Global Illumination

CS535

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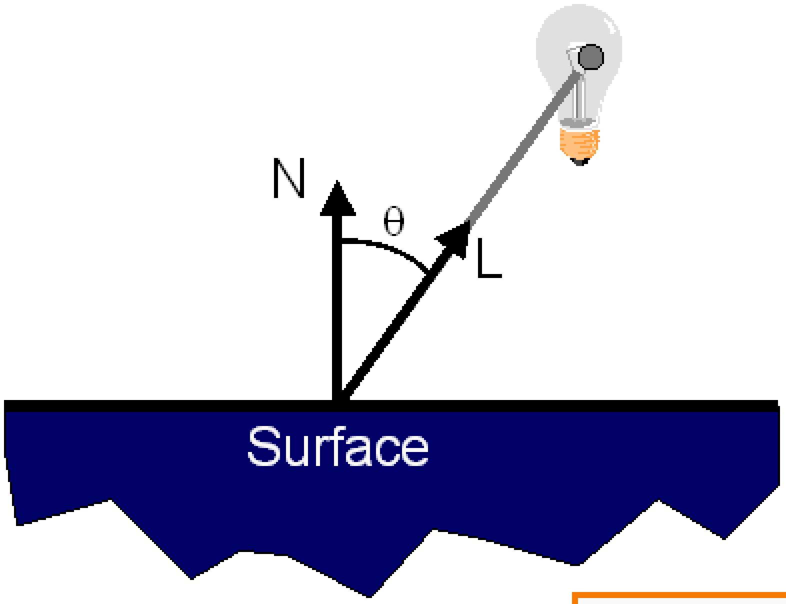
Recall: Lighting and Shading

- Light sources
  - Point light
    - Models an omnidirectional light source (e.g., a bulb)
  - Directional light
    - Models an omnidirectional light source at infinity
  - Spot light
    - Models a point light with direction

- Light model
  - Ambient light
  - Diffuse reflection
  - Specular reflection

Recall: Lighting and Shading

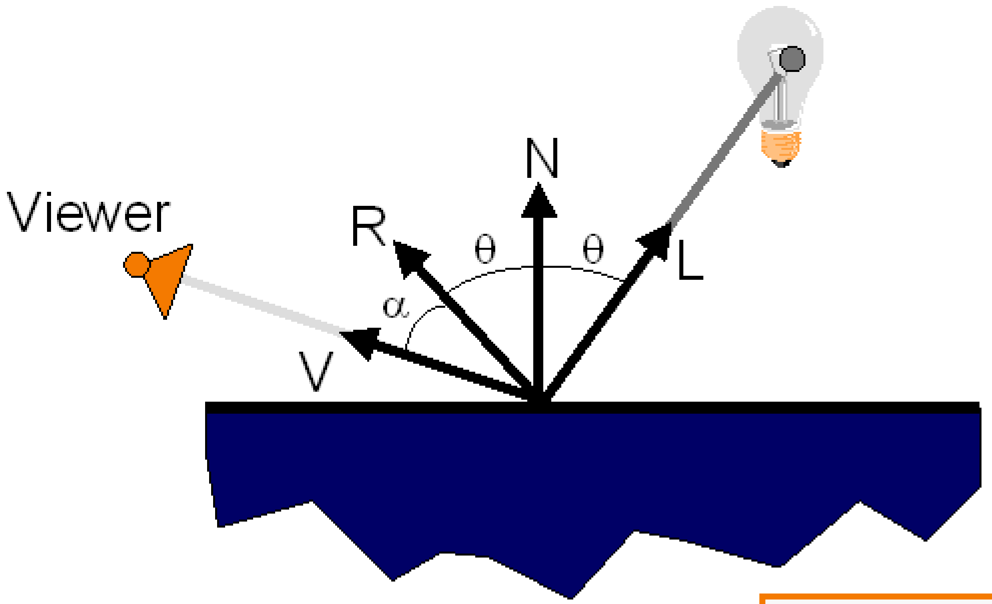
- Diffuse reflection
  - Lambertian model



