Photometric (and Photogeometric) Stereo

CS434

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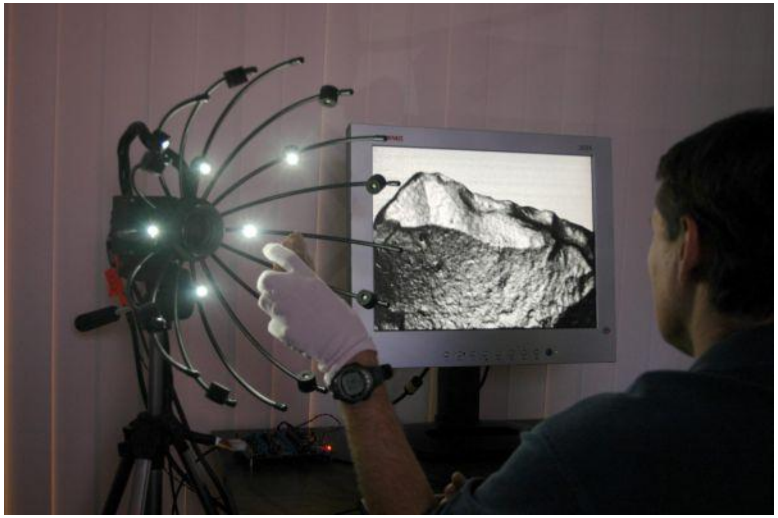
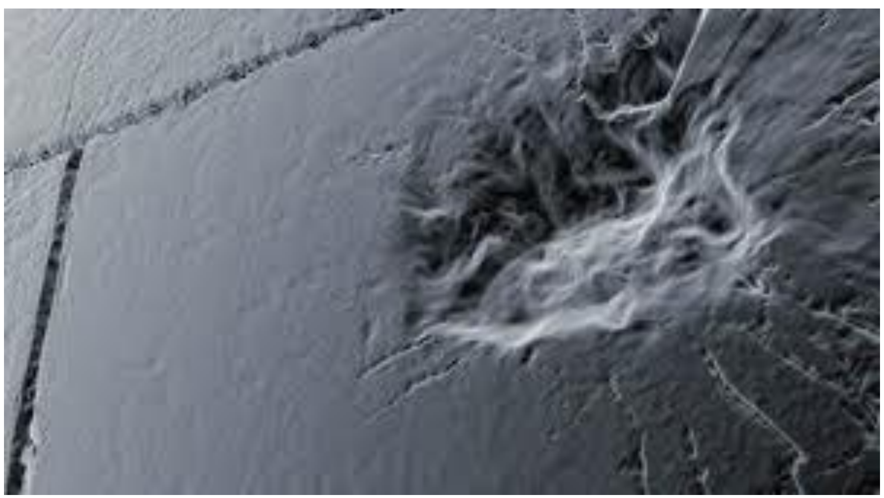
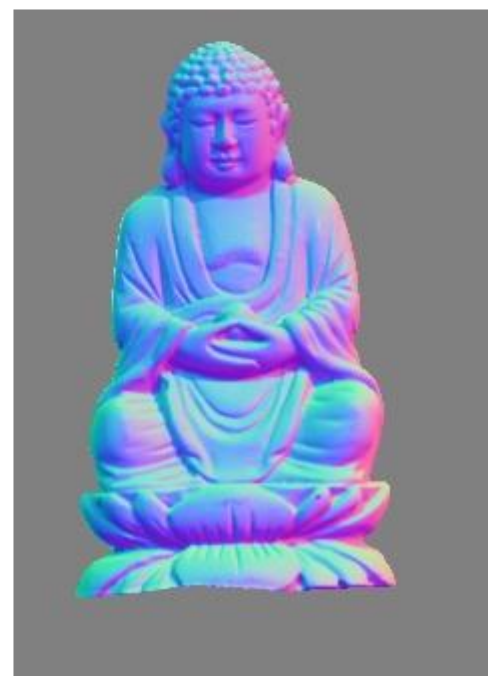
Photometric Stereo

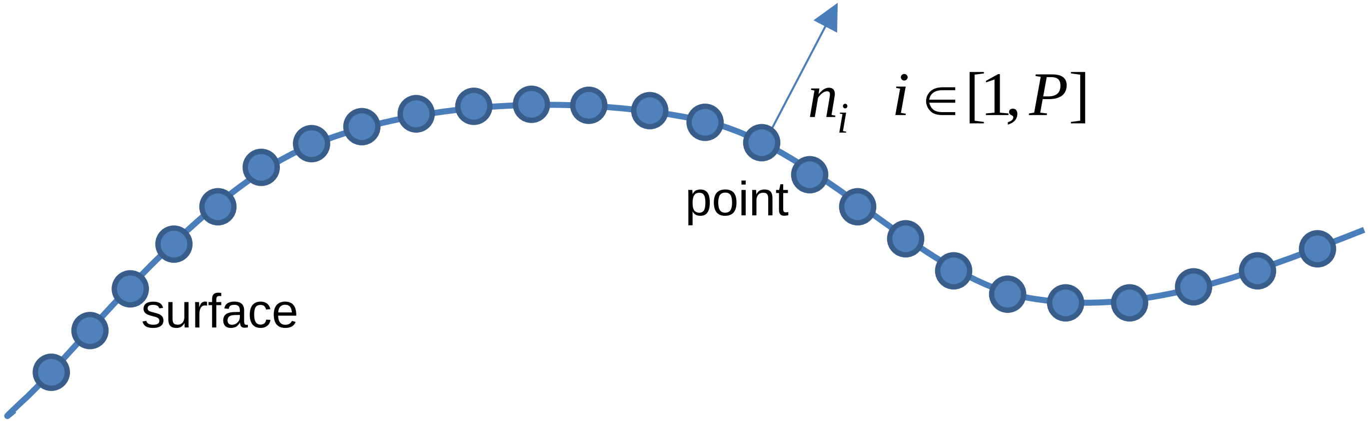
- A technique for estimating the surface normals of objects by observing that object under different lighting conditions
  - Then, using the surface normals, a plausible surface geometry can be reconstructed
  - Woodham in 1980
- Related: when using a single image, it is called *shape from shading*
  - B. K. P. Horn in 1989



