Graphics Pipeline: Transformation, Shading/Lighting, Projection, Texturing, and more!

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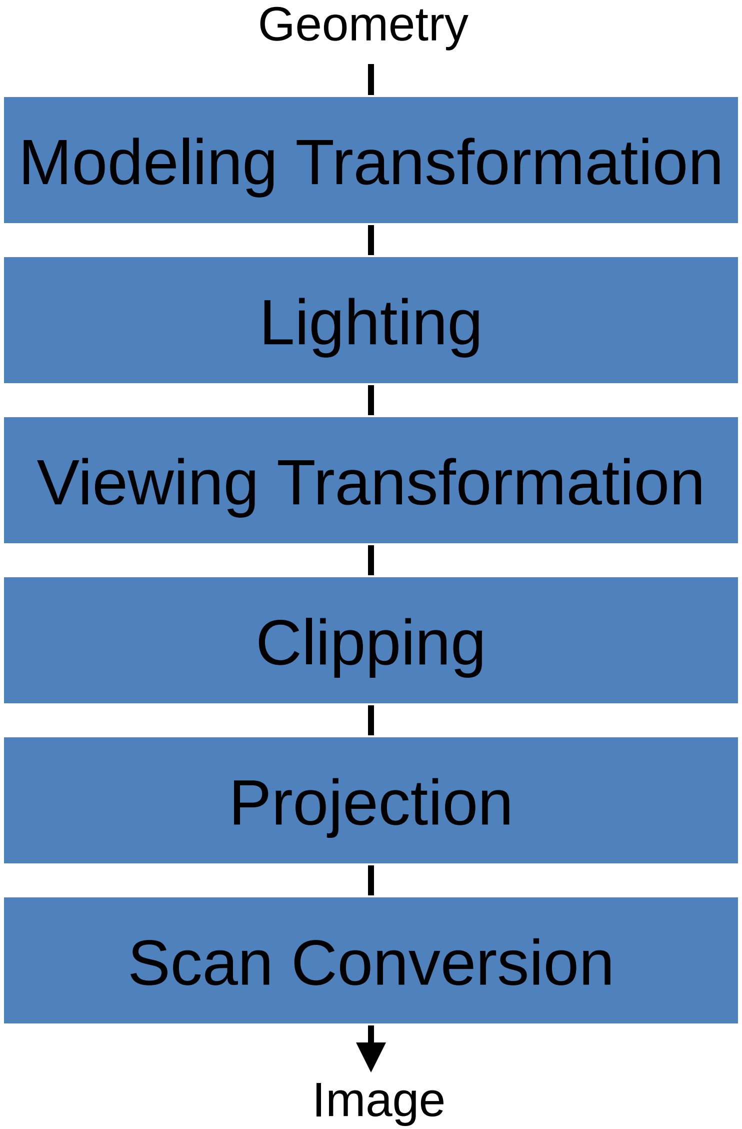
Computer Graphics Pipeline



Geometry

Modeling Transformation — Transform into 3D *world* coordinate system

Lighting — Simulate illumination and reflectance

Viewing Transformation — Transform into 3D *camera* coordinate system

Clipping — Clip primitives outside camera's view

Projection — Transform into 2D camera coordinate system

Scan Conversion — Draw pixels (incl. texturing, hidden surface…)

Image

Computer Graphics Pipeline

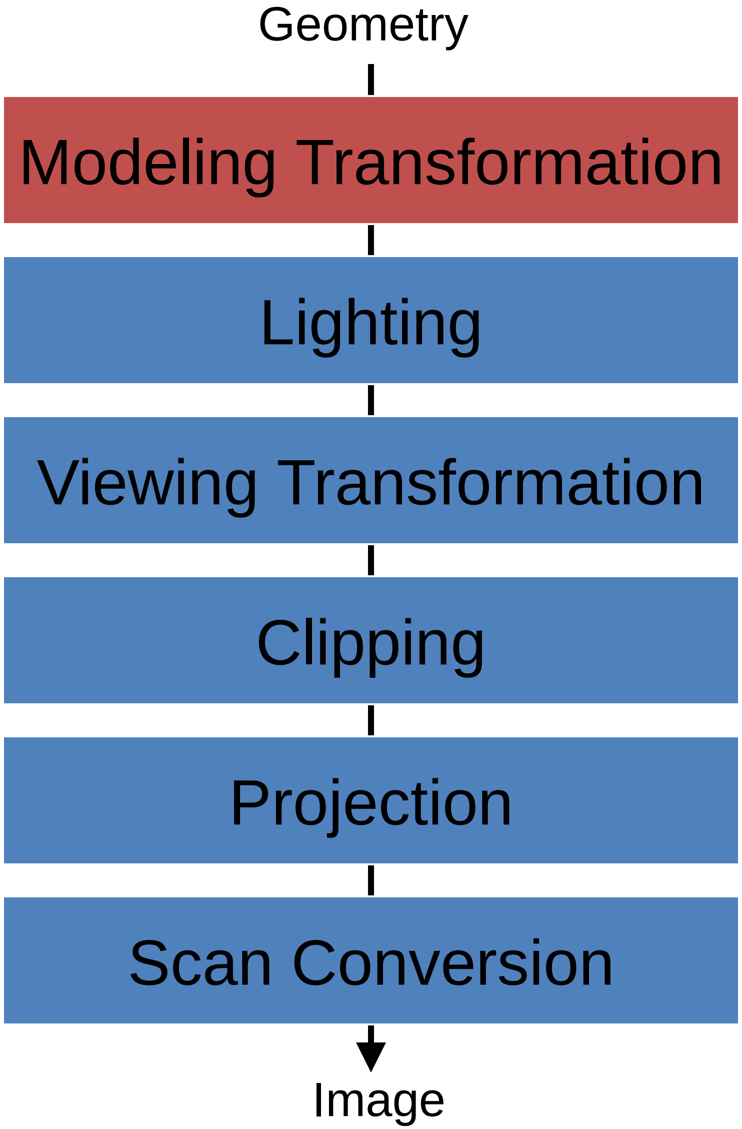


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Image

Modeling Transformations

- Most popular transformations in graphics
  - Translation
  - Rotation
  - Scale
  - Projection
- In order to use a single matrix for all, we use homogeneous coordinates…

Modeling Transformations

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror over X axis

Modeling Transformations

Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 & 0 \\ \sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

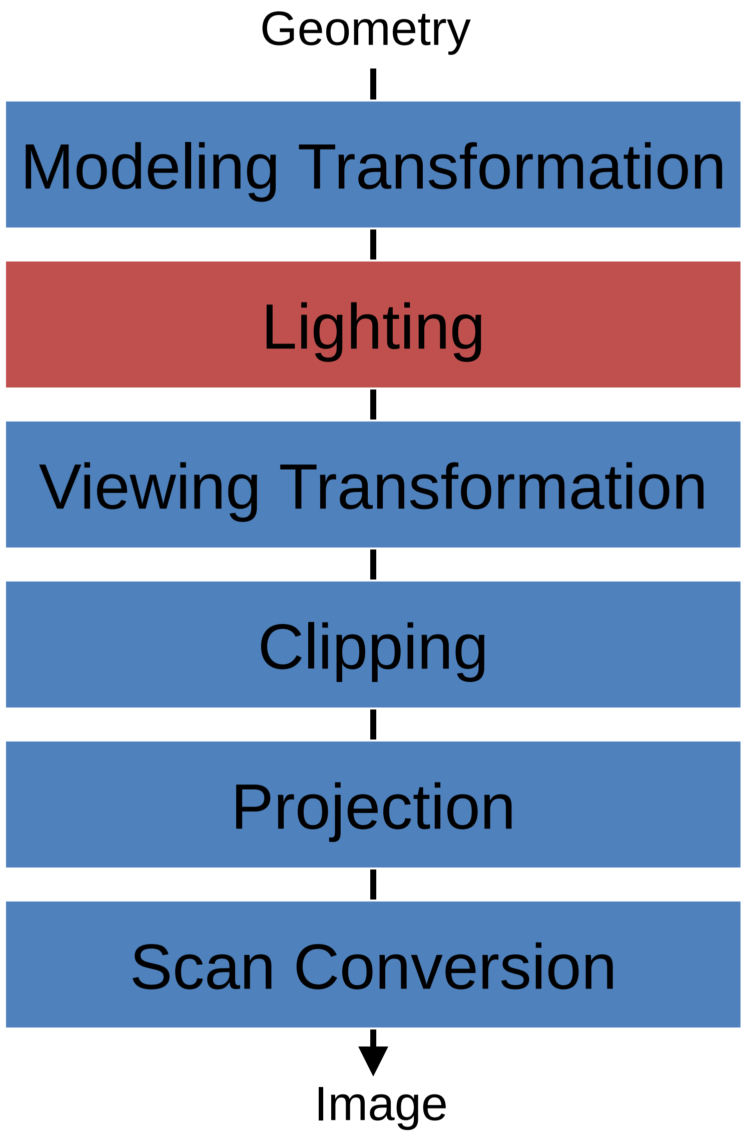
$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

And many more…

Rotate around X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & -\sin\Theta & 0 \\ 0 & \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Computer Graphics Pipeline



Geometry

Modeling Transformation

Transform into 3D *world* coordinate system

Lighting

Simulate illumination and reflectance

Viewing Transformation

Transform into 3D *camera* coordinate system

Clipping

Clip primitives outside camera’s view

Projection

Transform into 2D camera coordinate system

Scan Conversion

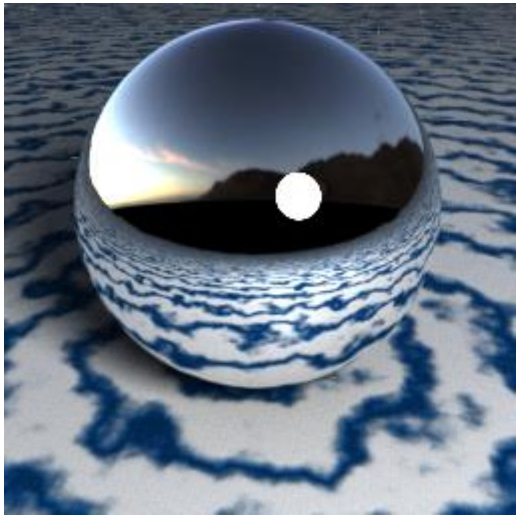
Draw pixels (incl. texturing, hidden surface…)

Image

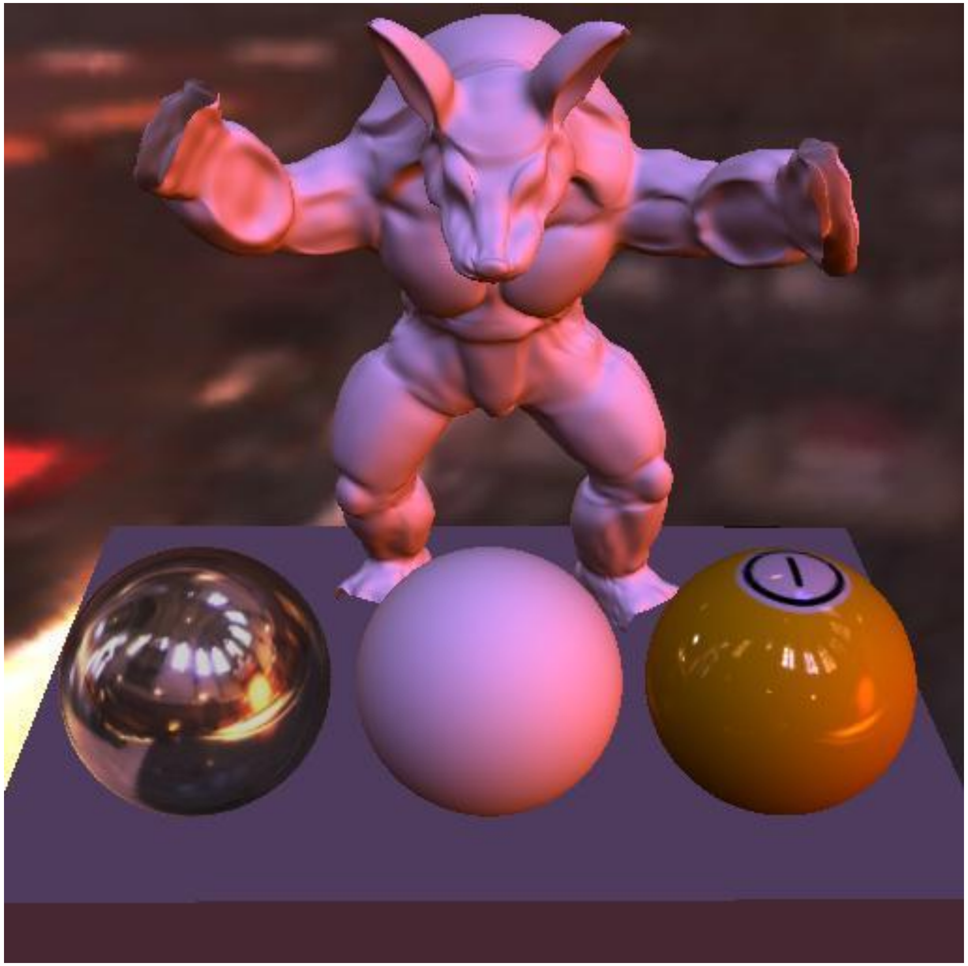
Diffuse

(mostly)

