

Ray Tracing (Part I)

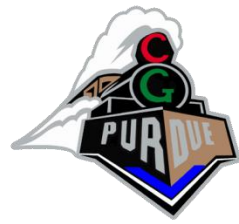
CS334 Fall 2026

Daniel G. Aliaga

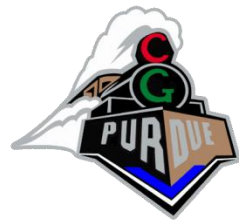
Department of Computer Science

Purdue University

Ray Casting and Ray Tracing



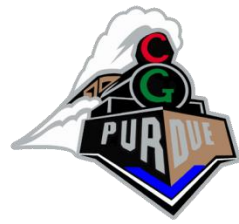
- Ray Casting
 - Arthur Appel, started around 1968...
- Ray Tracing
 - Turner Whitted, started around 1980...



Ray Tracing

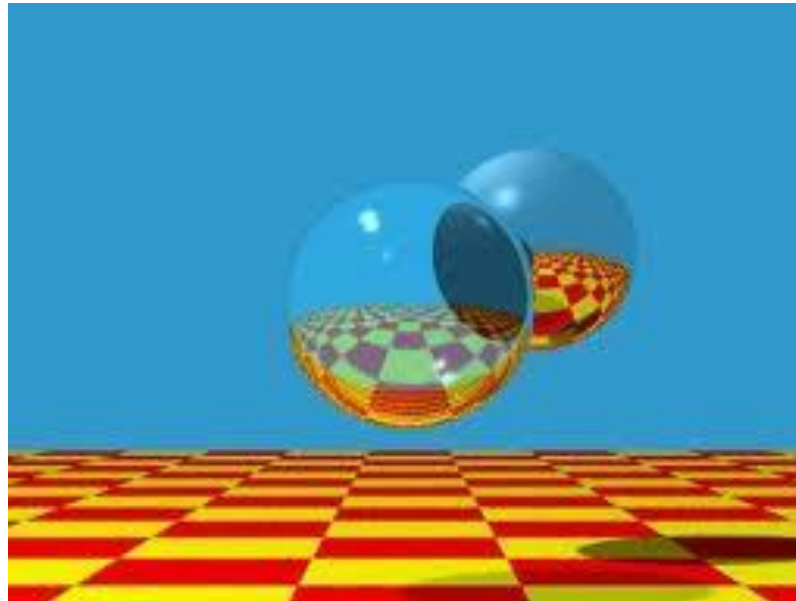
- Turner Whitted, Communications of the ACM, 23(6), 343-349, June 1980
- **ABSTRACT**

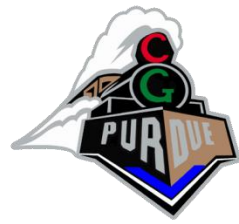
“To accurately render a two-dimensional image of a three-dimensional scene, global illumination information that affects the intensity of each pixel of the image must be known at the time the intensity is calculated. In a simplified form, this information is stored in a tree of “rays” extending from the viewer to the first surface encountered and from there to other surfaces and to the light sources. A visible surface algorithm creates this tree for each pixel of the display and passes it to the shader. The shader then traverses the tree to determine the intensity of the light received by the viewer. Consideration of all of these factors allows the shader to accurately simulate true reflection, shadows, and refraction, as well as the effects simulated by conventional shaders. Anti-aliasing is included as an integral part of the visibility calculations. Surfaces displayed include curved as well as polygonal surfaces.”