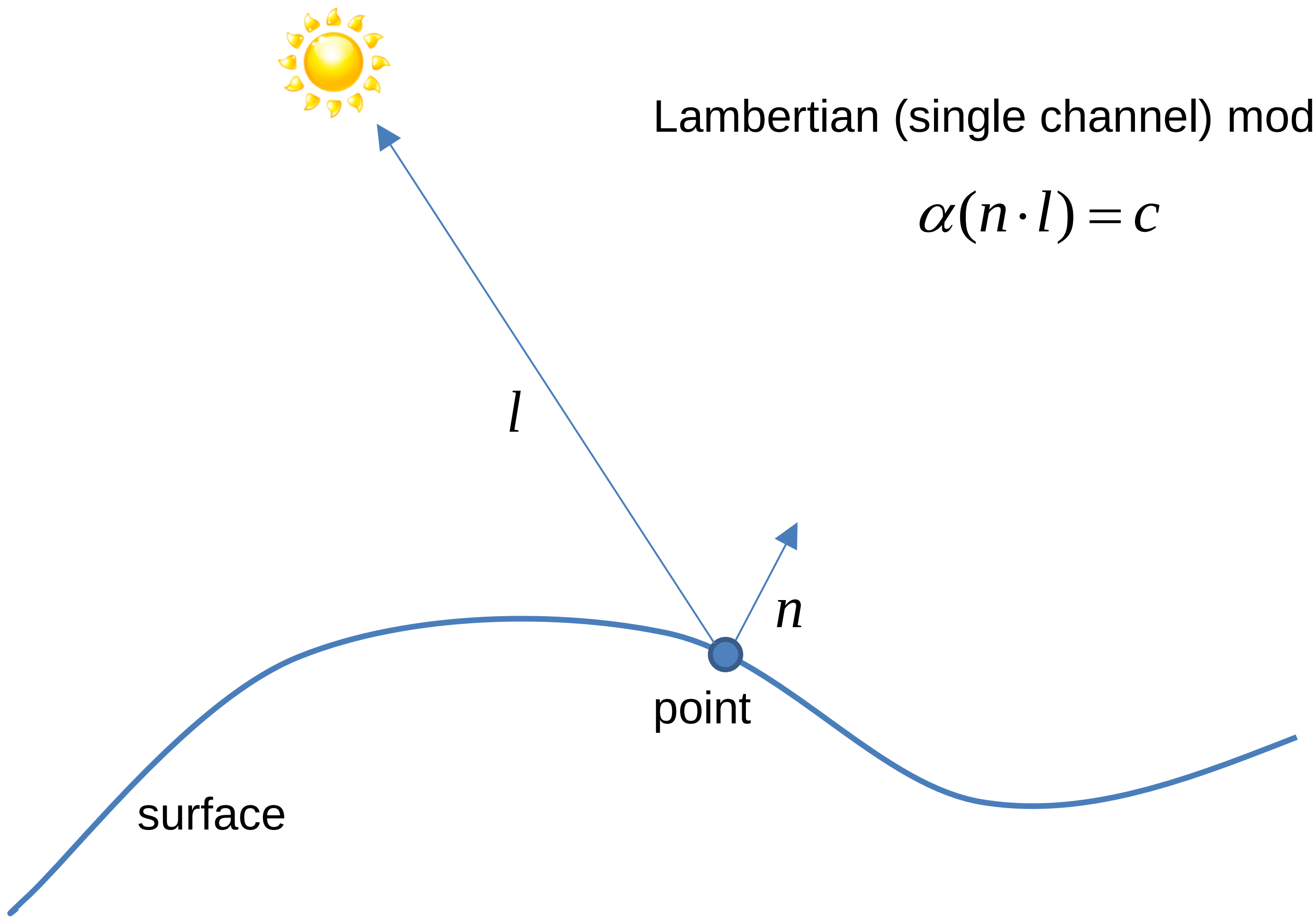



Photometric Stereo

What are the values for n_i ?



Photometric Stereo



Lambertian (single channel) model:

$$\alpha(n \cdot l) = c$$

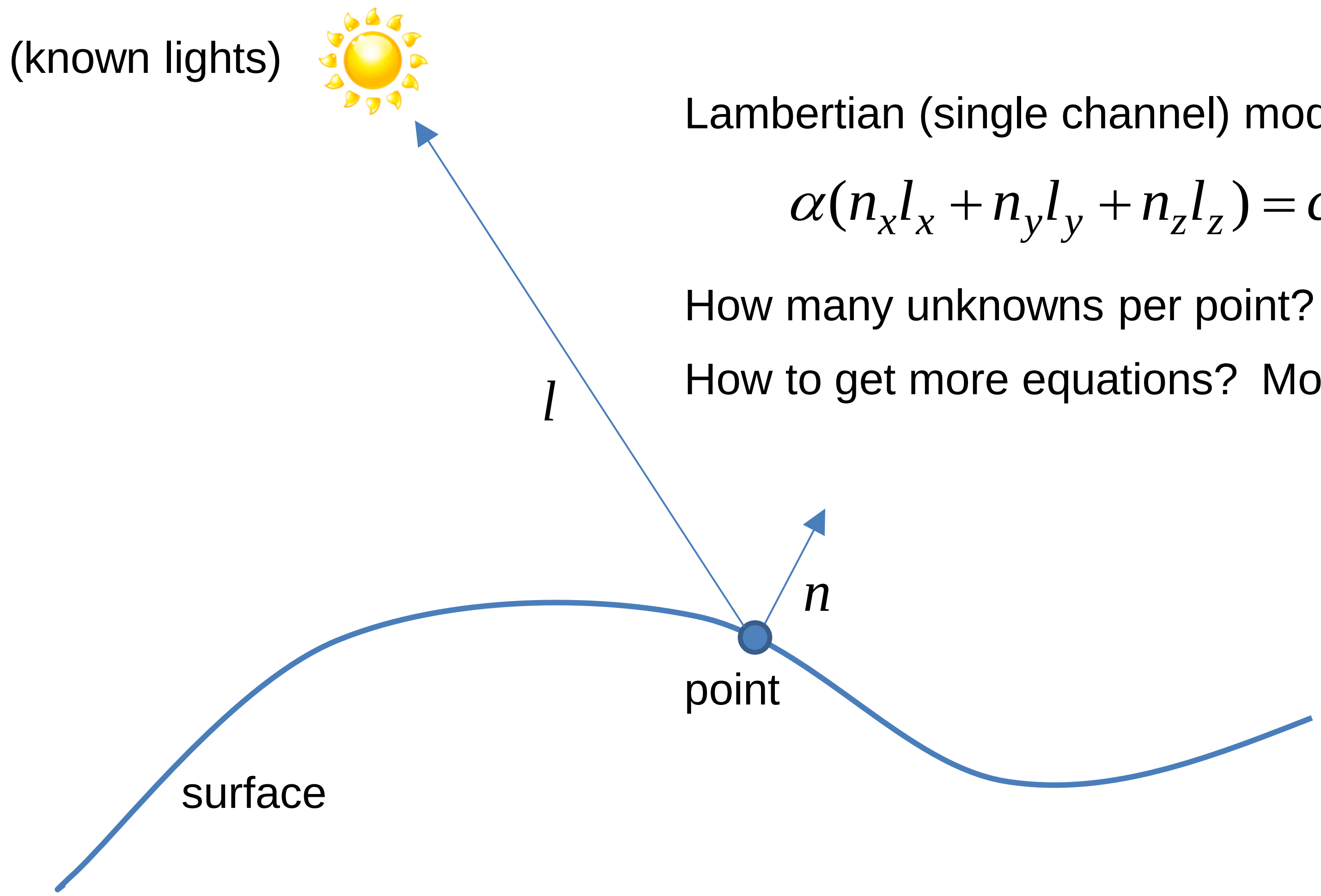
Photometric Stereo

Lambertian (single channel) model:

$$\alpha(n_x l_x + n_y l_y + n_z l_z) = c$$

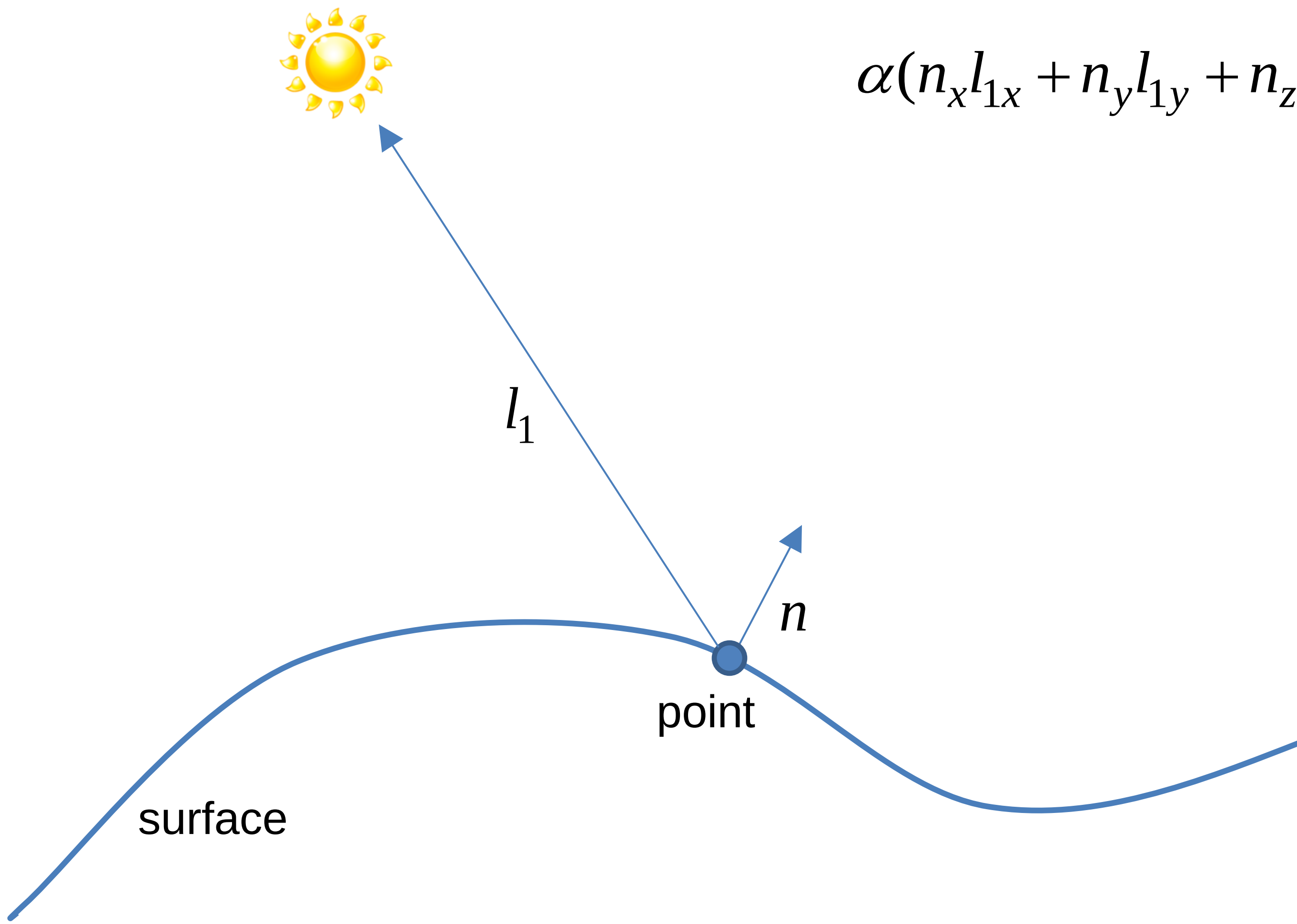
How many unknowns per point? 3+1

How to get more equations? More lights!



Photometric Stereo

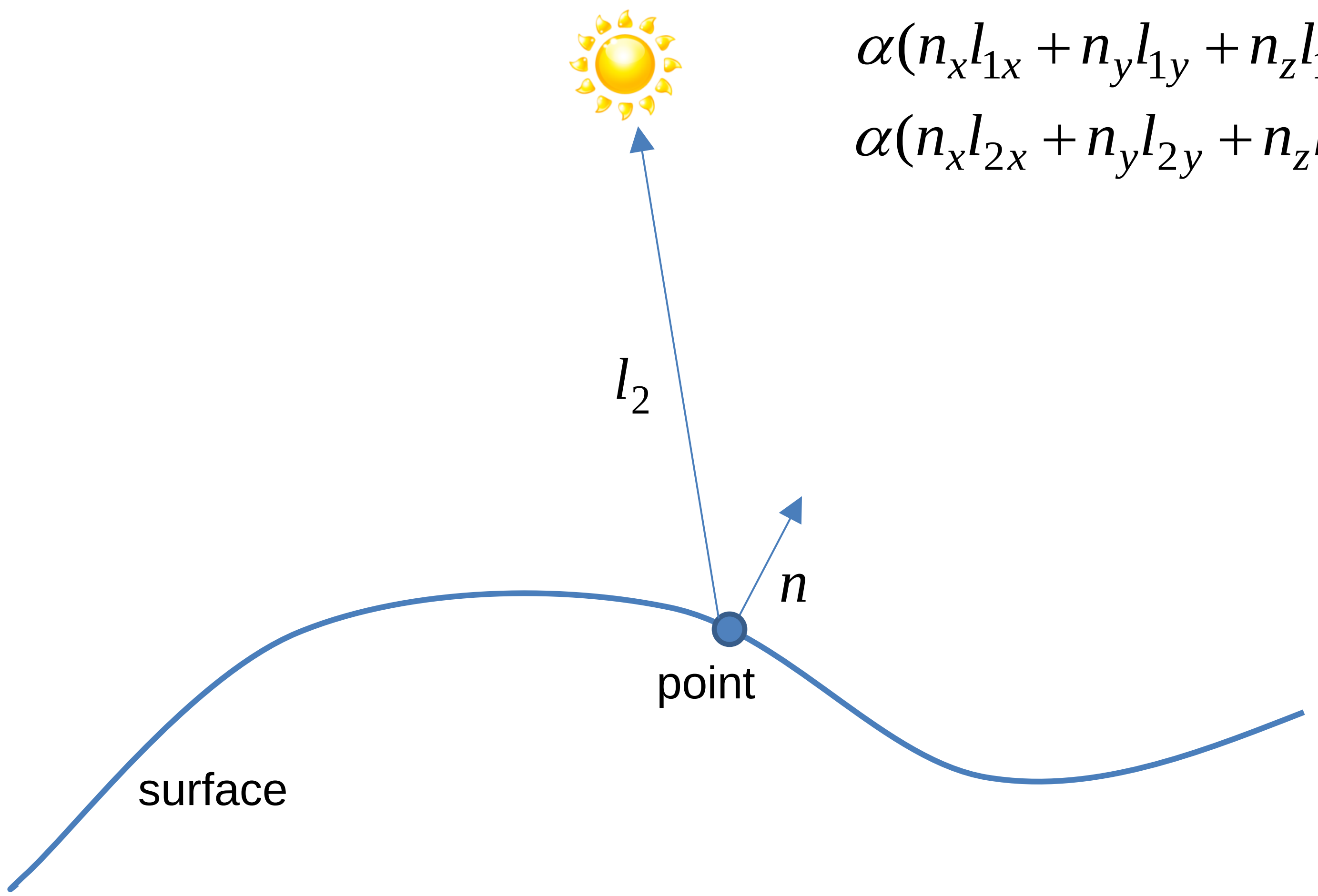
$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$