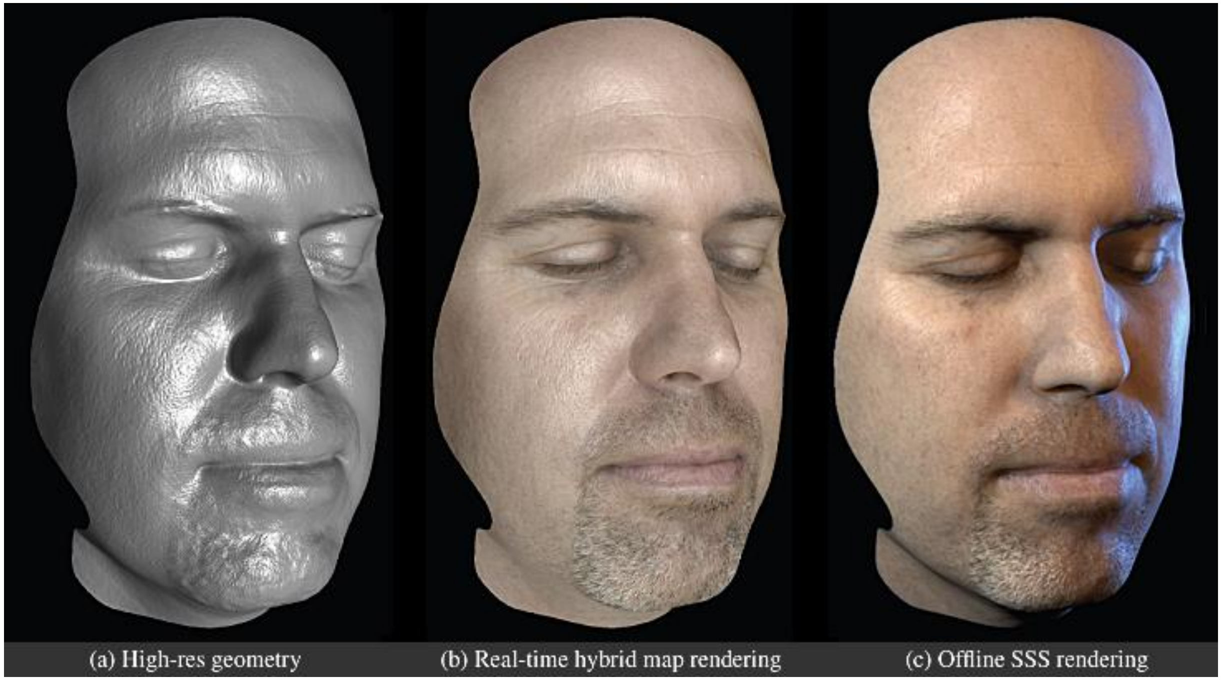
Specular++



Environment Mapping



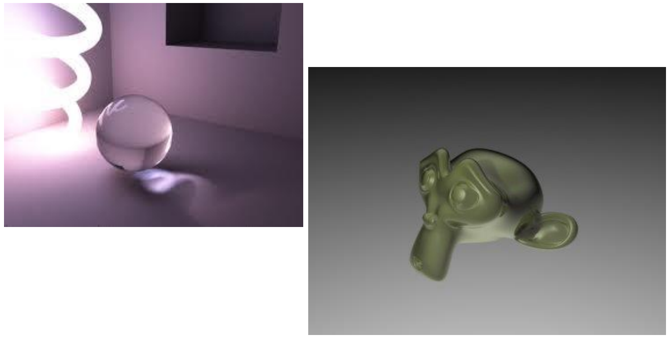
Subsurface Scatterring

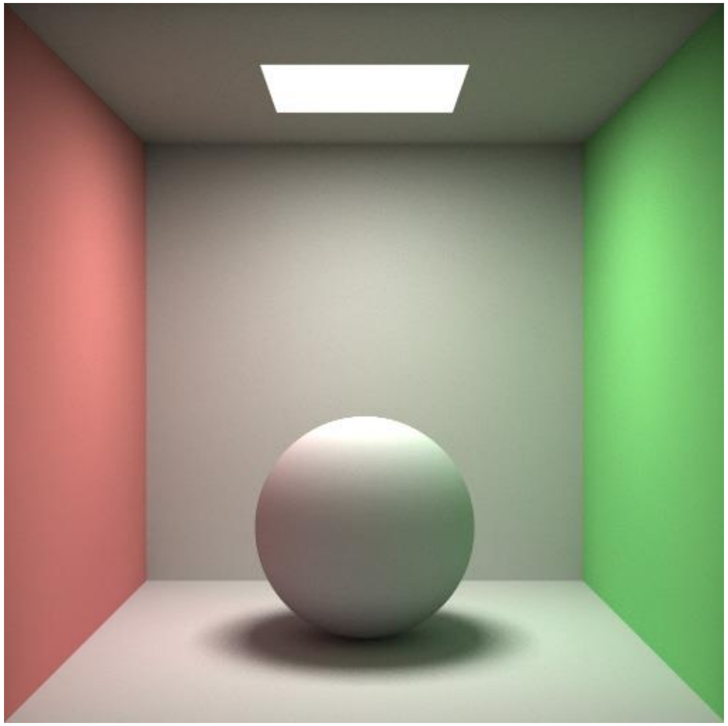


(a) High-res geometry (b) Real-time hybrid map rendering (c) Offline SSS rendering

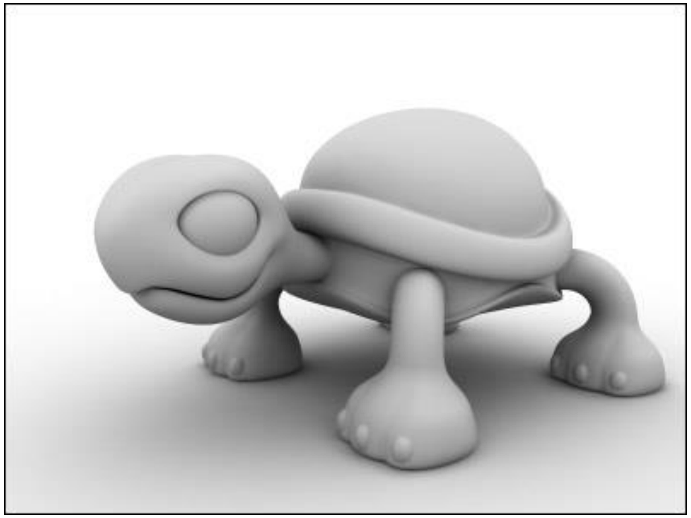
Others

Transparency



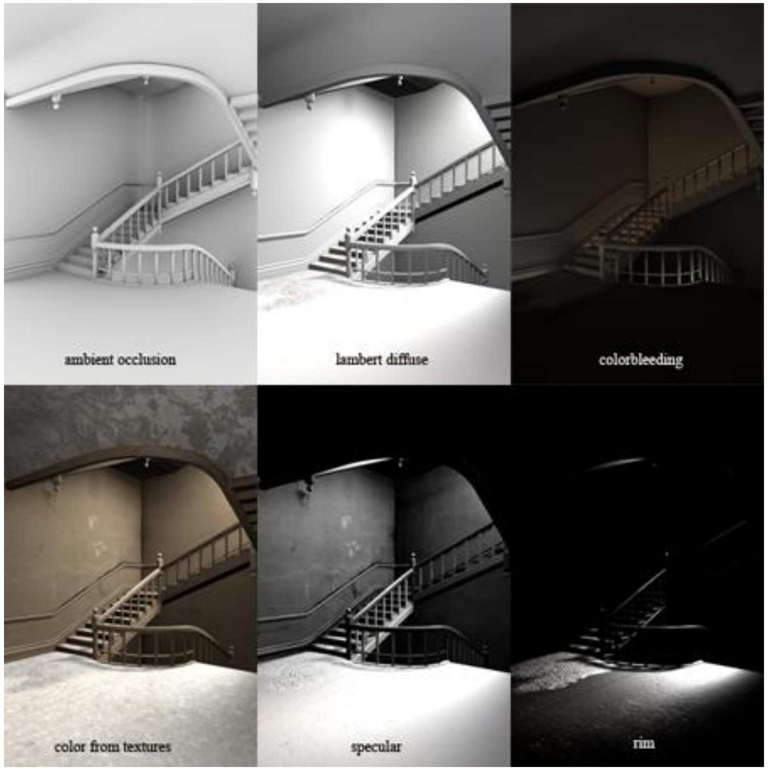


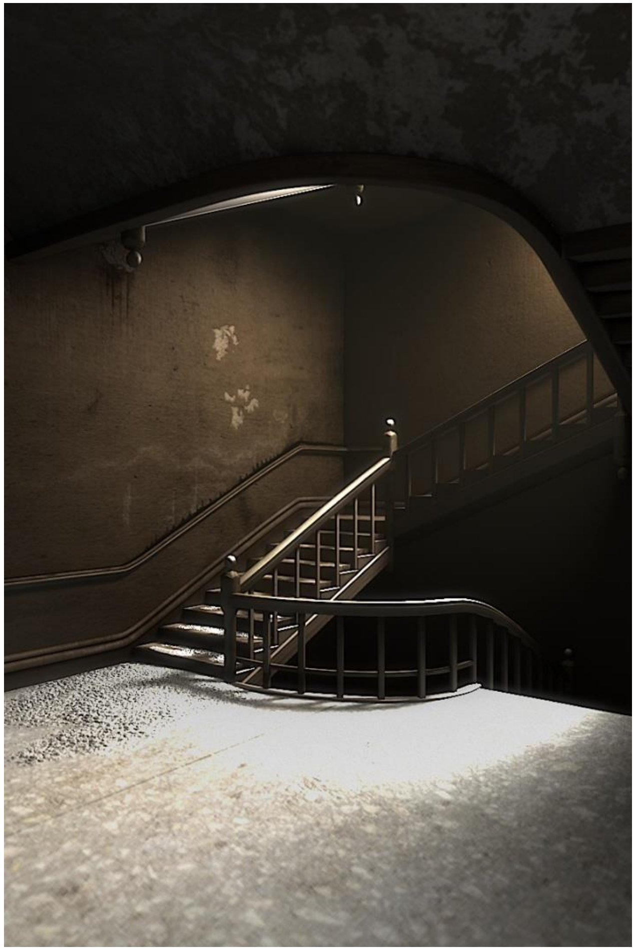
Radiosity



Ambient occlusion

Others


ambient occlusion
lambert diffuse
colorbleeding
color from textures
specular
rim



Lighting and Shading

- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction

- Light model
  - Ambient light
  - Diffuse reflection
  - Specular reflection

Lighting and Shading

- Diffuse reflection
  - – Lambertian model

Lighting and Shading

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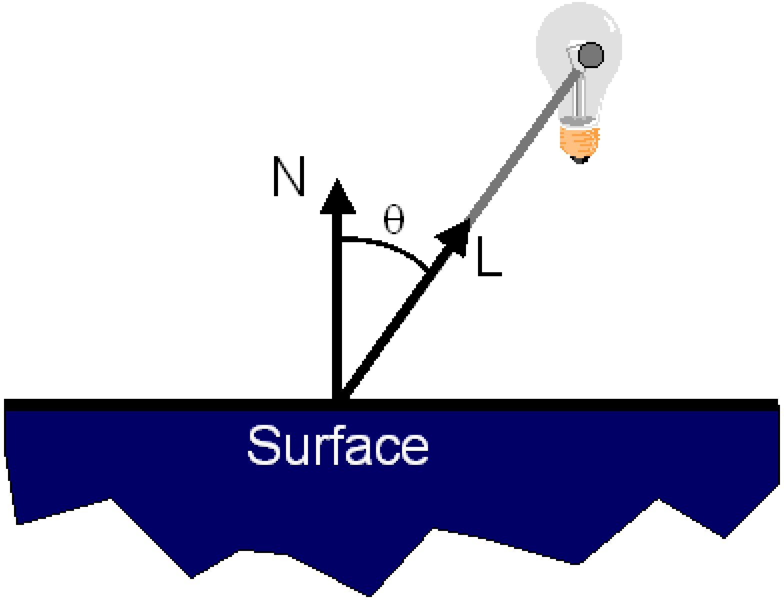