$$I_D = K_D(N \bullet L)I_L$$

Recall: Lighting and Shading

- Specular reflection
  - Phong model



$$I_S = K_S (V \bullet R)^n I_L$$

Recall: Lighting and Shading

- Well….there is much more

For example…

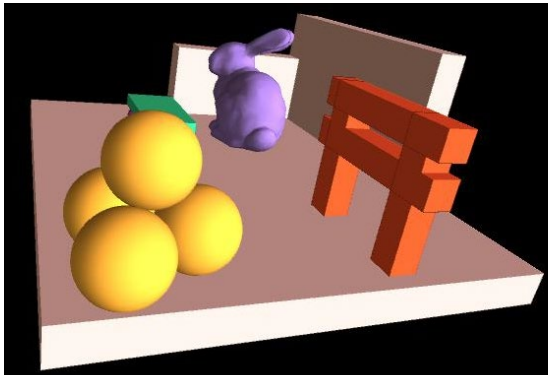
- Reflection -> Bidirectional Reflectance Distribution Functions (BRDF)
- Diffuse, Specular -> Diffuse Interreflection, Specular Interreflection
- Color bleeding
- Transparency, Refraction
- Scattering
  - Subsurface scattering
  - Through participating media
- And more!

Illumination Models

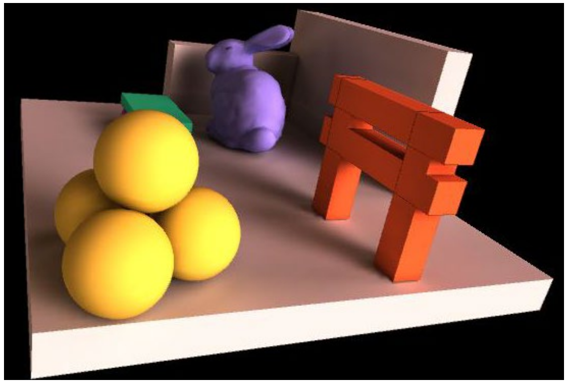
- So far, you considered mostly local (direct) illumination
  - Light directly from light sources to surface
  - No shadows (actually is a global effect)
- Global (indirect) illumination: multiple bounces of light
  - Hard and soft shadows
  - Reflections/refractions (you kinda saw already)
  - Diffuse and specular interreflections

Welcome to Global Illumination

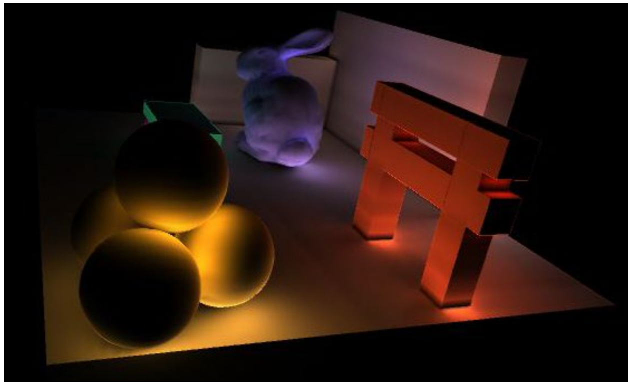
- *Direct illumination* + *indirect illumination*; e.g.
  - Direct = reflections, refractions, shadows, …
  - Indirect = diffuse and specular inter-reflection, …