Ray Tracing

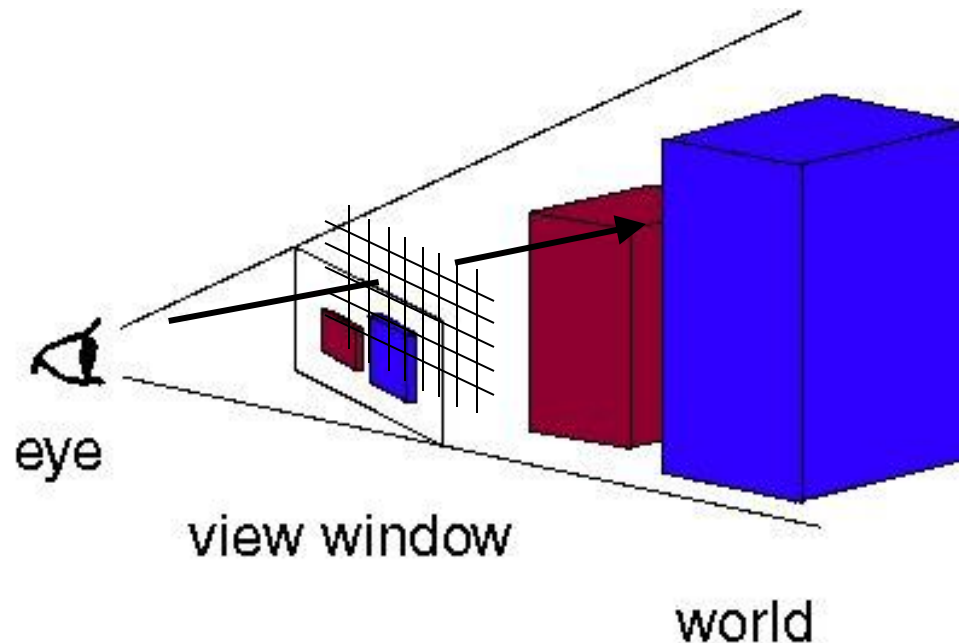
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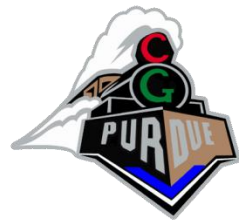




Ray Casting

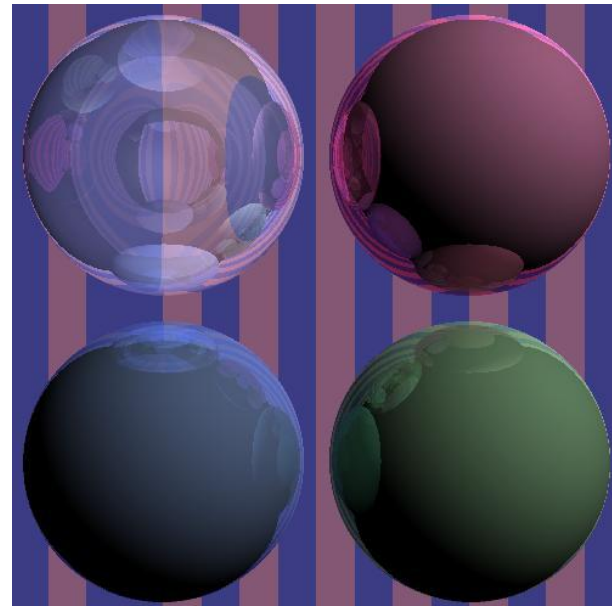
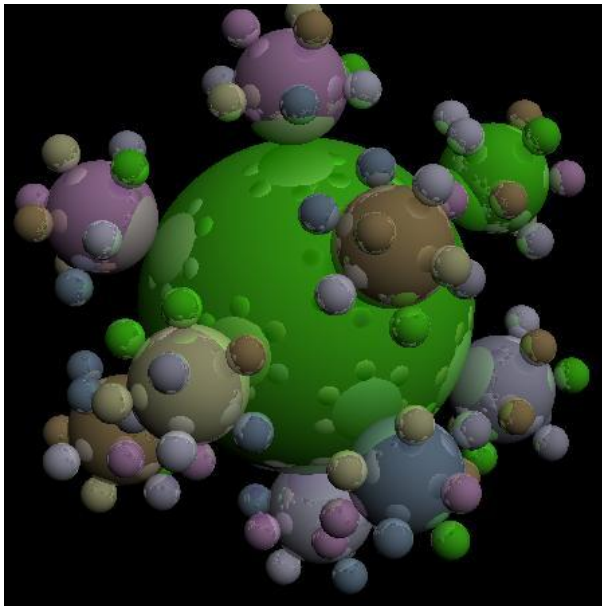
- To “cast” a ray for each pixel into the scene and color the ray based on which object it “hits”
 - This can be interpreted as the inverse of the feed-forward graphics pipeline which effectively “pushes” geometry onto the screen





Ray Tracing

- In addition to ray-casting, rays can reflect off objects, reflect, refract, identify objects in shadow, and numerous other effects...