Lighting and Shading

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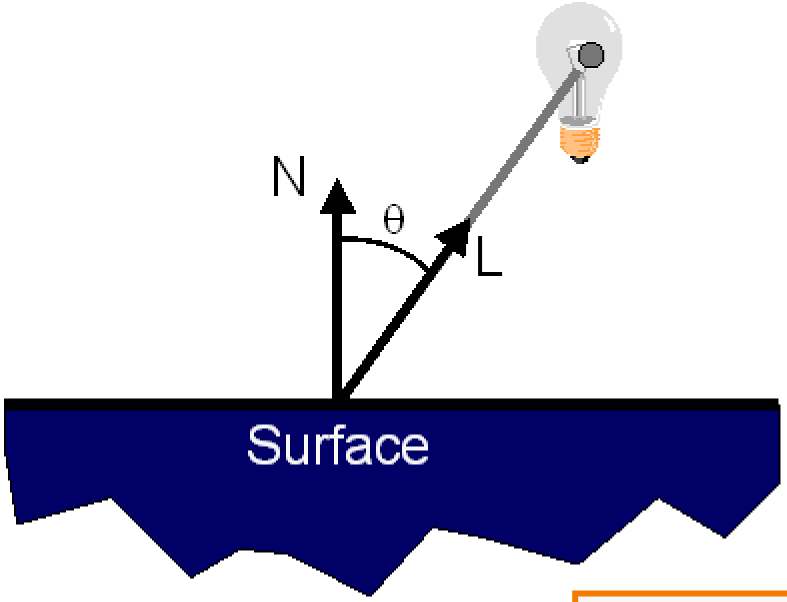
  - Lambertian model



$$\boxed{I_D = K_D (N \bullet L) I_L}$$

Lighting and Shading

- Specular reflection
  - Phong model

Lighting and Shading

- Specular reflection
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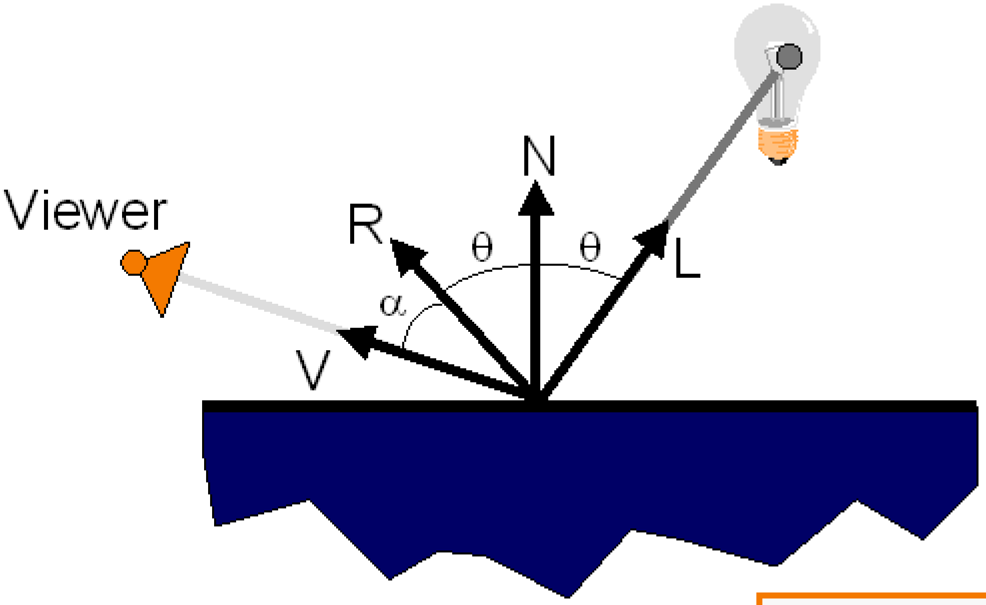
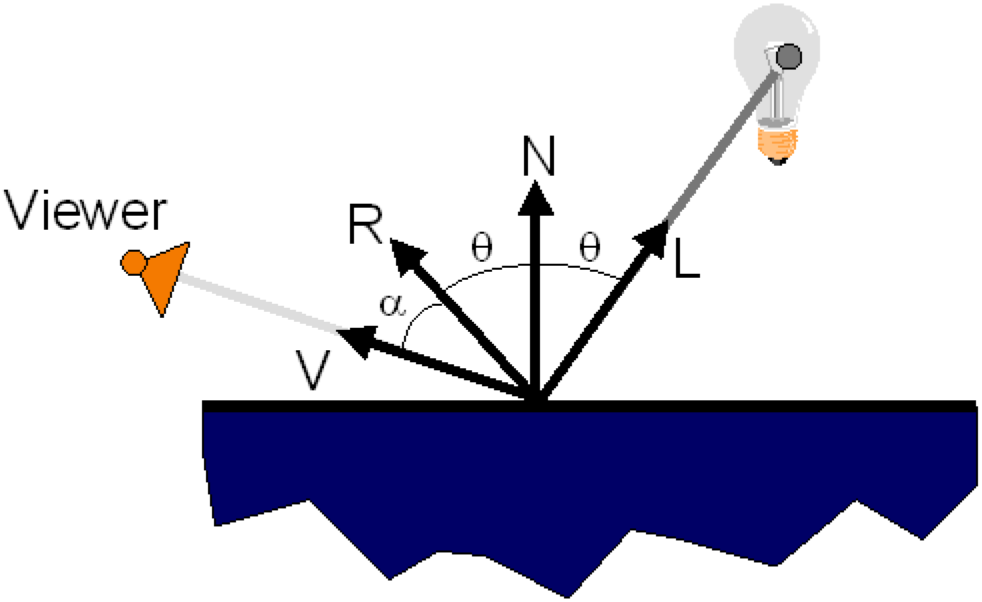


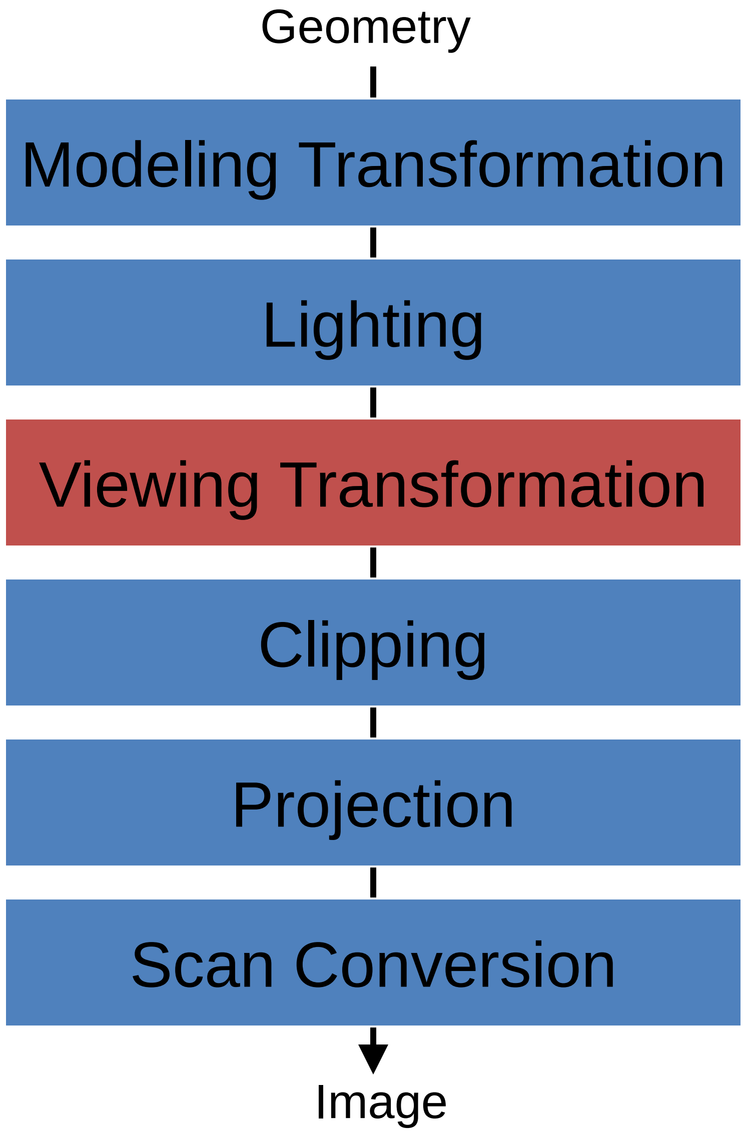
$$\boxed{I_S = K_S(V \bullet R)^n I_L}$$

Lighting and Shading

- Specular reflection
  - Phong model



Computer Graphics Pipeline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Modeling Transformation

Lighting

Viewing Transformation

Clipping

Projection

Scan Conversion

Image

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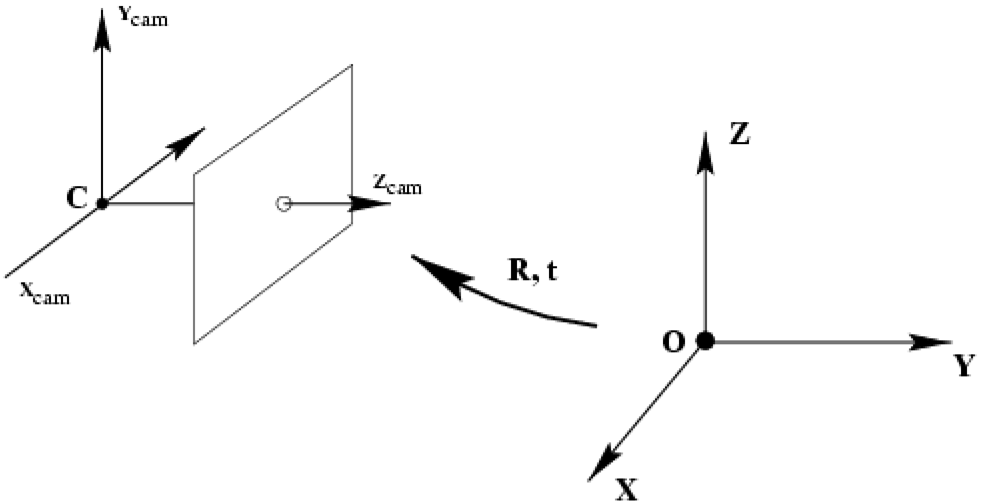
Viewing Transformation



$$\tilde{x}_c = R(\tilde{X} - C)$$

$$\tilde{x}_c = R\tilde{X} - \underset{\substack{\Downarrow \\ -t}}{RC}$$

$$\tilde{x}_c = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$R = R_x R_y R_z$$