direct illumination



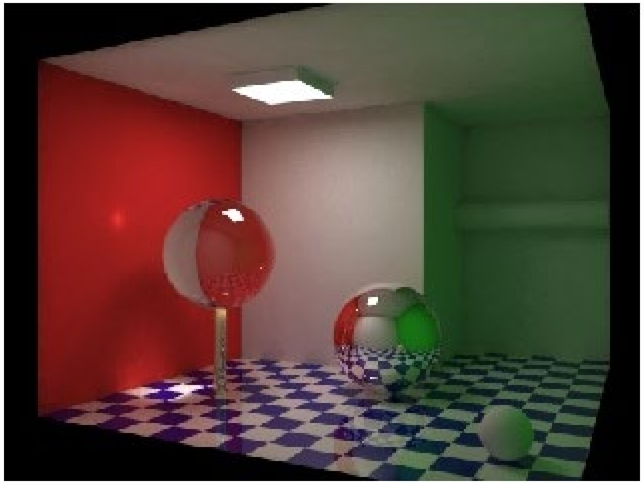
with global illumination



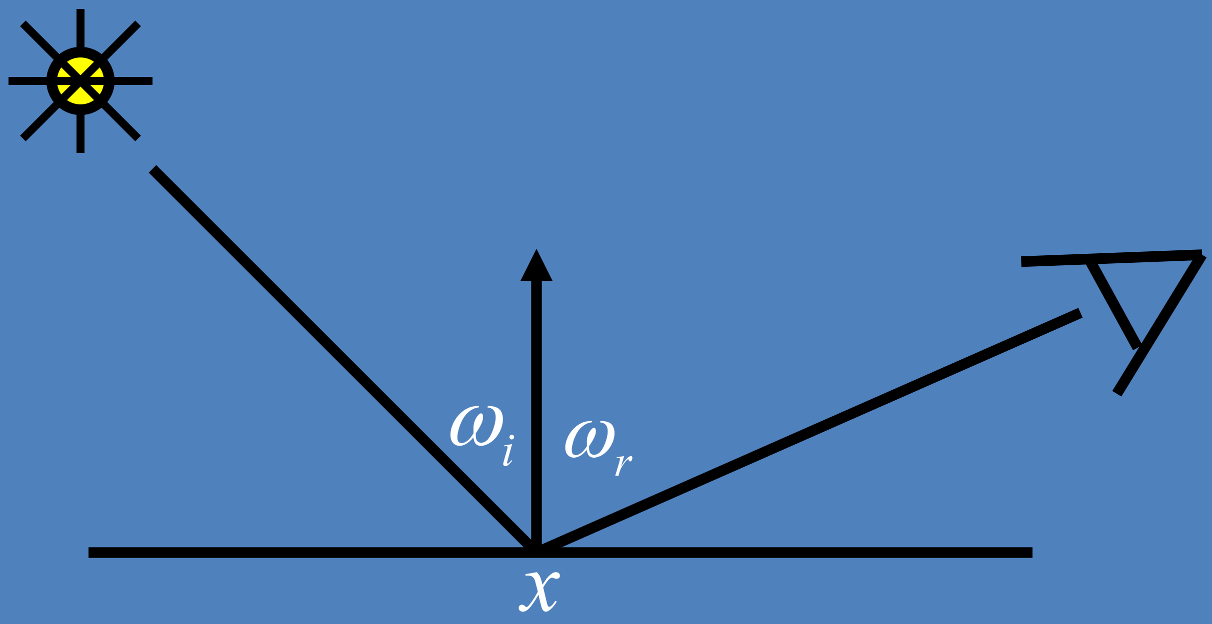
only diffuse inter-reflection

Global Illumination

- *Direct illumination* + *indirect illumination*; e.g.
  - Direct = reflections, refractions, shadows, …
  - Indirect = diffuse and specular inter-reflection, …



Reflectance Equation

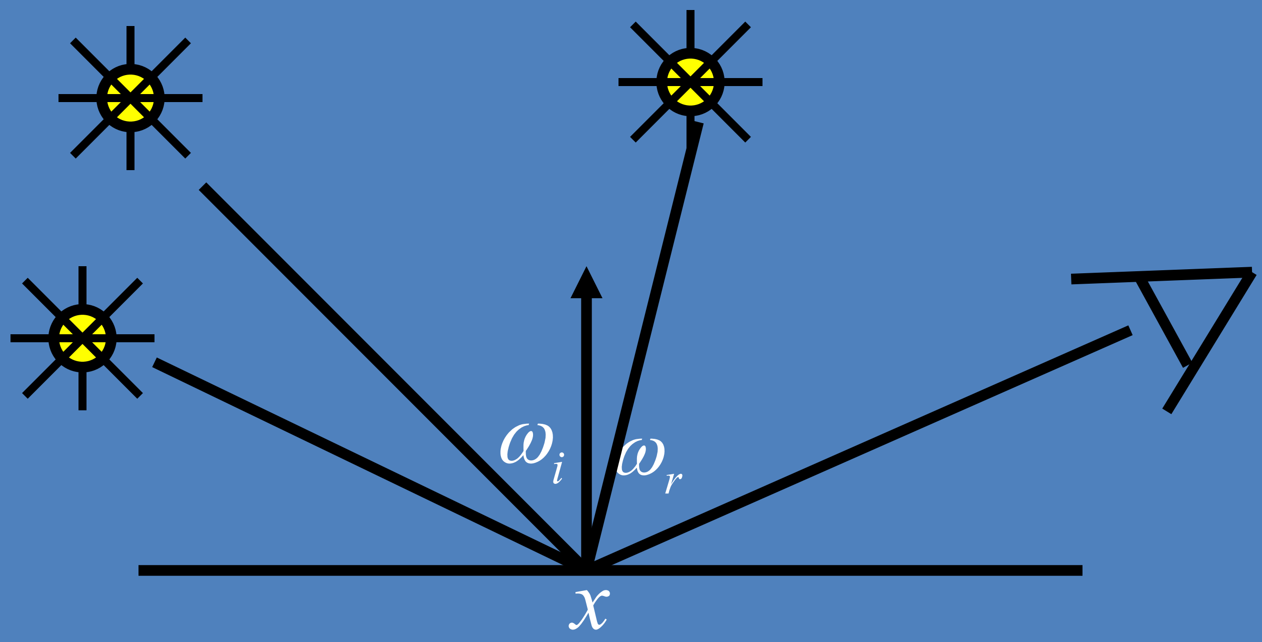


$$L_r(x,\omega_r) = L_e(x,\omega_r) + L_i(x,\omega_i)\, f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