Photometric Stereo

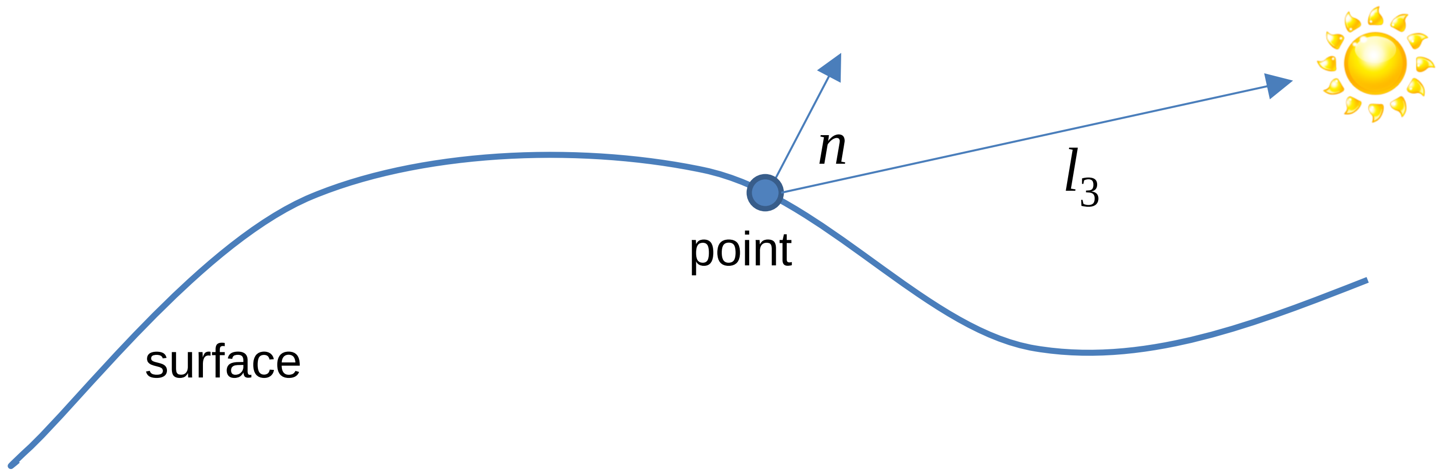
$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$

$$\alpha(n_x l_{2x} + n_y l_{2y} + n_z l_{2z}) = c_2$$



Photometric Stereo

$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$
$$\alpha(n_x l_{2x} + n_y l_{2y} + n_z l_{2z}) = c_2$$
$$\alpha(n_x l_{3x} + n_y l_{3y} + n_z l_{3z}) = c_3$$



Photometric Stereo

$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$
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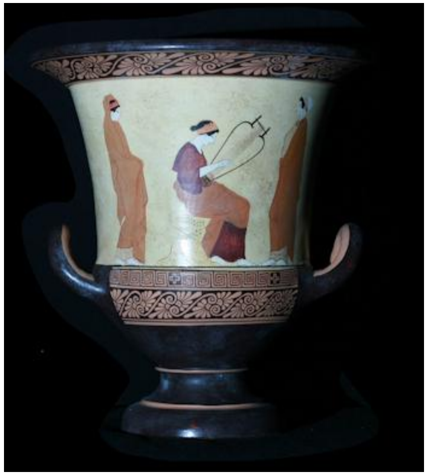
Using l_1



Using l_2

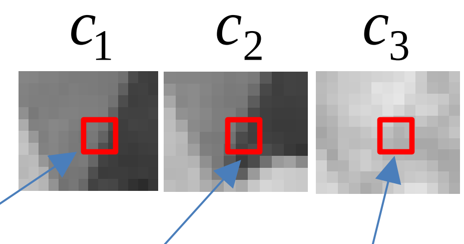
Using l_3

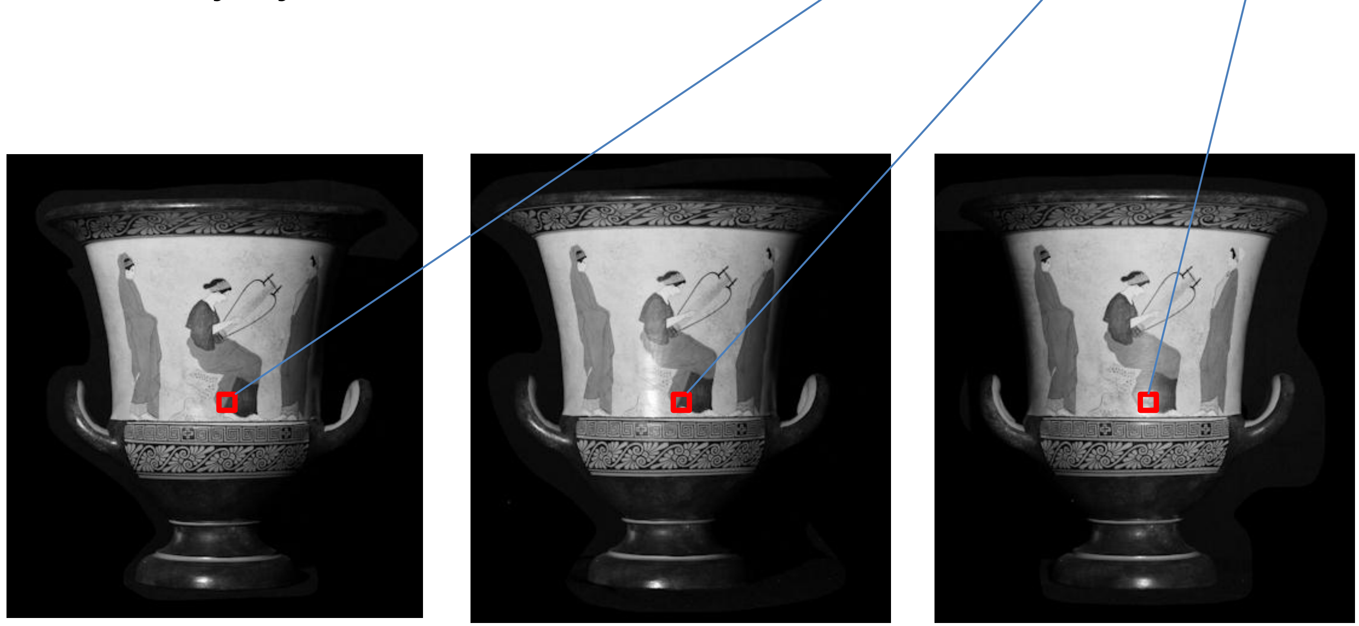
Photometric Stereo

$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$

$$\alpha(n_x l_{2x} + n_y l_{2y} + n_z l_{2z}) = c_2$$

$$\alpha(n_x l_{3x} + n_y l_{3y} + n_z l_{3z}) = c_3$$





Using l_1 Using l_2 Using l_3

Photometric Stereo

$$\alpha(n_x l_{1x} + n_y l_{1y} + n_z l_{1z}) = c_1$$
$$\alpha(n_x l_{2x} + n_y l_{2y} + n_z l_{2z}) = c_2$$
$$\alpha(n_x l_{3x} + n_y l_{3y} + n_z l_{3z}) = c_3$$

What is n_x, n_y, n_z ?