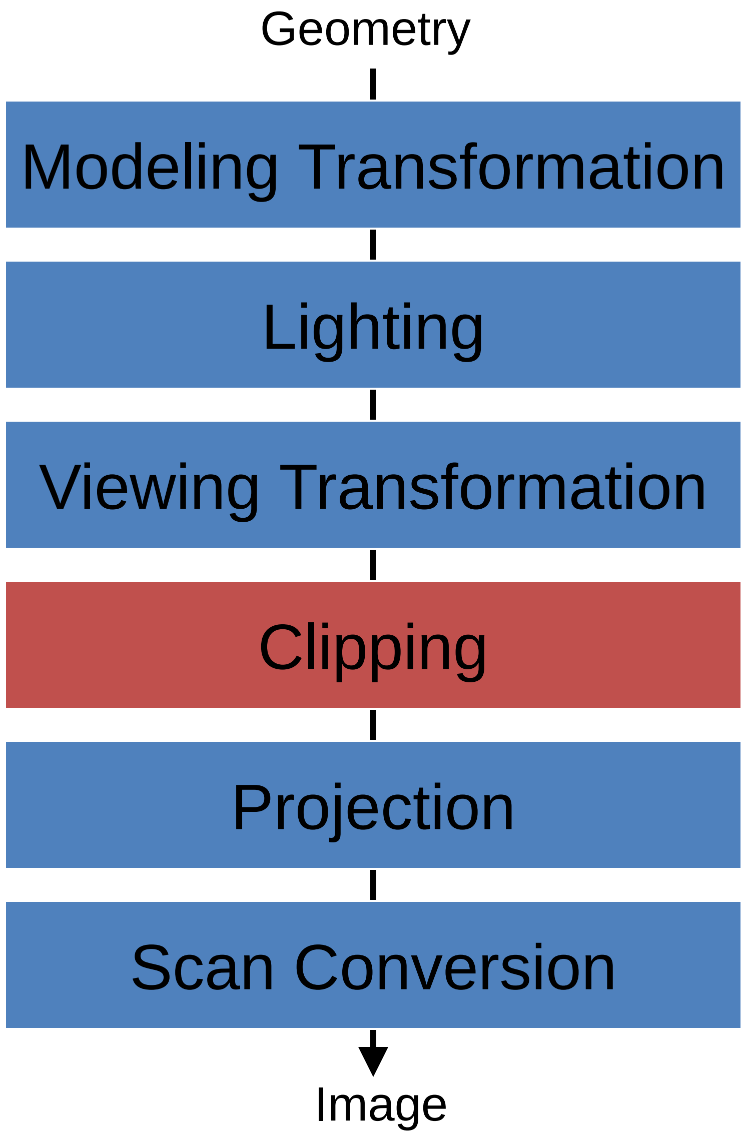
3x3 rotation matrices

$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

translation vector

World-to-camera matrix M

Computer Graphics Pipeline



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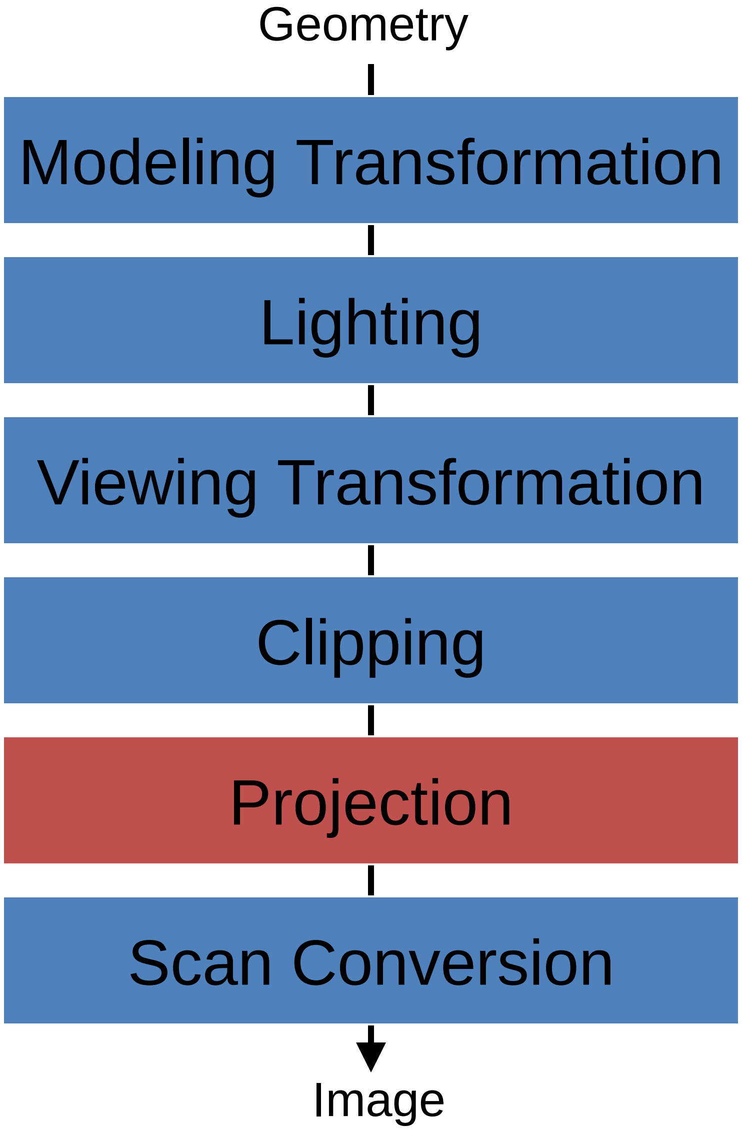
Computer Graphics Pipeline



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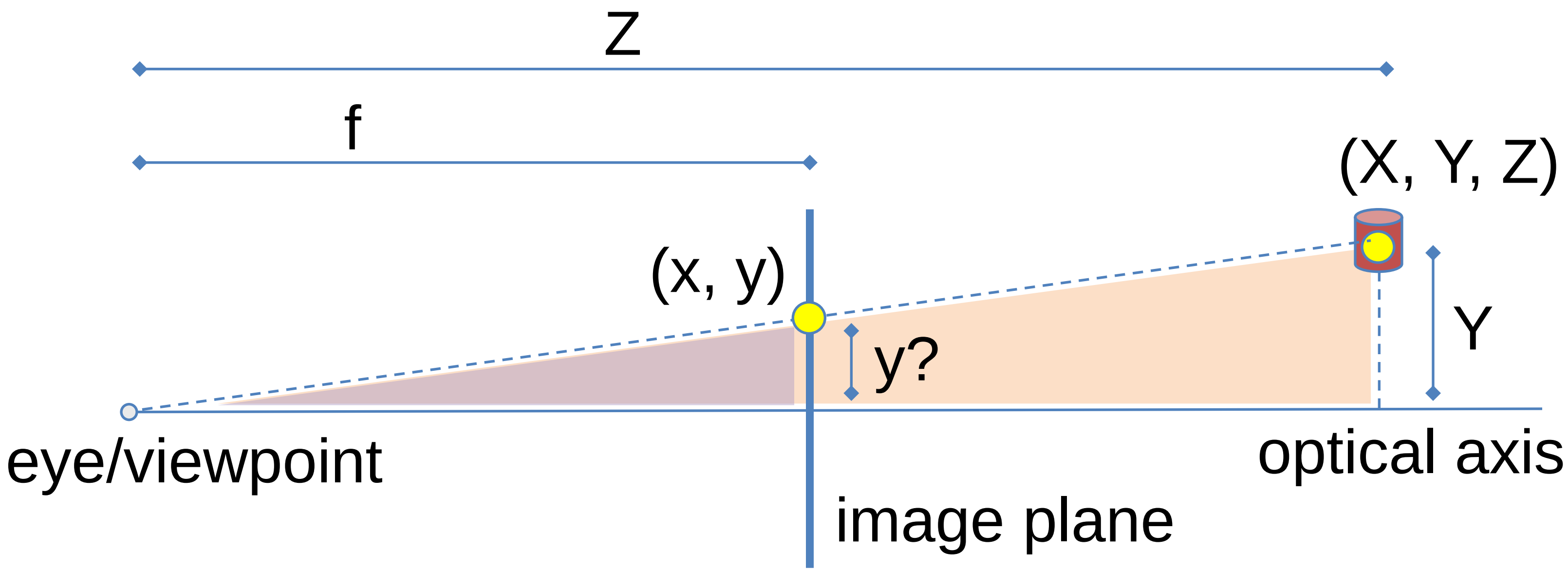
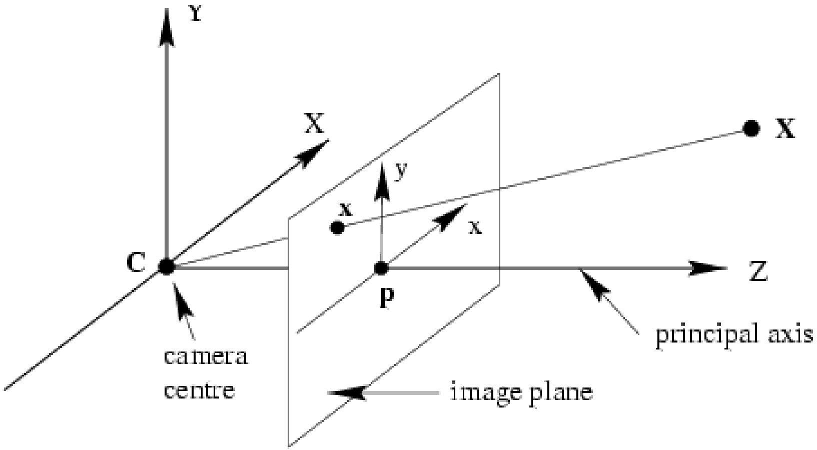
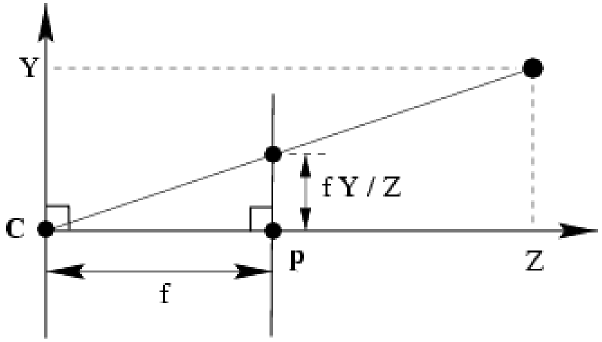
Perspective projection



$$\frac{y}{f} = \frac{Y}{Z} \Rightarrow y = f\frac{Y}{Z} \quad \& \quad x = f\frac{X}{Z}$$

