end{bmatrix}$$

As seen from camera... As seen from projector!!!

# Dual photography from diffuse reflections



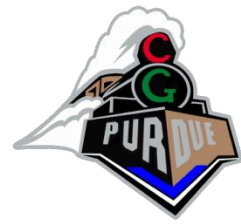
the camera's view



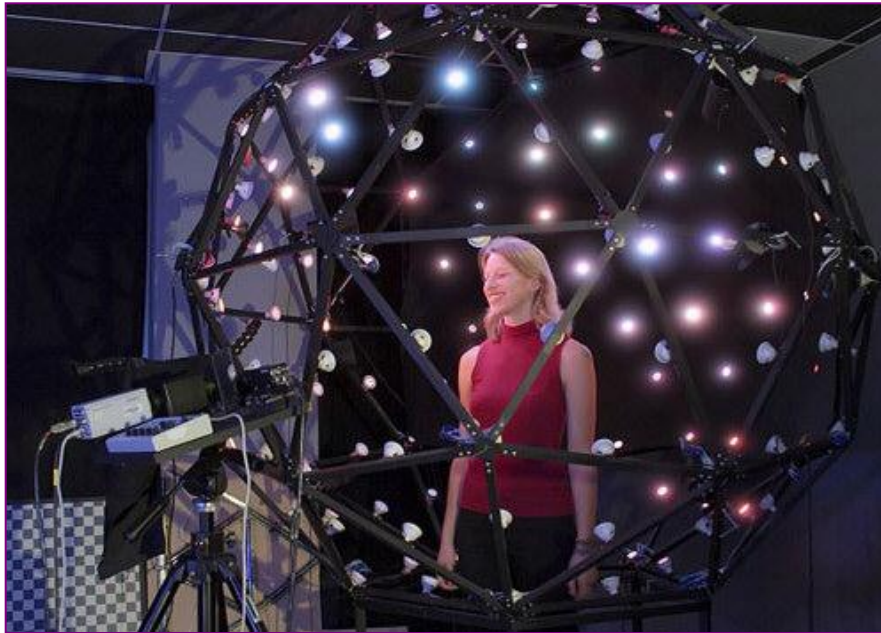
# Properties of the transport matrix

- little inter-reflection
  - sparse matrix
- many inter-reflections
  - dense matrix
- convex object
  - diagonal matrix
- concave object
  - full matrix

Can we create a dual photograph entirely from diffuse reflections?



# Relighting

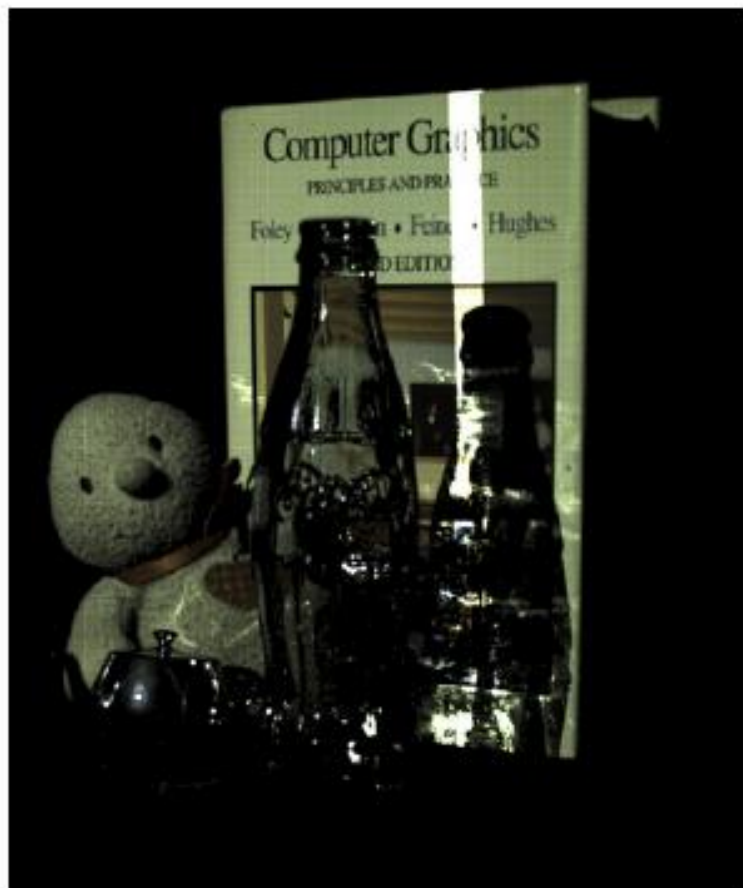


Paul Debevec's  
Light Stage 3

- subject captured under multiple lights
- one light at a time, so subject must hold still
- point lights are used, so can't relight with cast shadows