Write as $Ln = c$ and solve $n = L^{-1}c$

Where $n = \begin{bmatrix} \alpha n_x & \alpha n_y & \alpha n_z \end{bmatrix}^T$

$c = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}^T$

What is the surface normal at the point? $n / \|n\|$

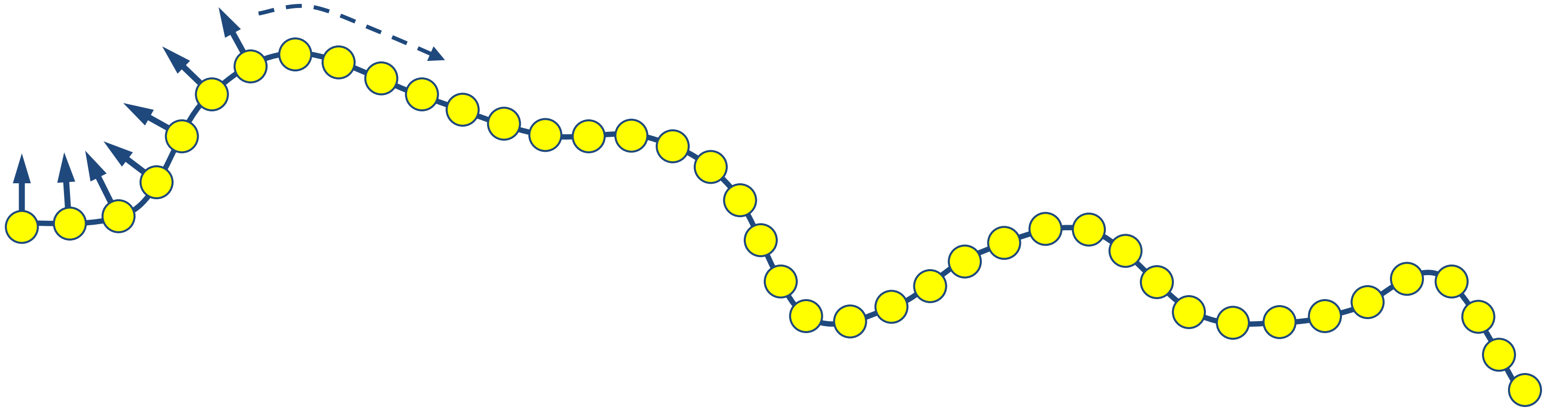
What is α (the albedo at the surface point)? $\alpha = \sqrt{n \cdot n}$

Lambertian Photometric Stereo with Known Lights

- Take three pictures of a static Lambertian object with a static camera
- In each picture move the light to a different but known position
  - For distant lights, could know light direction instead of light position
- At pixel i, solve $n_i = L^{-1} c_i$
- Use normals to "integrate" a surface

Normals -> Surface

- How?



Normals -> Surface

Surface (height field) is $z(x, y)$

Surface normal is $n(x, y) = \begin{bmatrix} z_x & z_y & -1 \end{bmatrix}^T$

and $z_x = (z(x+1, y) - z(x, y))$

$z_y = (z(x, y+1) - z(x, y))$

As we saw before $\begin{bmatrix} n_x & n_y & n_z \end{bmatrix} = \dfrac{\alpha}{\sqrt{{z_x}^2 + {z_y}^2 + 1}} \begin{bmatrix} z_x & z_y & -1 \end{bmatrix}$

So $\dfrac{n_x}{n_z} = \dfrac{z_x}{-1}$ and $\dfrac{n_y}{n_z} = \dfrac{z_y}{-1}$ or $z_x = -n_x / n_z$ and $z_y = -n_y / n_z$

Altogether:

$$n_z z(x+1, y) - n_z z(x, y) = n_x$$
$$n_z z(x, y+1) - n_z z(x, y) = n_y$$

Normals -> Surface

$$n_z z(x+1, y) - n_z z(x, y) = n_x$$
$$n_z z(x, y+1) - n_z z(x, y) = n_y$$

Can setup as a large over-constrained linear system: $Az = b$

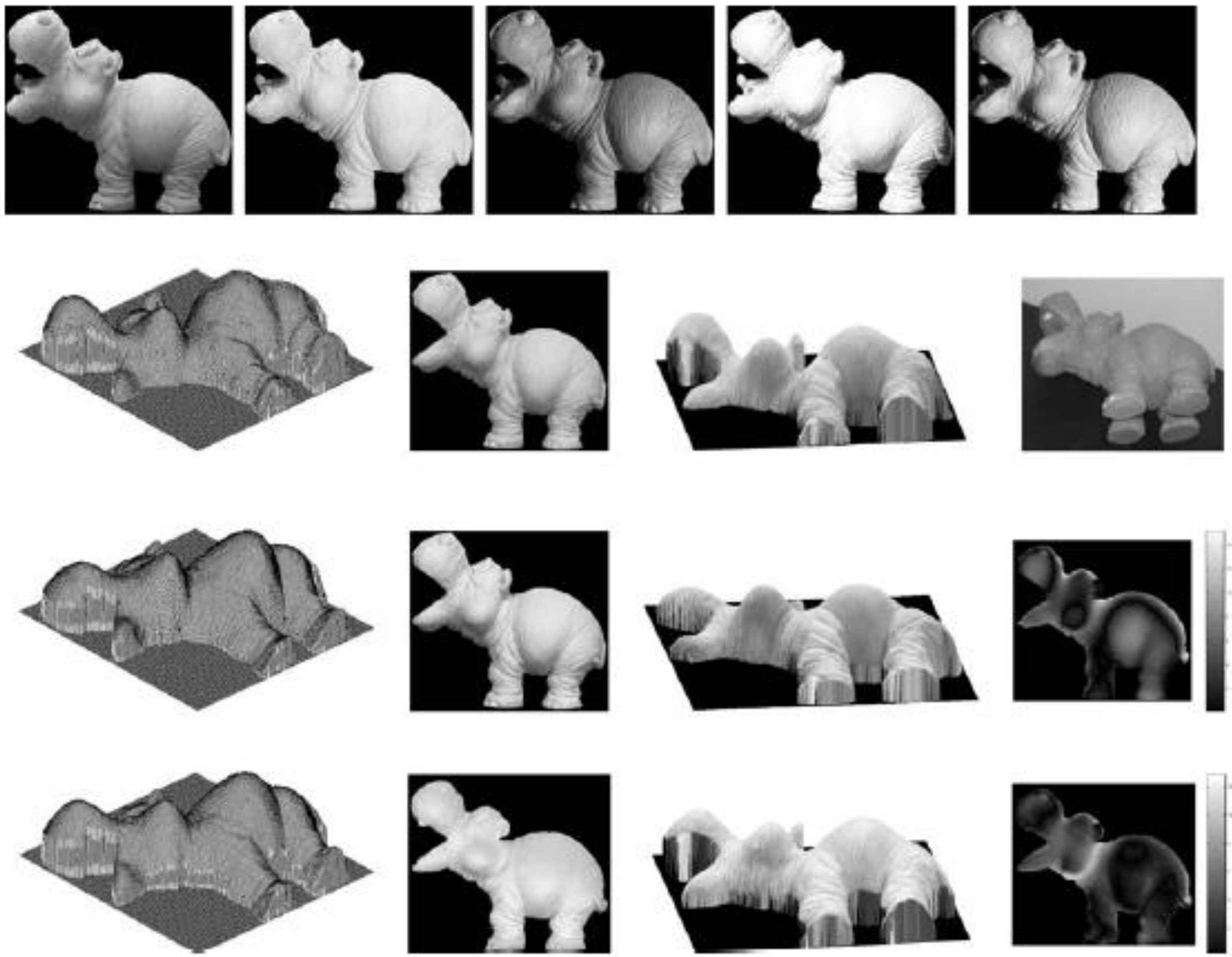
where A has a bunch of normal values

z are the unknown heights (z-values)

b is a zero vector

and solve $z = A^{-1}b$ (e.g., use “lsqr” method)

Normals -> Surface



[Basri et al., IJCV, 2007]

Fundamental Ambiguity

$$NL = C$$

$$NRR^{-1}L = C$$

$$NAA^{-1}L = C$$

$$(NA)(A^{-1}L) = C$$

Rotation matrix

$$A = RG$$

Generalized Bas Relief (GBR) Ambiguity matrix

Fundamental Ambiguity

Belhumeur et al. 1999

Fundamental Ambiguity

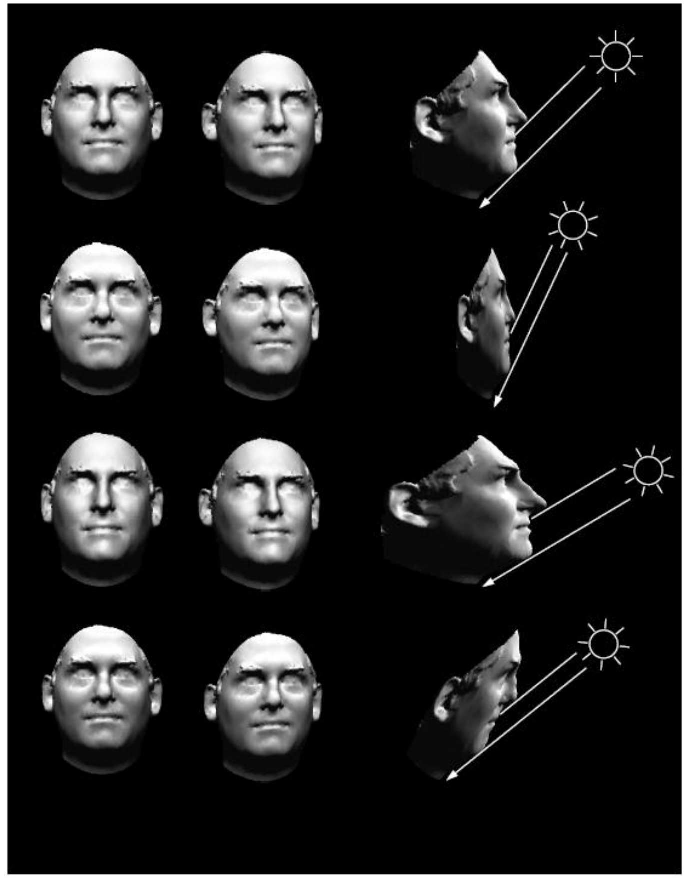


Fig. 2. Three-dimensional data for the human head (top row) was obtained using a laser scan (Cyberware) and rendered as a Lambertian surface with constant albedo (equal grey values for all surface points). The subsequent three rows show images of heads whose shapes have been transformed by different generalized bas-relief transformations, but whose albedos have not been transformed. The profile views of the face in the third column reveal the nature the individual transformations and the direction of the light source. The top row image is the true shape; the second from top is a flattened shape ($\lambda = 0.5$) (as are classical bas-reliefs); the third is an elongated shape ($\lambda = 1.5$); and the bottom is a flattened shape plus an additive plane ($\lambda = 0.7$, $\nu = 0.5$, and $\mu = 0.0$). The first column shows frontal views of the faces in the third column. From this view the true 3-d structure of the objects cannot be determined; in each image the shadowing patterns are identical, and even though the albedo has not been transformed according to Eq. 3, the shading patterns are so close as to provide few cues as to the true structure. The second column shows near frontal views of the faces from the same row, after having been separately rotated to compensate for the degree of the flattening or elongation. The rotation about the vertical axis is 7° for the first row of the second column; 14° for the second row; 4.6° for the third; and 14° for the fourth row. To mask the shearing produced by the additive plane, the fourth row has also been rotated by 5° about the line of sight.

Belhumeur et al. 1999

GBR Transform

- Consider $\bar{z}(x,y) = \lambda z(x,y) + \mu x + \nu y$
  - This means "flatten" by λ and add a plane (μ, ν)
- When $\mu = \nu = 0$ this is classical "bas-relief"
- Else, it is a generalized bas-relief that can be written as

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix} \qquad G^{-1} = \frac{1}{\lambda} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ -\mu & -\nu & 1 \end{bmatrix}$$

Photometric Stereo Ambiguity

$$(NA)(A^{-1}L) = C$$

$$A = RG = R\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix}$$

- Interpretation: given a solution for the normals, we can transform the normals by A and the lights by A^{-1} and the resulting picture looks the same…

Photometric Stereo with Unknown Lights

- Question:
  - What if both the surface normals and the light directions are unknown?
  - Can we reconstruct the light directions, surface normals, and thus surface geometry (up to the aforementioned ambiguity)?
- Answer:
  - Yes, and the problem is still linear!