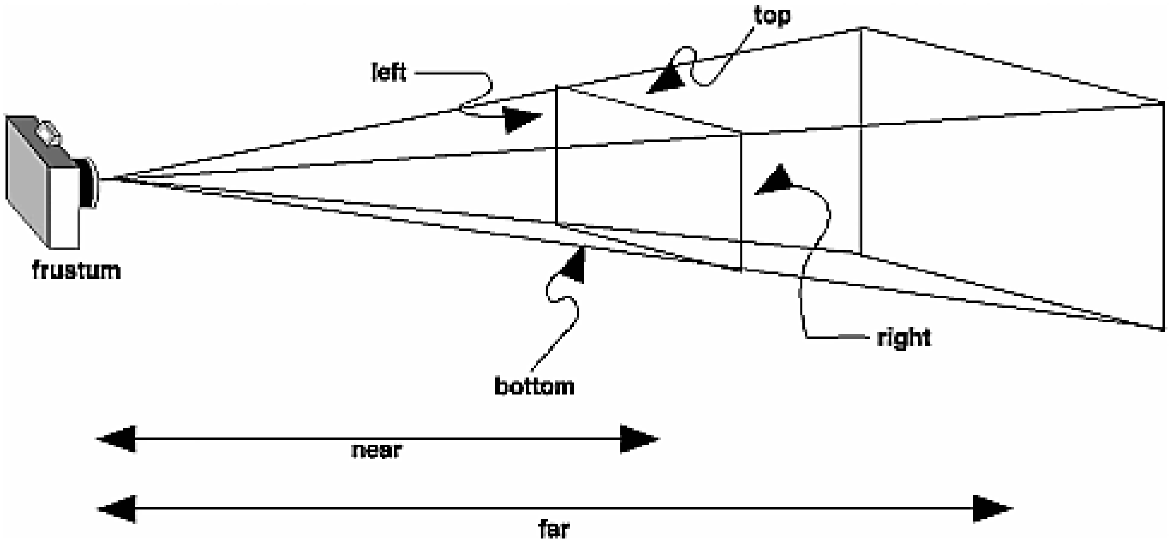
Perspective Projection





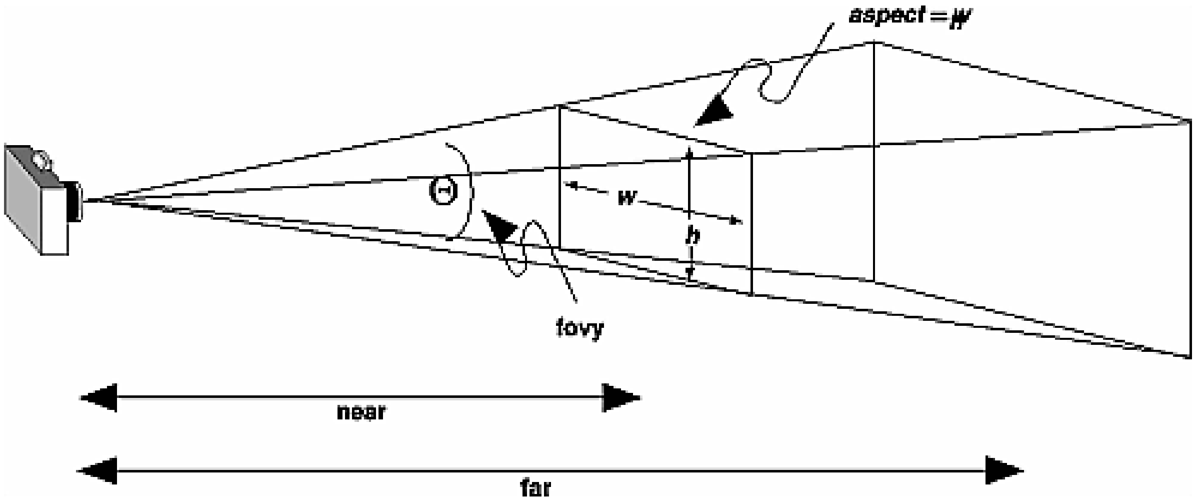
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Projection Transformations



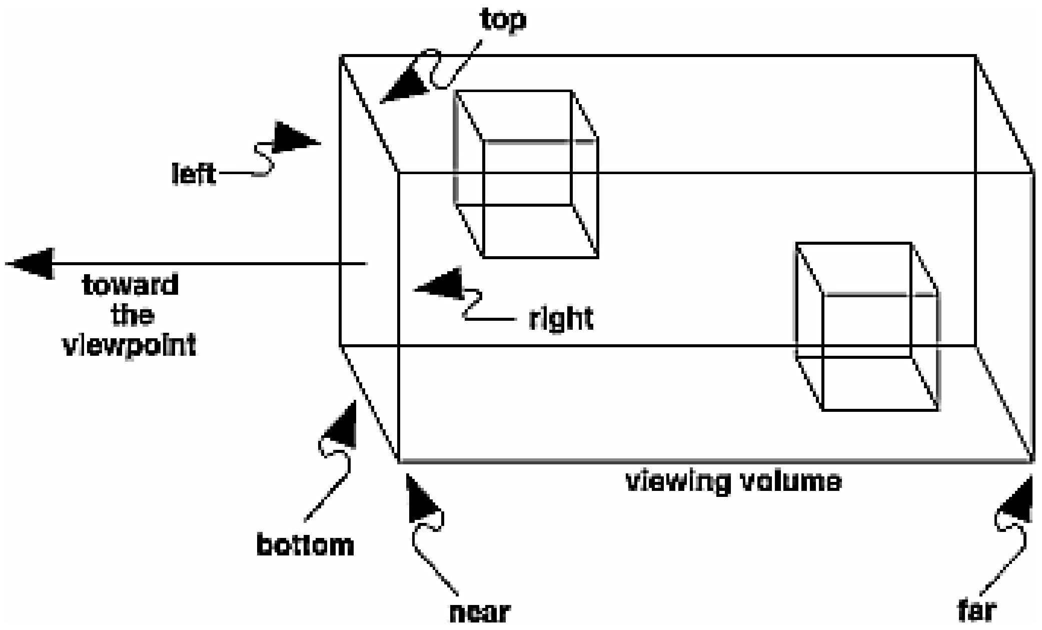
```
void glFrustum(GLdouble left, GLdouble right, GLdouble
  bottom, GLdouble top, GLdouble near, GLdouble far);
```

Projection Transformations



```
void gluPerspective(GLdouble fovy, GLdouble aspect, GLdouble
   near, GLdouble far);
```

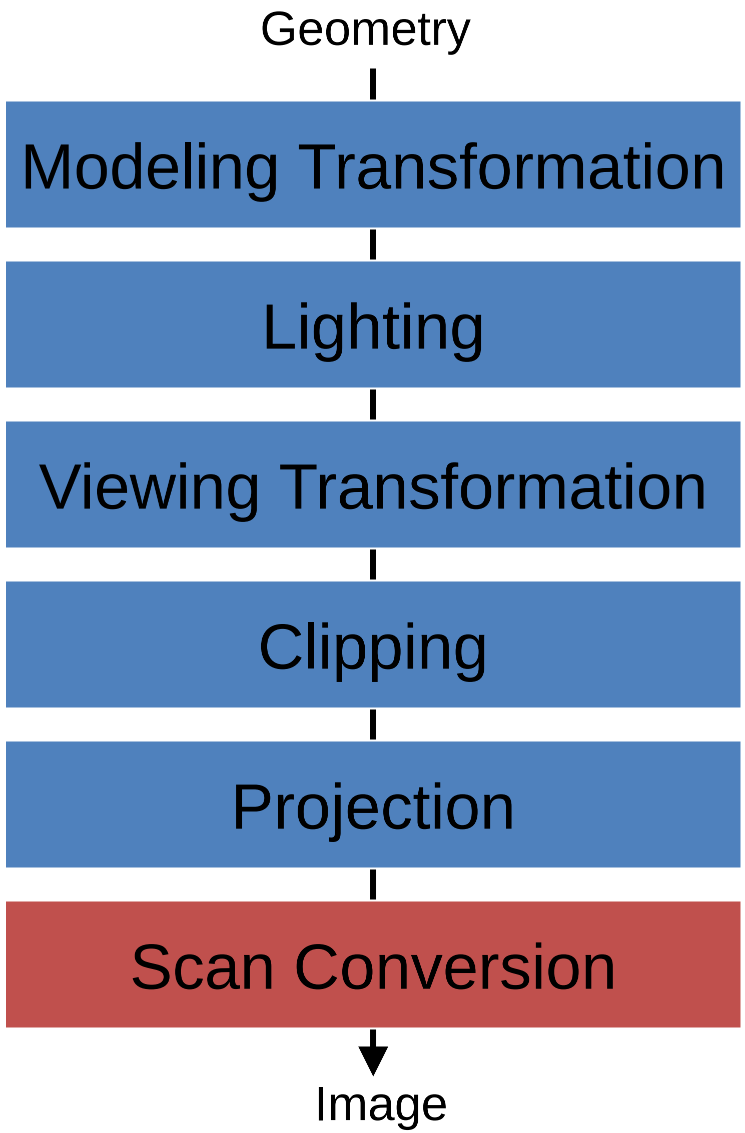
Projection Transformations



```
void glOrtho(GLdouble left, GLdouble right, GLdouble
  bottom,
  GLdouble top, GLdouble near, GLdouble far);

void gluOrtho2D(GLdouble left, GLdouble right,
  GLdouble bottom, GLdouble top);
```

Computer Graphics Pipeline



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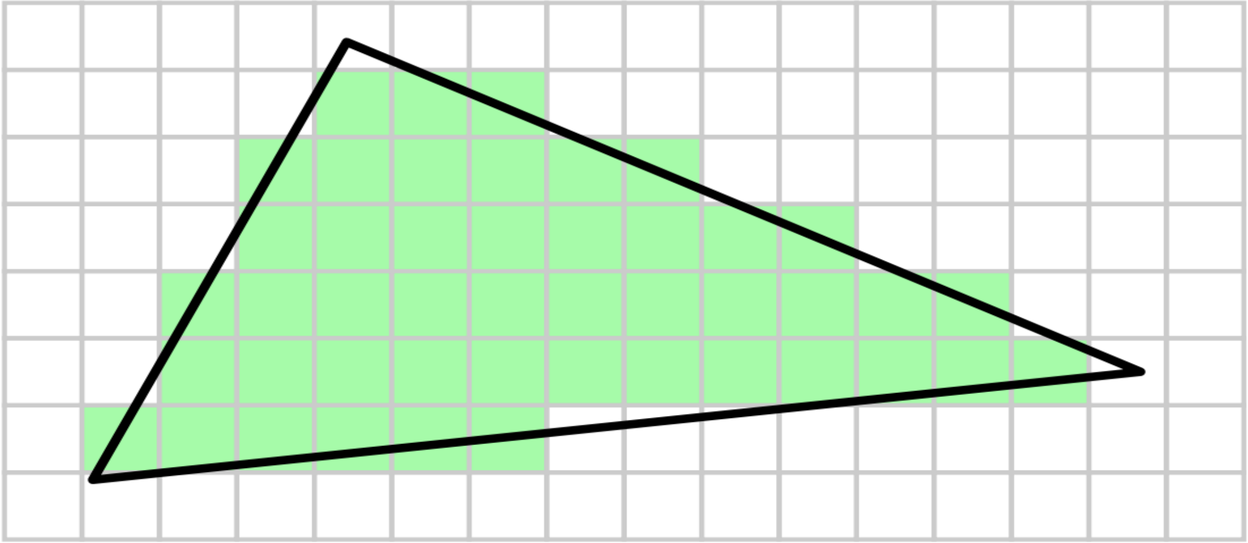
Scan Conversion/Rasterization

- Determine which fragments get generated
- Interpolate parameters (colors, textures, normals, etc.)