[Slides with help from Pat Hanrahan and Henrik Jensen]

Reflectance Equation



Sum over all light sources

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \sum L_i(x,\omega_i) f(x,\omega_i,\omega_r)(\omega_i \bullet n)$$

Reflected Light (Output Image)

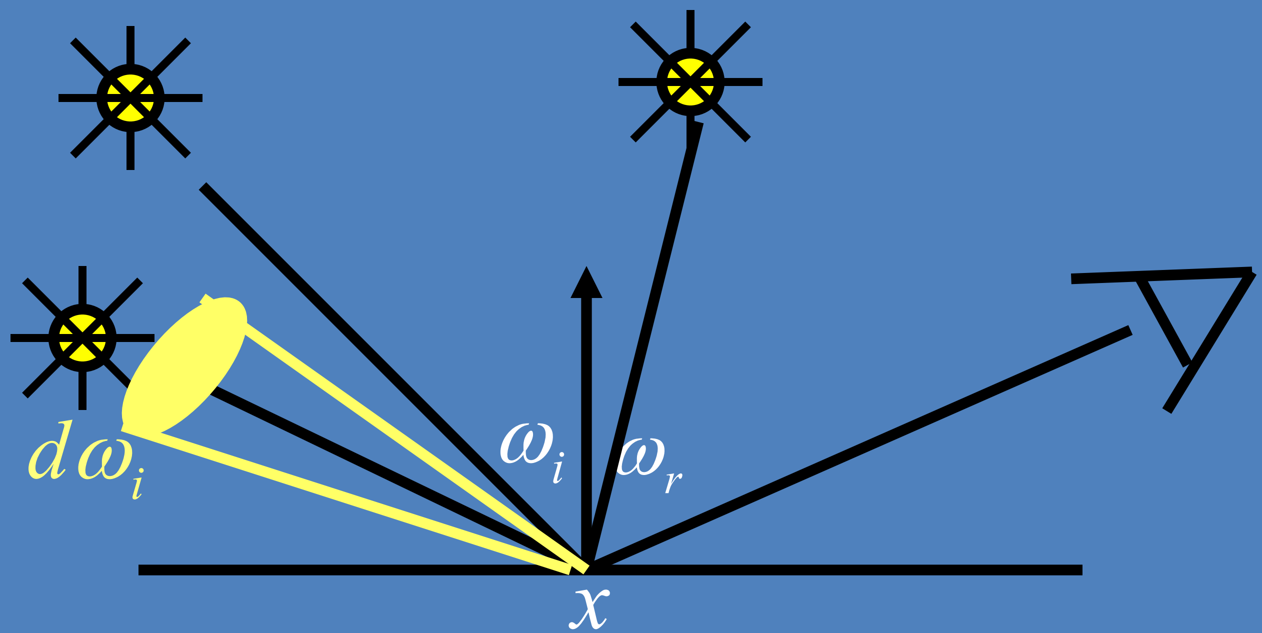
Emission

Incident Light (from light source)