PS with Unknown Lighting

Recall $Ln = c$

Think of the normals as located at the origin, then we care about a linear transformation of the sphere $(n^T n)^{1/2} = 1$

Note $n = L^{-1}c = Pc$

So $(Pc)^T Pc = c^T P^T Pc = c^T Qc = 1$

And Q is a 3x3 symmetric positive definite matrix: $= \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{bmatrix}$

Geometrically, this means intensity's c lie on an ellipsoid whose equation is

$$q_{11}c_1^{\;2} + q_{22}c_2^{\;2} + q_{33}c_3^{\;2} + 2q_{12}c_1c_2 + 2q_{13}c_1c_3 + 2q_{23}c_2c_3 - 1 = 0$$

which only has 6 unknowns

PS with Unknown Lighting

- Given at least six pixels, can solve for q's
- In general, a large overconstrained linear solution is used

$$M = \begin{bmatrix} c_{11}^{2} & c_{12}^{2} & c_{13}^{2} & 2c_{11}c_{12} & 2c_{11}c_{13} & 2c_{12}c_{13} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ c_{n1}^{2} & c_{n2}^{2} & c_{n3}^{2} & 2c_{n1}c_{n2} & 2c_{n1}c_{n3} & 2c_{n2}c_{n3} \end{bmatrix}$$

$$q = \begin{bmatrix} q_{11} & q_{22} & q_{33} & q_{12} & q_{13} & q_{23} \end{bmatrix}^{T}$$

$$b = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}$$

$$q = M^{-1}b \quad \text{or} \quad q = (M^{T}M)^{-1}M^{T}b$$

PS with Unknown Lighting

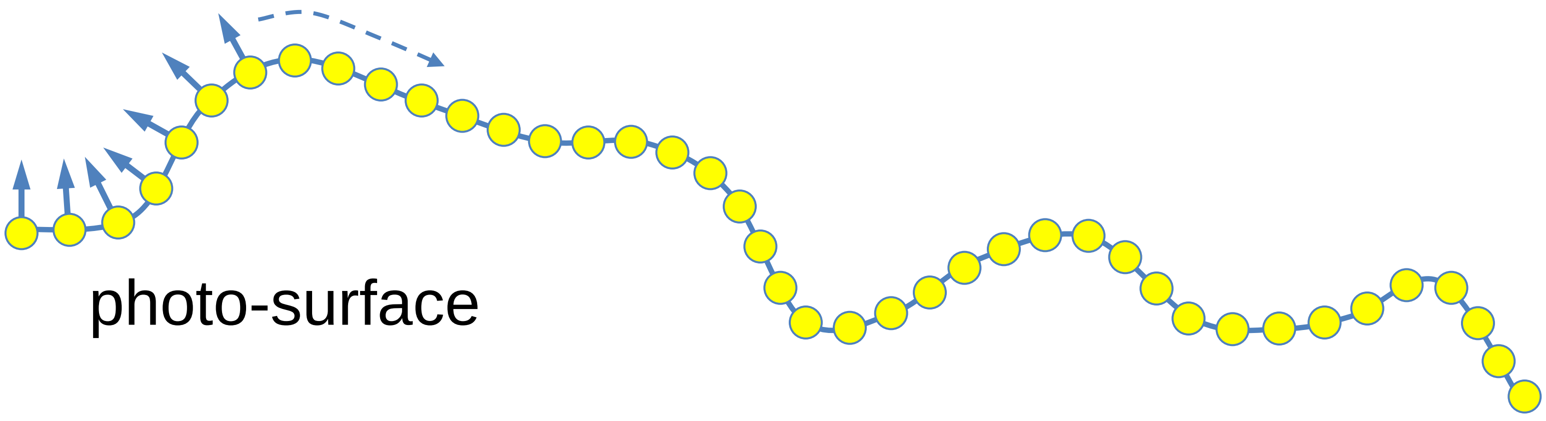
- Given Q, we can plug back and get the angles between light directions and their strengths
- Then "lights are known" and solve for normals as before…
  - but need to choose a reasonable R and G

Photogeometric Approach

- Combine photometric stereo with geometric stereo
  - High resolution of photometric stereo
  - Accuracy of geometric method
  - Can lead to self-calibration of entire acquisition process