Scan Conversion/Rasterization

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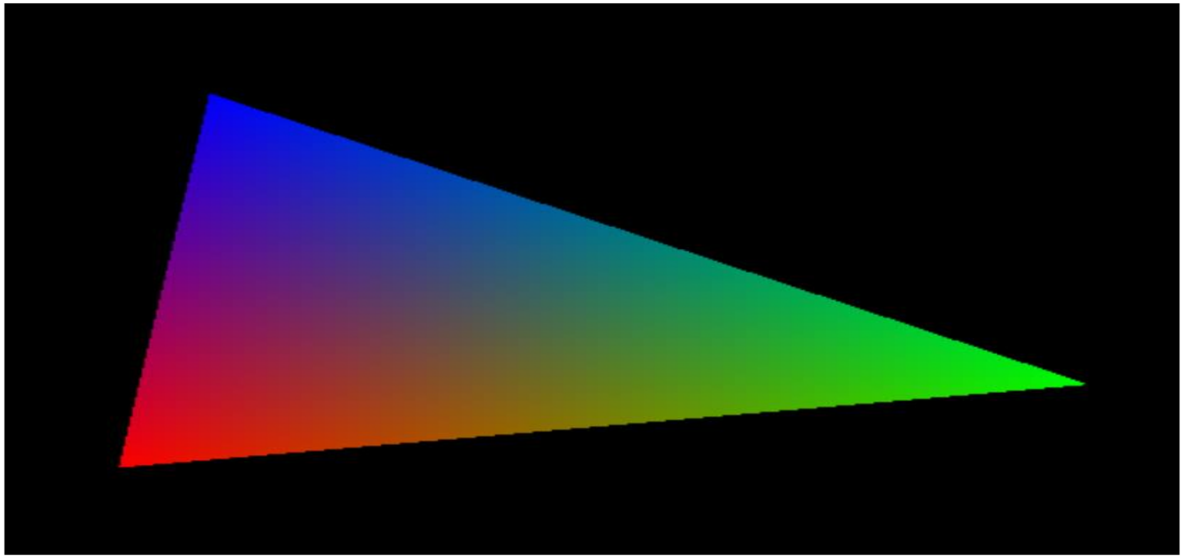

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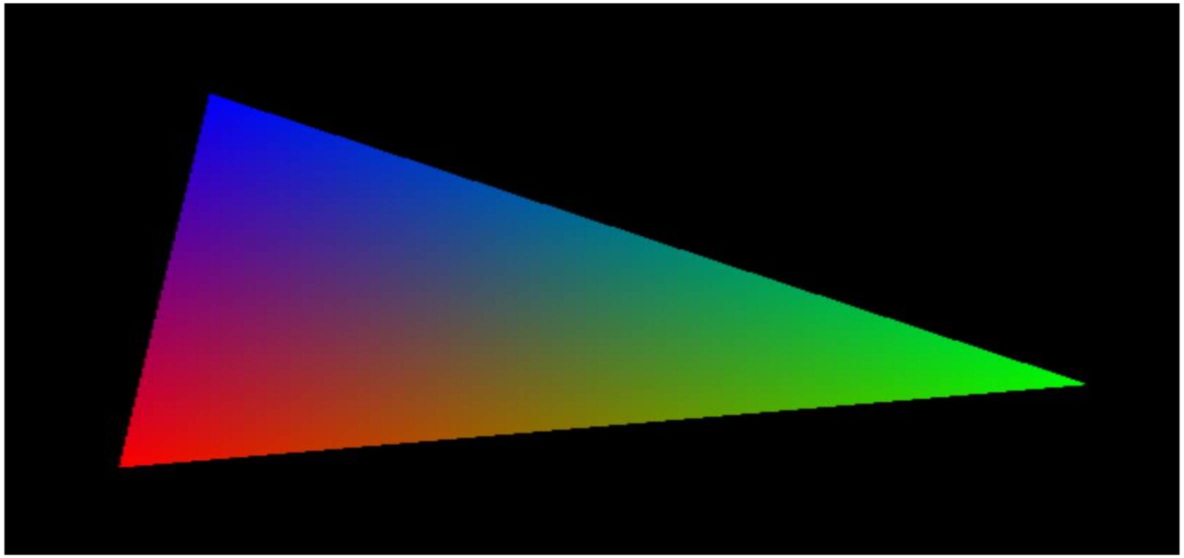

- How?

Scan Conversion/Rasterization

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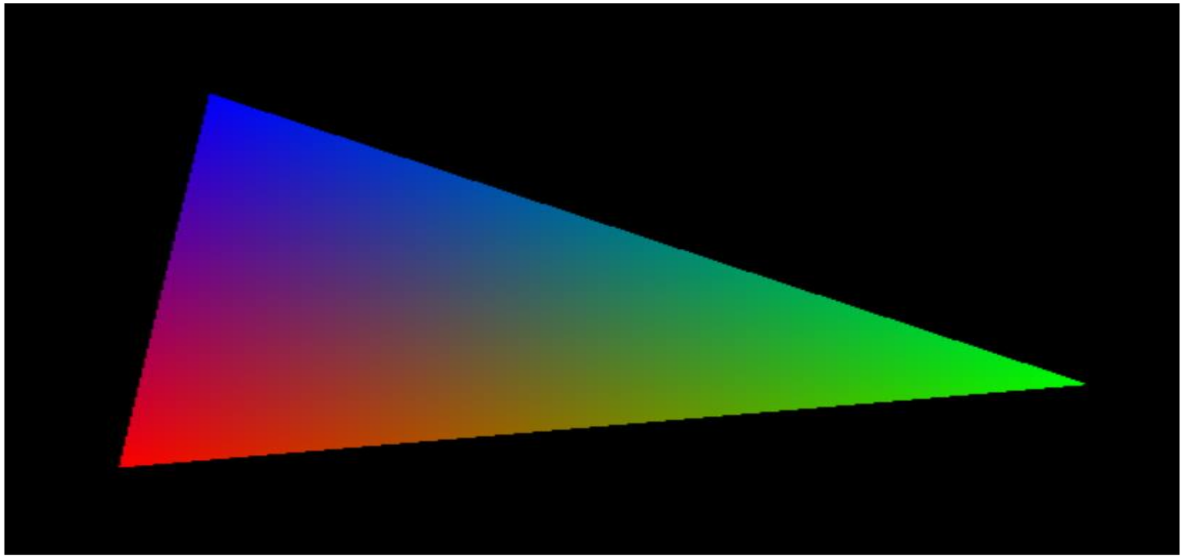
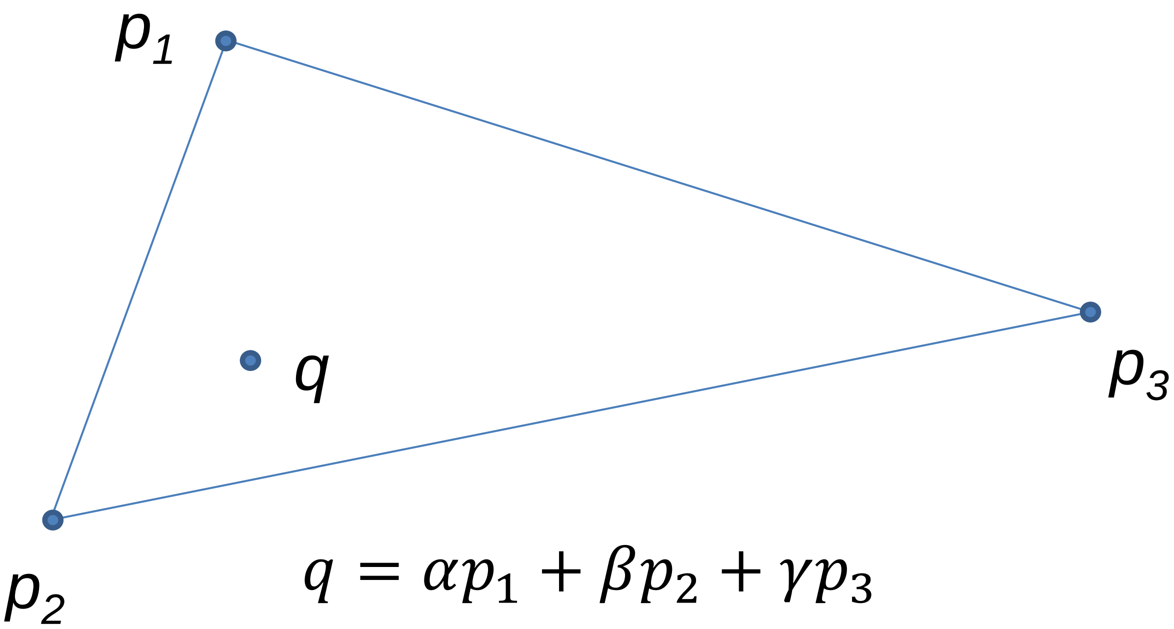


- Barycentric coords amongst many other ways…

Barycentric coordinates

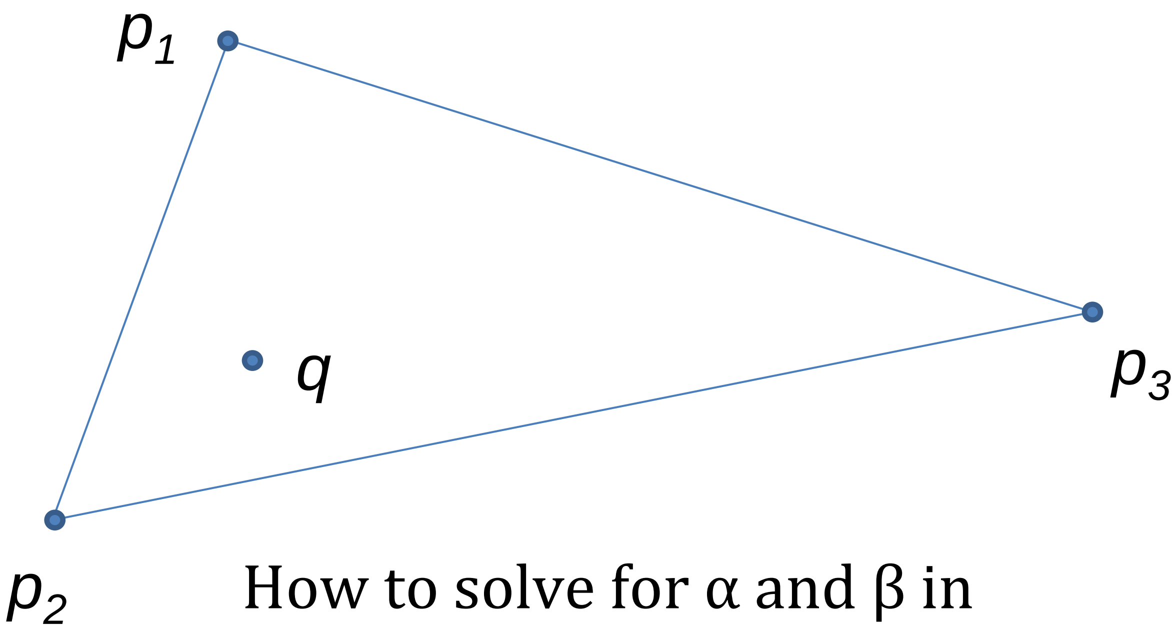


$$q = \alpha p_1 + \beta p_2 + \gamma p_3$$

If $[\alpha + \beta + \gamma = 1 \ \ and \ \ \{\alpha, \beta, \gamma\} \geq 0]$,
then q inside triangle (p_1, p_2, p_3)

Can also write:
$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3$

Barycentric coordinates



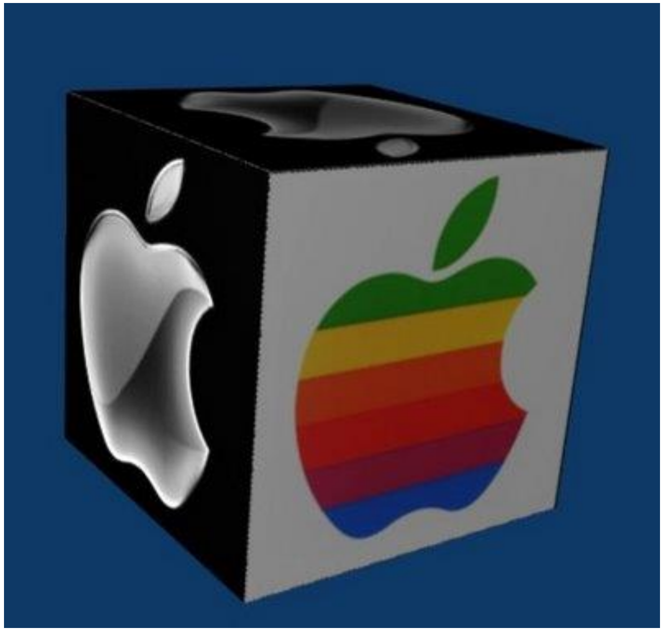
How to solve for α and β in

$$q = \alpha p_1 + \beta p_2 + (1 - \alpha - \beta) p_3?$$

Two equations, two unknowns:
use 2x2 matrix inversion…

Additional concept: Texture mapping

- Model surface-detail with images
  - wrap object with photograph(s)
  - graphics object itself is a simpler model but "looks" more complex