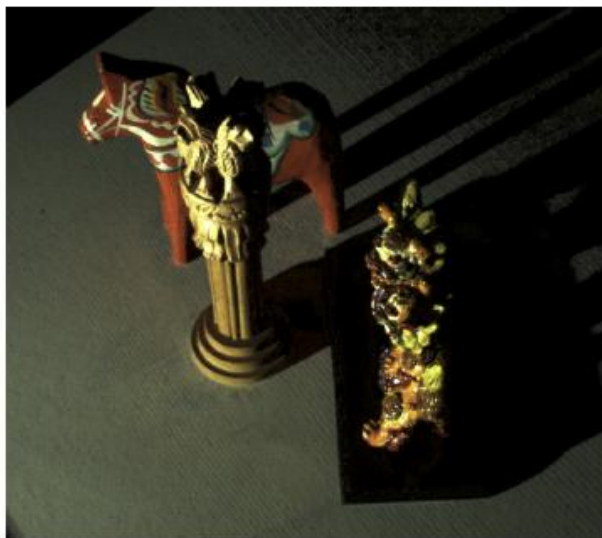
# Relighting



With Dual Photography...



# Relighting



With Dual  
Photography...





# Relighting



(a)



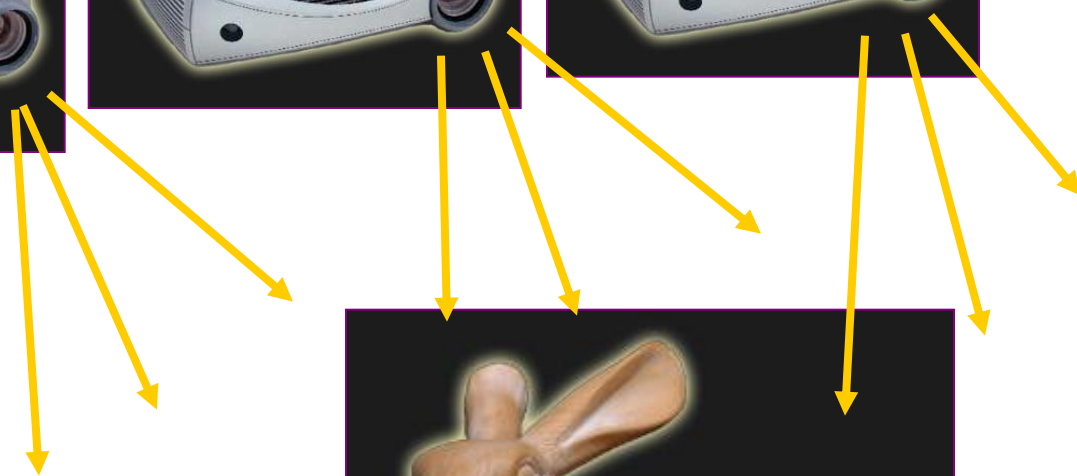
(b)



With Dual  
Photography...

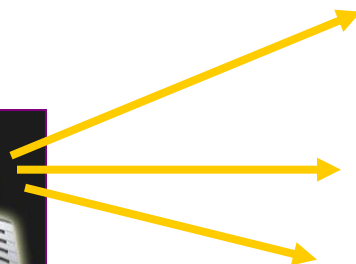
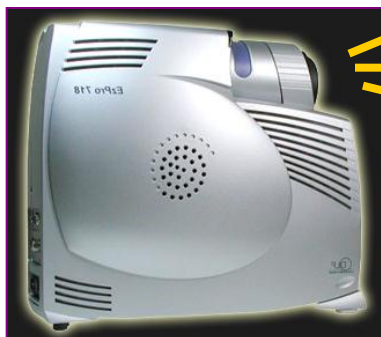