Photogeometric Upsampling

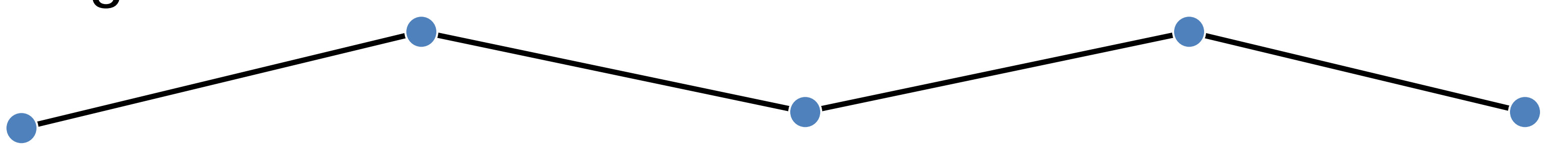
1. Integrate surface normals



Photogeometric Upsampling

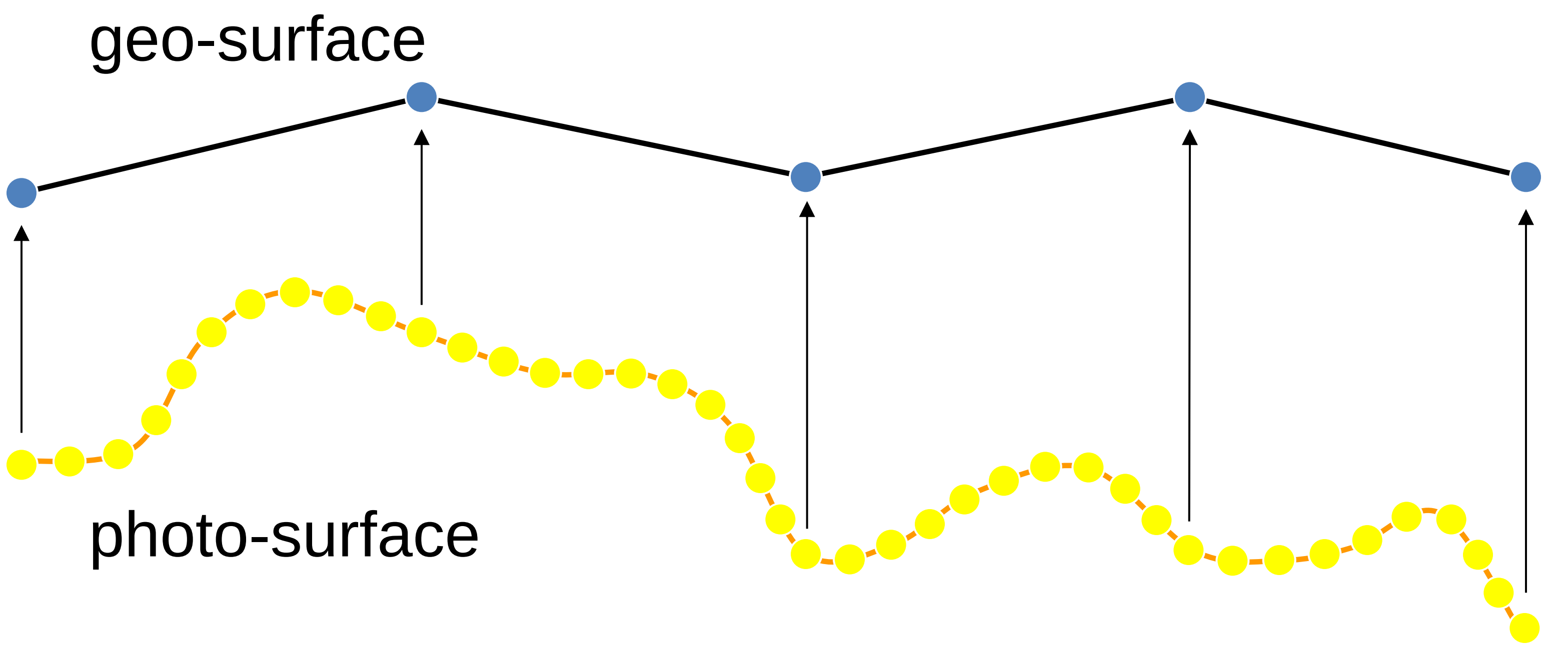
2. Compute sparse geometric model

geo-surface



Photogeometric Upsampling

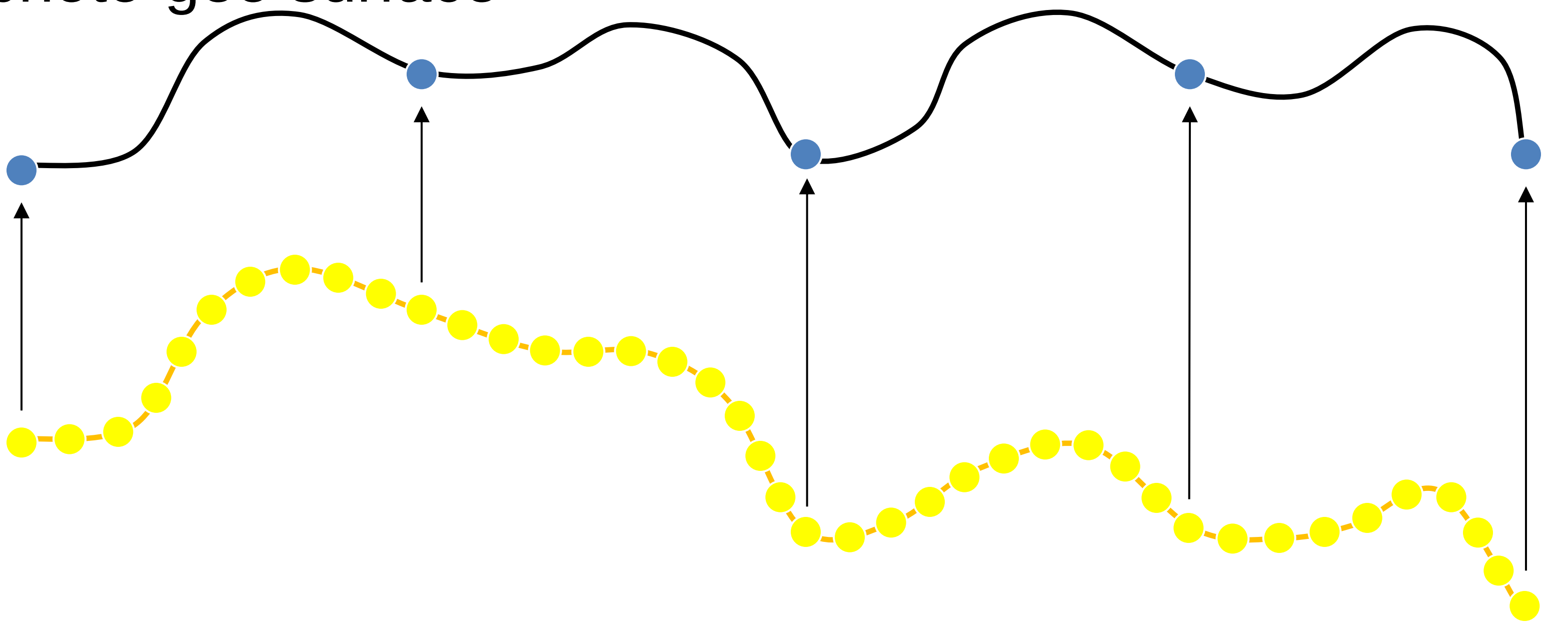
3. Warp photometric surface to geometric surface



Photogeometric Upsampling

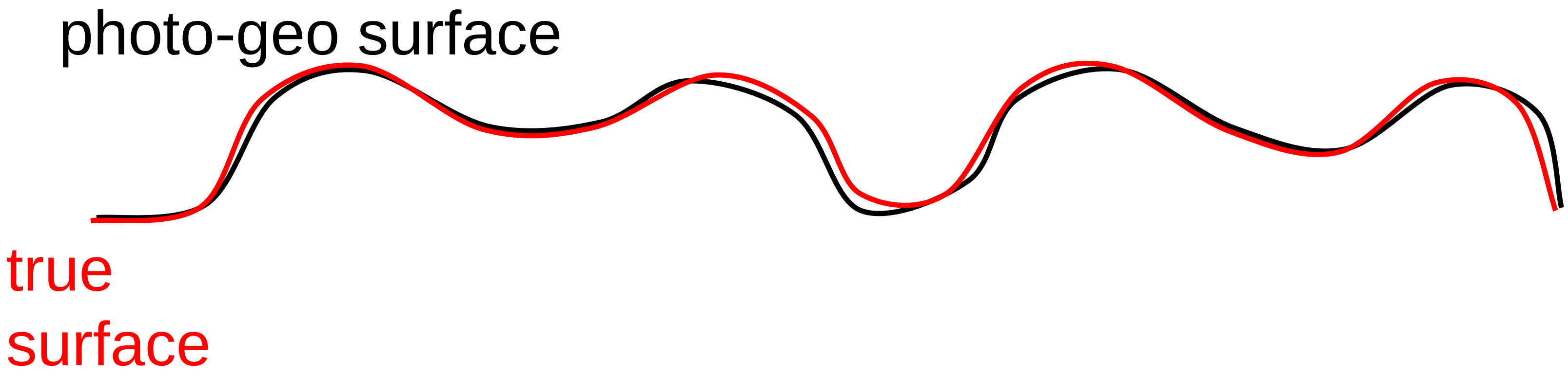
3. Warp photometric surface to geometric surface

photo-geo surface



Photogeometric Upsampling

4. Triangulate and proceed to optimization



Photogeometric Optimization

- Linear system in the unknown 3D points (p_i)
- Supports multi-view reconstruction
- Weighted combination of three error terms:

$$e = (1-\lambda)(1-\tau)\kappa_g e_g + \lambda\kappa_p e_p + \tau\kappa_r e_r \rightarrow 0$$

where

e_g = error of reprojection

e_p = error of perpendicularity of normal-to-tangent

e_r = error of relative distance change

Photogeometric Optimization

- Linear system in the unknown 3D points (p_i)
- Supports multi-view reconstruction
- Weighted combination of three error terms:

$$e = (1-\lambda)(1-\tau)\kappa_g e_g + \lambda \kappa_p e_p + \tau \kappa_r e_r \to 0$$

where

$$e_g = \sum_j \sum_i \begin{bmatrix} \hat{p}_{ij_x} - \left(\frac{u_{ij}\hat{p}_{ij_z}}{f}\right) \\ \hat{p}_{ij_y} - \left(\frac{v_{ij}\hat{p}_{ij_z}}{f}\right) \end{bmatrix}$$

$$e_p = \sum_i \delta_{ik}(n_i \cdot (p_i - p_k))$$

$$e_r = \sum_i \delta_{ik}((p_i - p_{ik}) - d_{ik})$$

Photogeometric Reconstruction