Ray Tracing

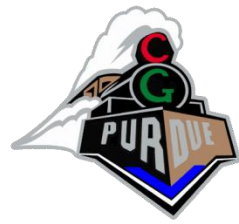
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ray trace example

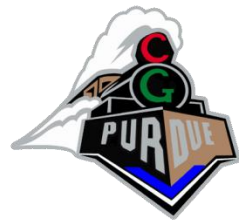


using CUDA/GPU programming



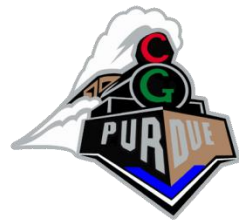
Ray Tracing

- Fluid Animation:
 - <https://www.youtube.com/watch?v=3rlbnqayGZs>
- Real-time
 - <https://www.youtube.com/watch?v=2ow7cbtzg1E>



Ray Tracers

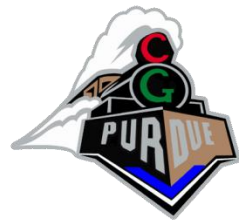
- POV-Ray
 - <http://www.povray.org>
- RayShade
 - <http://www-graphics.stanford.edu/~cek/rayshade/rayshade.html>
- NVIDIA OptiX
 - <https://developer.nvidia.com/optix>



Basic Ray Tracing Algorithm

- Setup image plane
 - Center-of-projection
 - Field-of-view
- Define objects
 - Functional descriptions
 - E.g., sphere, box, $f(x,y,z)=0$
 - Polygonal descriptions
- Foreach pixel...

Basic Ray Tracing Algorithm



Foreach pixel

define ray from eye through pixel and into scene

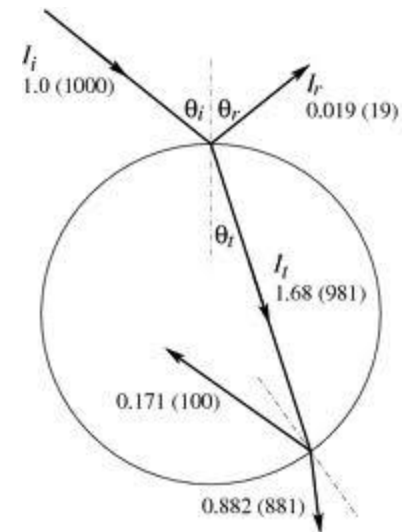
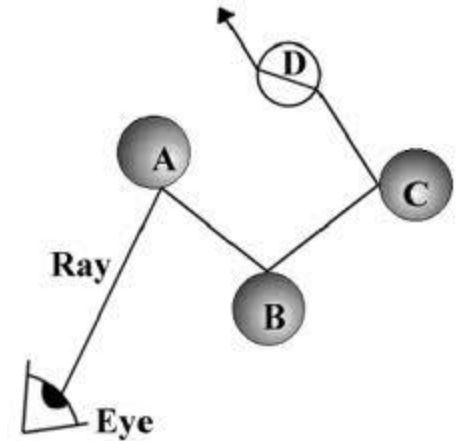
get intersection with scene

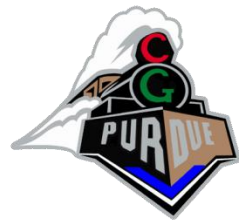
trace reflected ray...

trace refracted ray...

trace to-light ray...

combine resulting colors...

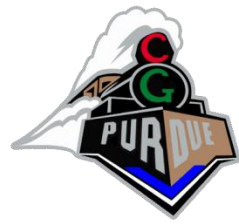




Ray tracing

- Ray tracing (intersection computations)
 - Sphere, plane, polygon, box, quadric
- Reflections
- Refractions
- Shadows

(more on the board...)



Ray Object Intersections

- A ray is defined as

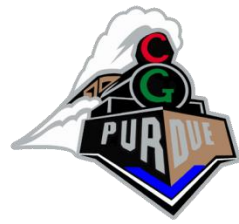
$$R_0 = [x_0, y_0, z_0] \quad \text{- origin of ray}$$

$$R_d = [x_d, y_d, z_d] \quad \text{- direction of ray}$$

Then, a set of points on the ray are defined as

$$R(t) = R_0 + R_d * t \quad \text{where } t > 0$$

If R_d is normalized, then t equals the distance of the ray from origin in World Coordinates,



Ray Sphere Intersection

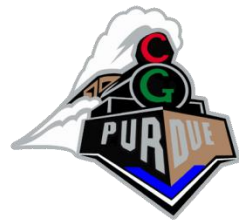
- Sphere is defined by its center ($S_c = [x_c, y_c, z_c]$) and radius S_r

Thus, the equation of sphere is

$$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = S_r^2$$

Substitute equation of ray and solve quadratic eqn for **t**,

- If discriminant $< 0 \Rightarrow$ No intersection
- If discriminant $\geq 0 \Rightarrow$ Intersection. Smaller **t** gives the nearest intersection point



Ray Plane Intersection

- Plane is defined by $Ax+By+Cz+D=0$

Substitute the equation for ray and solve for $\mathbf{t}$,

$$\begin{aligned}\mathbf{t} &= -(AX_0 + BY_0 + CZ_0 + D) / (AX_d + BY_d + CZ_d) \\ &= -(\mathbf{P}_n \cdot \mathbf{R}_0 + D) / (\mathbf{P}_n \cdot \mathbf{R}_d)\end{aligned}$$