# The 6D transport matrix





# The 6D transport matrix



# The advantage of dual photography



- capture of a scene as illuminated by different lights cannot be parallelized
- capture of a scene as viewed by different cameras can be parallelized

# Measuring the 6D transport matrix



projector



camera array



scene



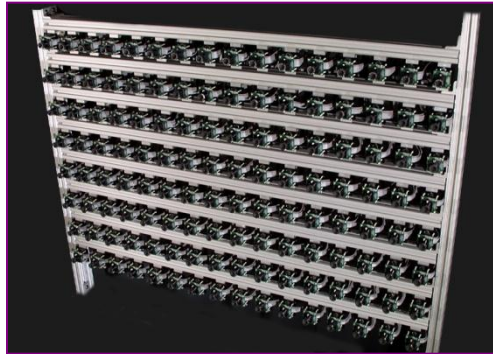
# Relighting with complex illumination



projector



camera array



scene

$$\begin{matrix} & & pq \times mn \times uv \\ \left[ \begin{matrix} C' \end{matrix} \right] & = & \left[ \begin{matrix} T^T \end{matrix} \right] \left[ \begin{matrix} P' \end{matrix} \right] \\ pq \times 1 & & mn \times uv \times 1 \end{matrix}$$

- step 1: measure 6D transport matrix T
- step 2: capture a 4D light field
- step 3: relight scene using captured light field

# Running time

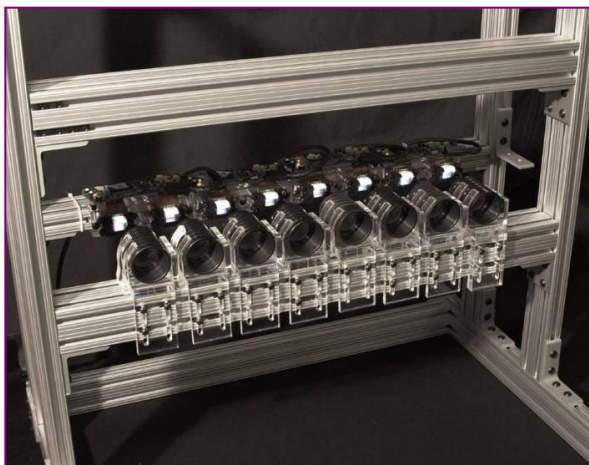


- the different rays within a projector can in fact be parallelized to some extent
- this parallelism can be discovered using a coarse-to-fine adaptive scan
- can measure a 6D transport matrix in 5 minutes

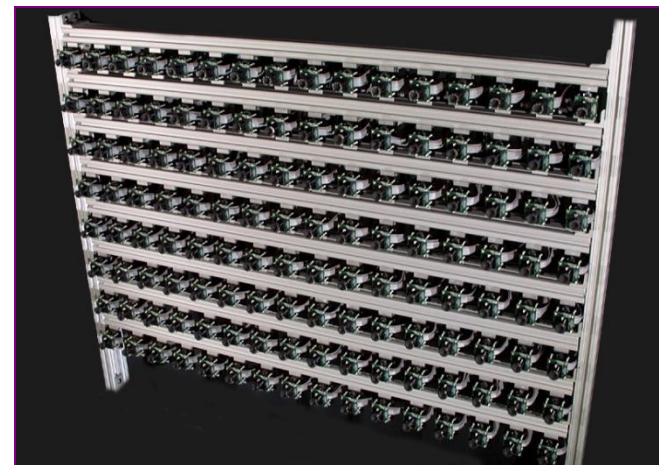
# Can we measure an 8D transport matrix?



projector array



camera array



scene





# Demos

- Metropolis Light Transport
  - <http://www.youtube.com/watch?v=3Xo0qVT3nxg>
  - [http://www.youtube.com/watch?v=GMDfy\\_B0rvQ](http://www.youtube.com/watch?v=GMDfy_B0rvQ)
- Faster acquisition:
  - [http://www.youtube.com/watch?v=fVBICVBEGVU&playnext=1&list=PL361744591665D18D&feature=results\\_video](http://www.youtube.com/watch?v=fVBICVBEGVU&playnext=1&list=PL361744591665D18D&feature=results_video)