Texture mapping

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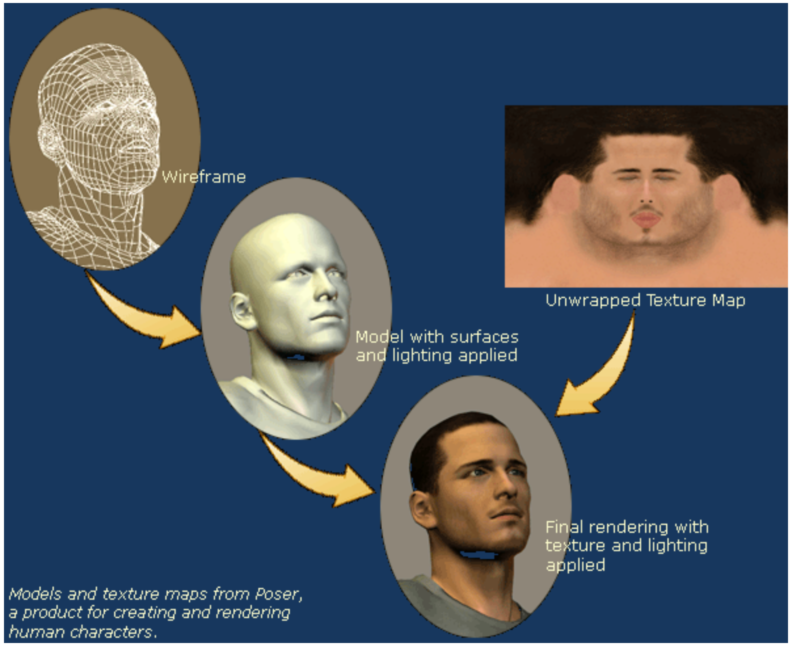
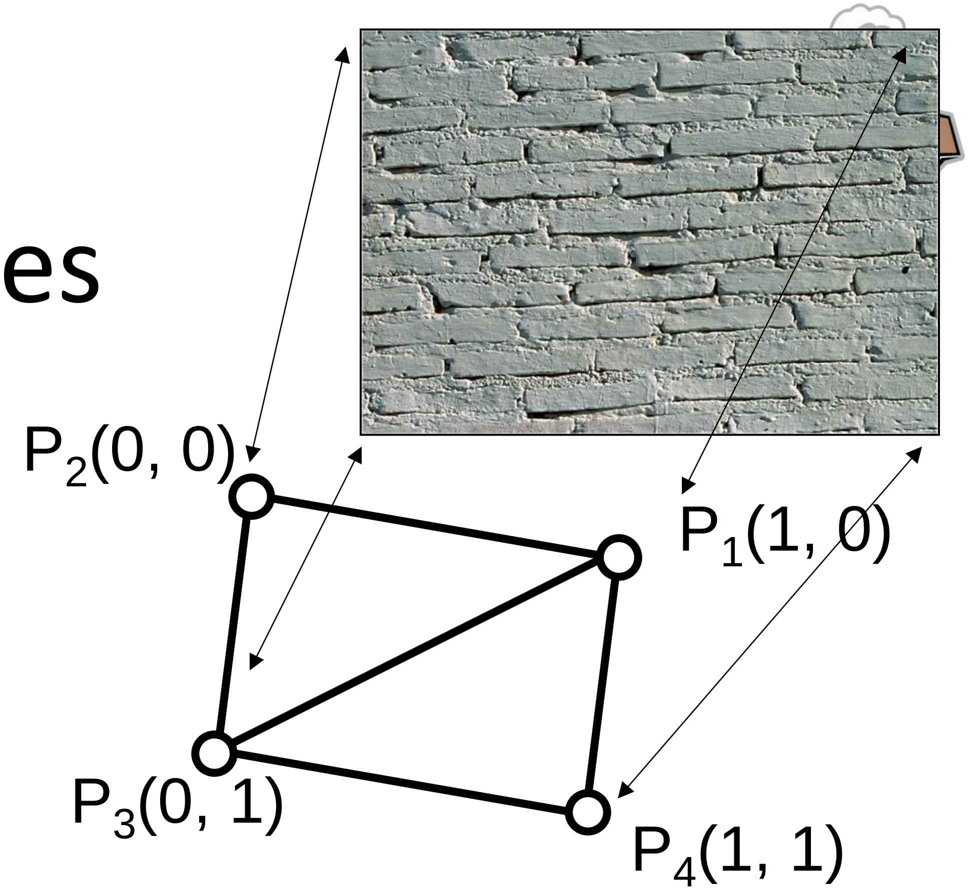

Wireframe
Unwrapped Texture Map
Model with surfaces and lighting applied
Final rendering with texture and lighting applied

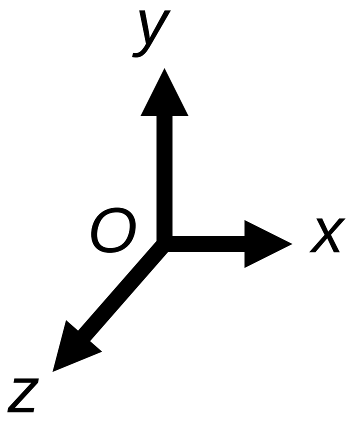
Models and texture maps from Poser, a product for creating and rendering human characters.

Texture coordinates

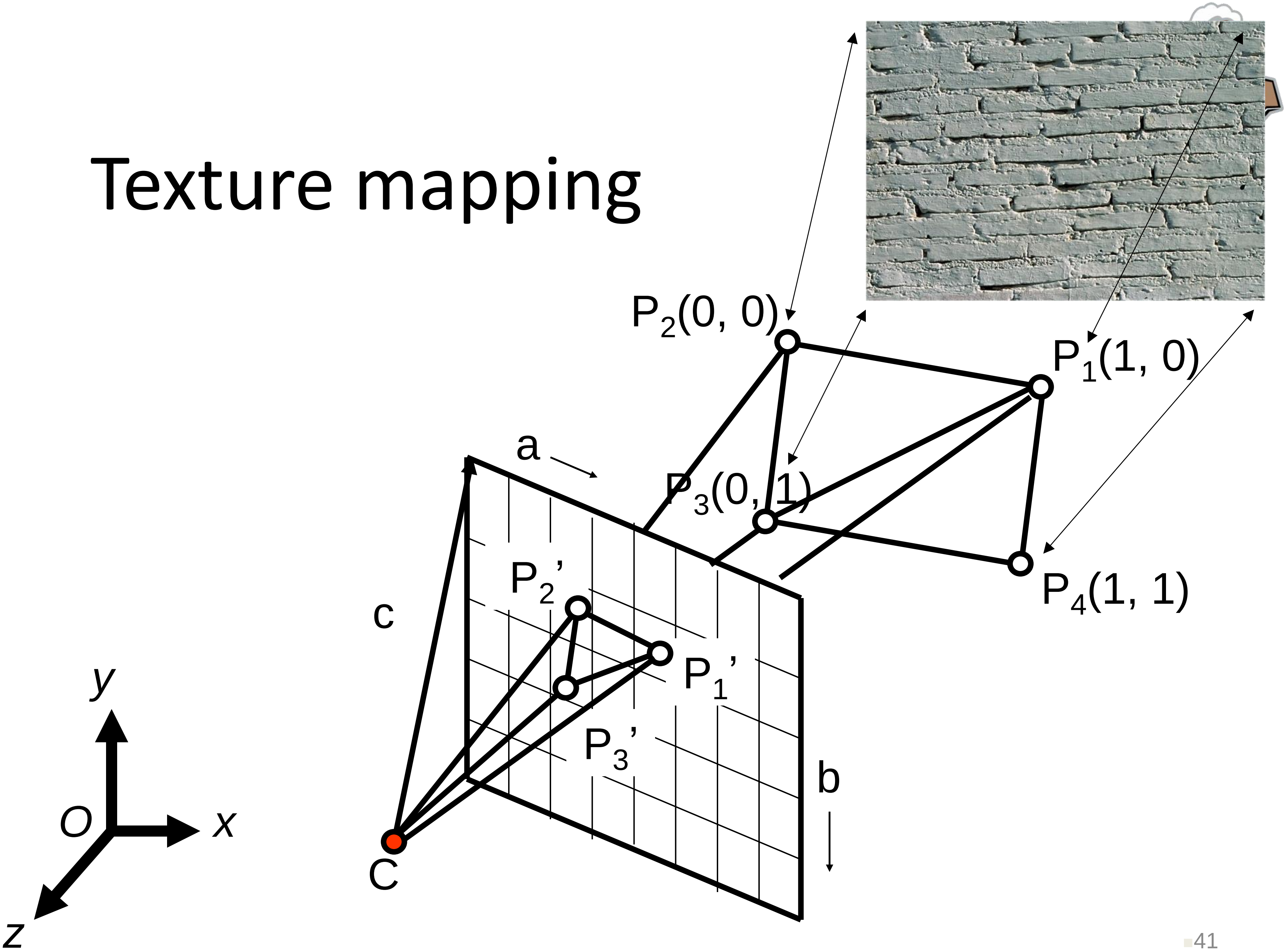
- Mechanism for attaching the texture map to the surface modeled
  - a pair of floats (s, t) for each triangle vertex
  - corners of the image are (0, 0), (0, 1), (1, 1), and (1, 0)
  - tiling indicated with tex. coords. > 1
  - *texels* – color samples in texture maps