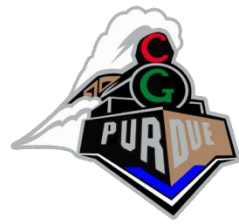
where $\mathbf{P}_n$ is the plane normal of unit length

If $\mathbf{P}_n \cdot \mathbf{R}_d = 0 \Rightarrow$ Ray is parallel to the plane

If $\mathbf{P}_n \cdot \mathbf{R}_d > 0 \Rightarrow$ Normal of plane points away from the ray

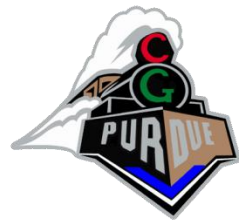
If $\mathbf{t} < 0 \Rightarrow$ Ray intersects plane behind origin

If $\mathbf{t} > 0 \Rightarrow$ Substitute $\mathbf{t}$ in equation for intersection point



Ray Polygon Intersection

- Check whether the ray intersects the polygon plane
- Calculate the point of intersection
- Check whether the point is within the polygon. **How?**
 - Cross products
 - Shoot a ray from the point and count edges crossed, if odd then inside
 - or one of several other approaches



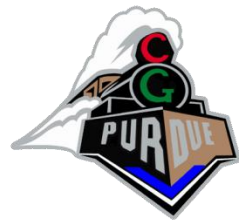
Ray Box Intersection

- Transform box vertices ($\mathbf{V}_i$) to align with the coordinate axes
- Solve for intersection with $x_{\min}$ and $x_{\max}$ planes; similarly solve for Y and Z.
- Check for common interval or intersection

If, $\max(t_{\min}) \leq \min(t_{\max})$

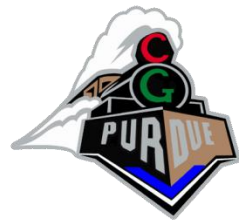
Then, $t_0 = \max(t_{\min})$

$t_1 = \min(t_{\max})$



Ray Quadric Intersection

- Class of quadrics (surfaces that can be defined by a quadratic equation) include cylinders, cones, ellipsoids, paraboloids, etc
- The general quadric surface equation is
$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$
- Substitute the equation of ray, we get the form,
$$A_q t^2 + B_q t + C_q = 0$$
- Solve for **t** using quadratic formula

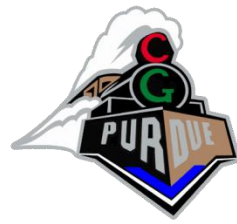


Ray tracing

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Refraction (or Transmission)



- Need refraction indices:

k_1 =outside object (e.g., air)

k_2 =inside object (e.g., glass)

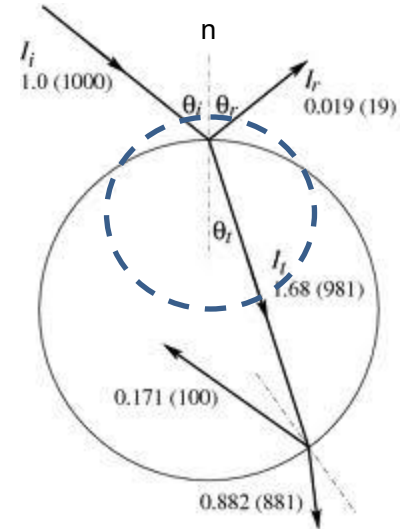
- Snell's Law:

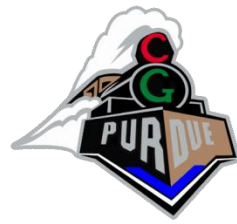
$$k_1 \sin(\theta_i) = k_2 \sin(\theta_t)$$

- After some rearrangement:

$$l_t = -n \cos(\theta_t) + b \sin(\theta_t)$$

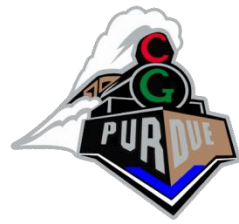
$$\text{where } b = n \times (n \times l_i)$$





Ray Traced Movies

- First:
 - <https://www.youtube.com/watch?v=0fy2bmXfOFs>
- Recent:
 - <https://www.youtube.com/watch?v=KJRZTkttgLw>



Ray Tracers Online

- <https://b-z.github.io/ray/>
- <https://rayground.com/view/LYnWPJNFuPU>