Texture coordinates

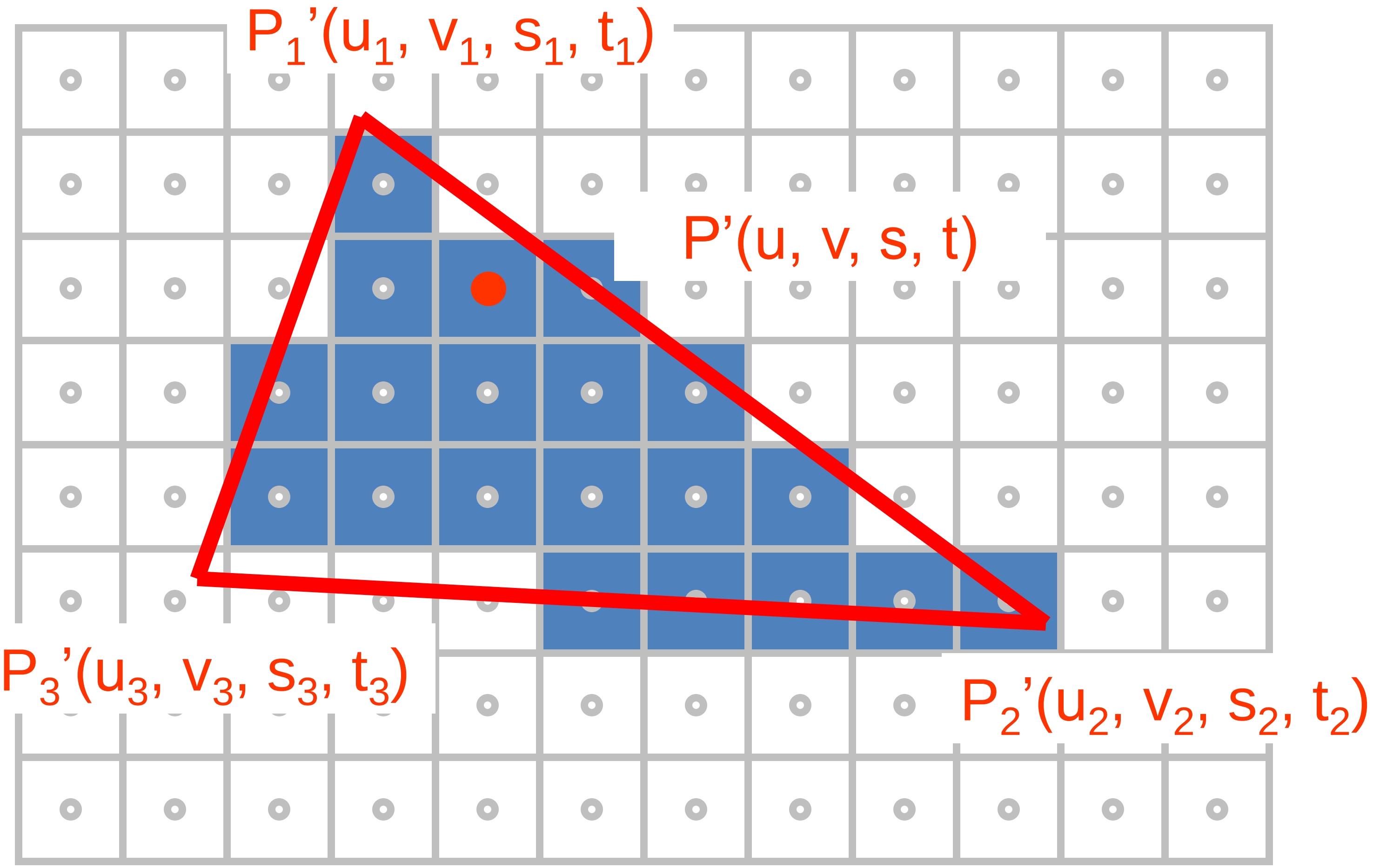




Texture mapping

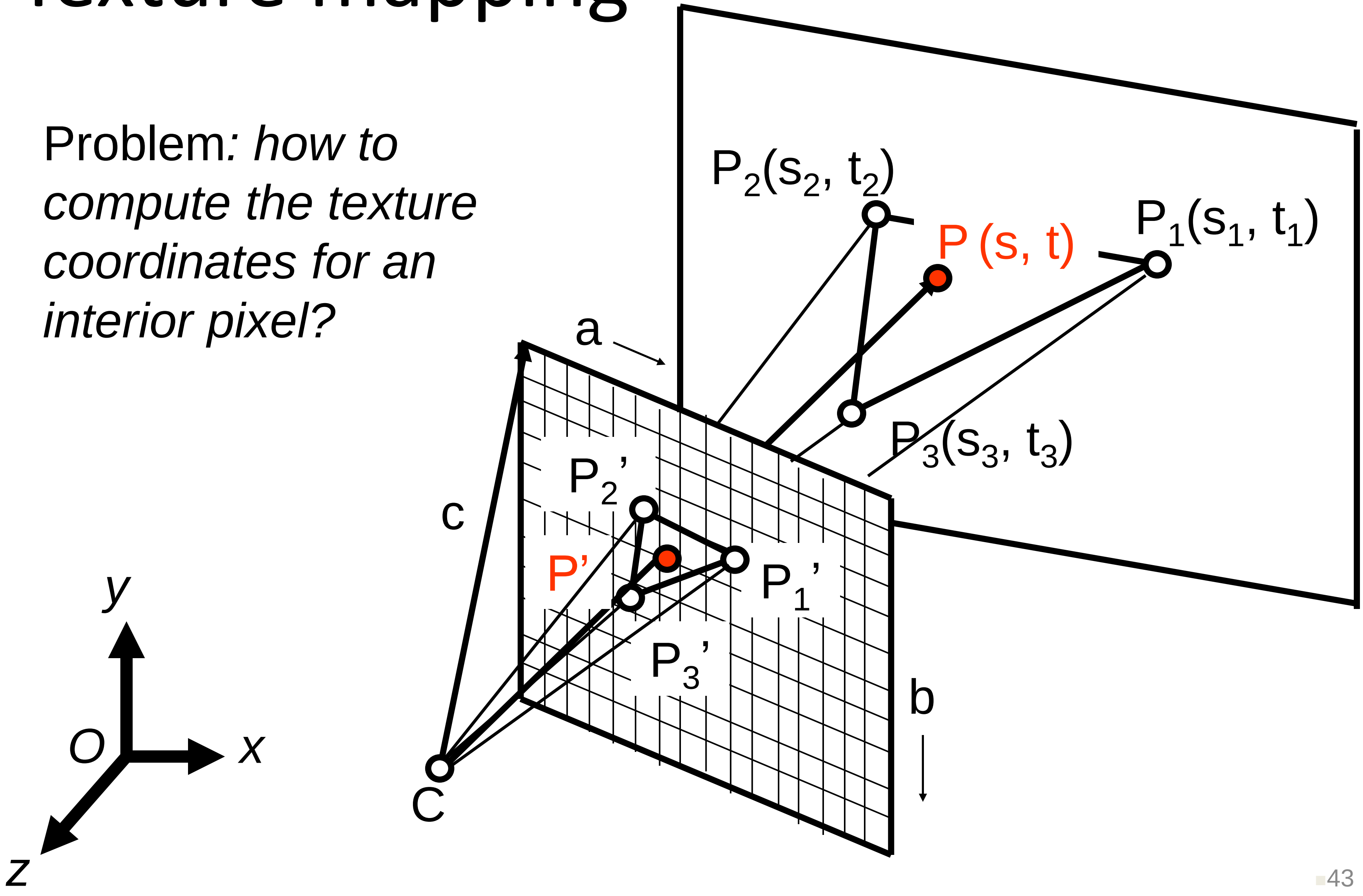


Texels: texture elements


$P_1'(u_1, v_1, s_1, t_1)$
$P'(u, v, s, t)$
$P_3'(u_3, v_3, s_3, t_3)$
$P_2'(u_2, v_2, s_2, t_2)$

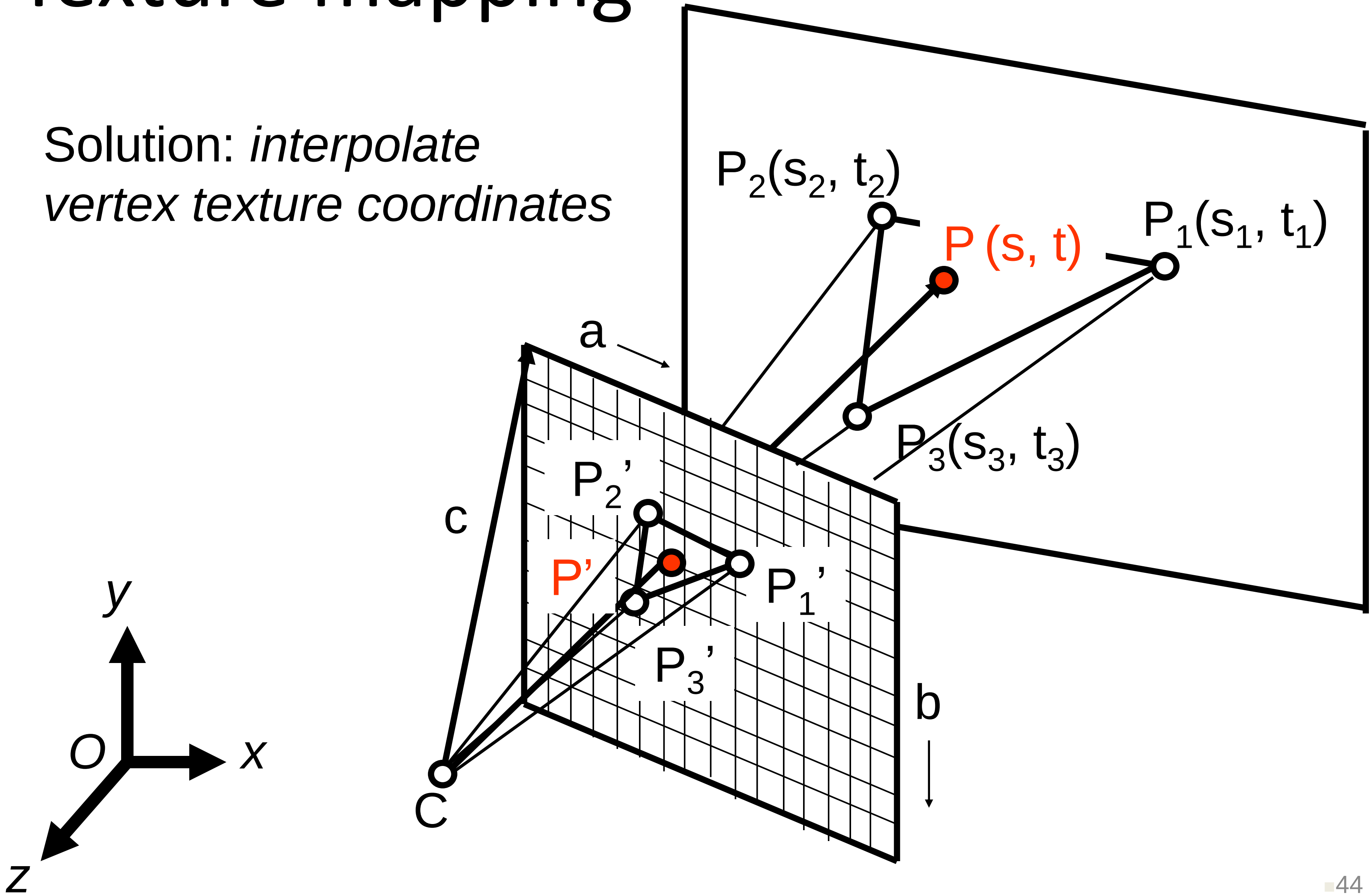
Texture mapping

Problem*: how to compute the texture coordinates for an interior pixel?*



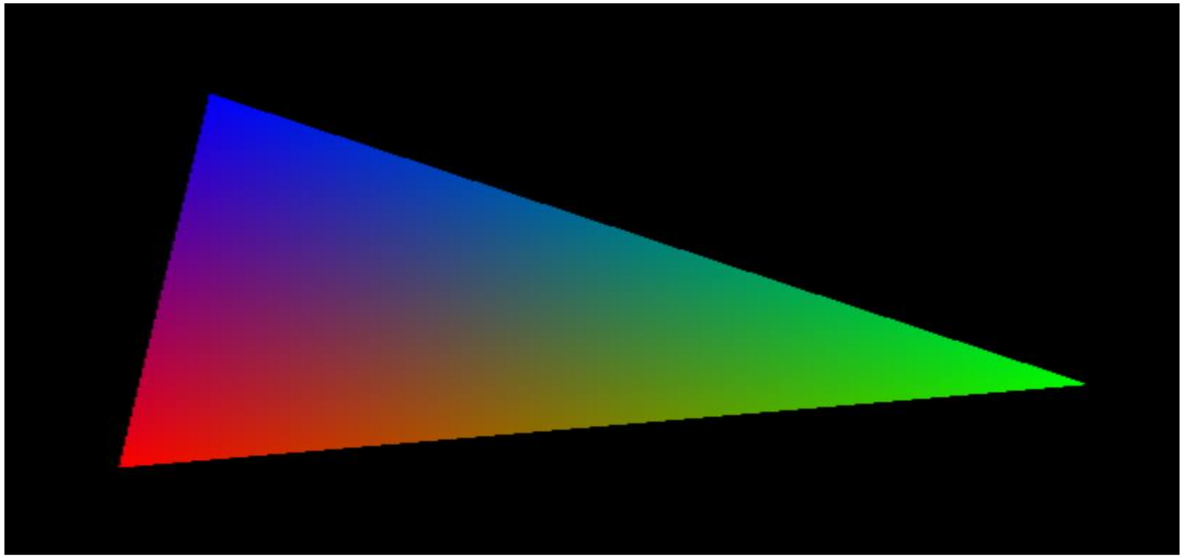
Texture mapping

Solution: *interpolate vertex texture coordinates*



Parameter Interpolation

- Texture coordinates, colors, normals, etc.



- How?
  - Again, use barycentric coordinates…