BRDF

Cosine of Incident angle

Reflectance Equation

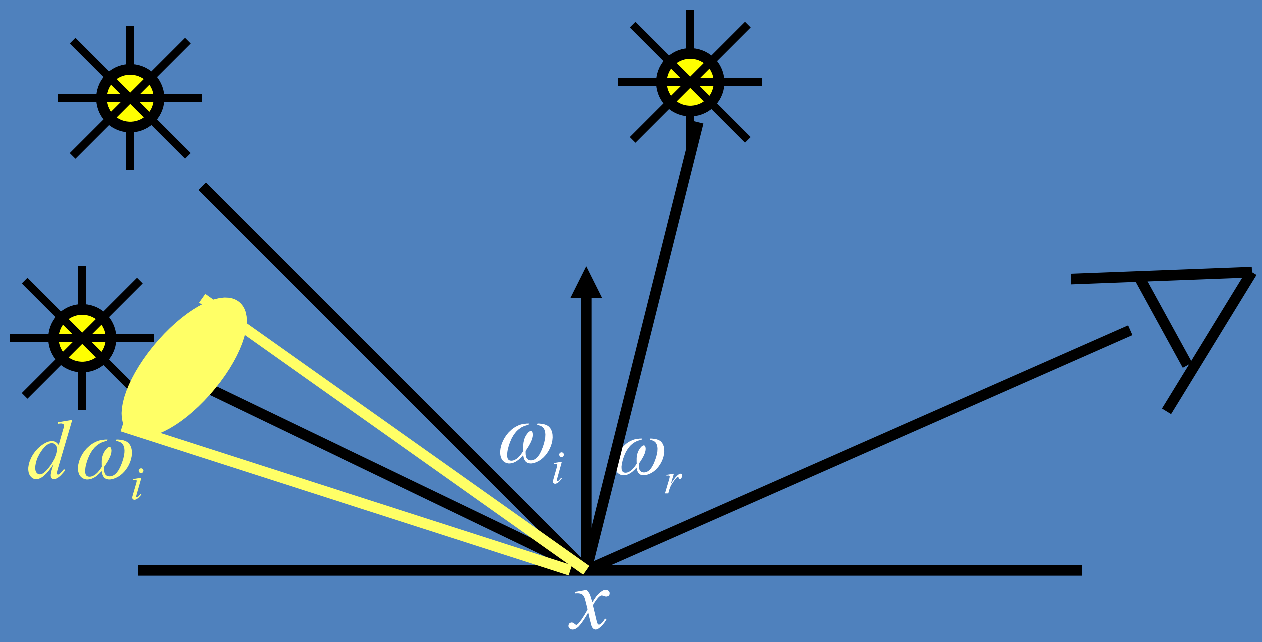


Replace sum with integral

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

Reflected Light (Output Image)	Emission	Incident Light (from light source)	BRDF	Cosine of Incident angle

Reflectance Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

The Challenge

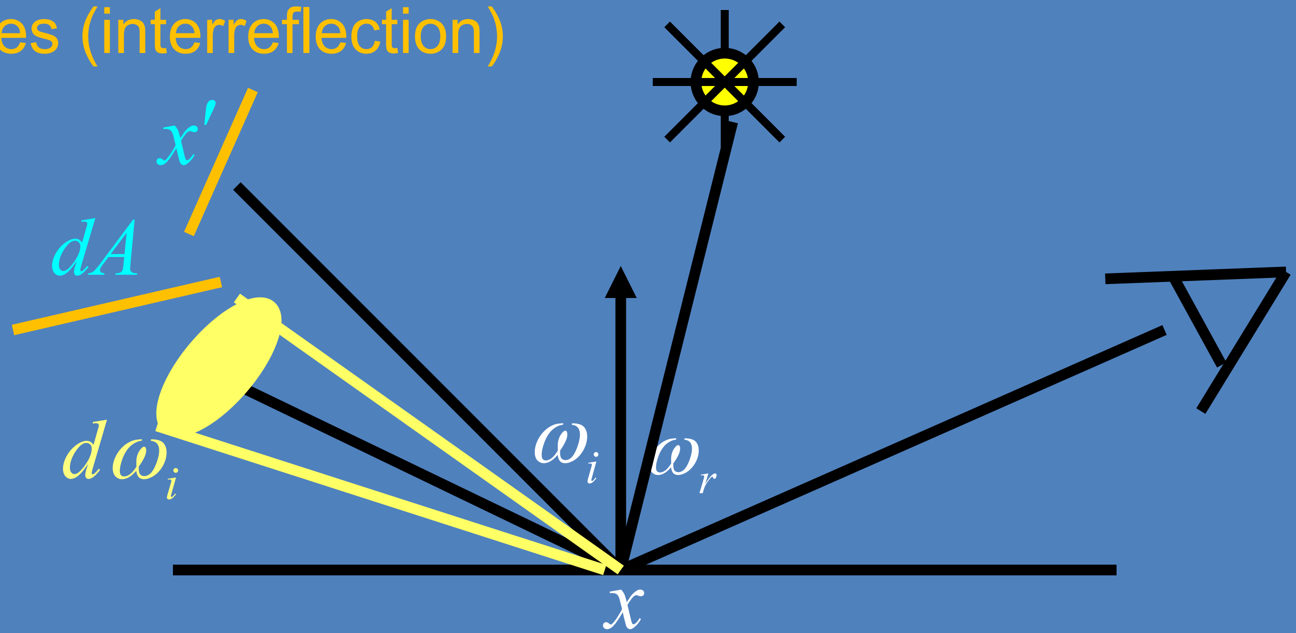
$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_i(x,\omega_i) f(x,\omega_i,\omega_r) \cos\theta_i d\omega_i$$

- Computing reflectance equation requires knowing the incoming radiance from surfaces

- …But determining incoming radiance requires knowing the reflected radiance from surfaces

Global Illumination

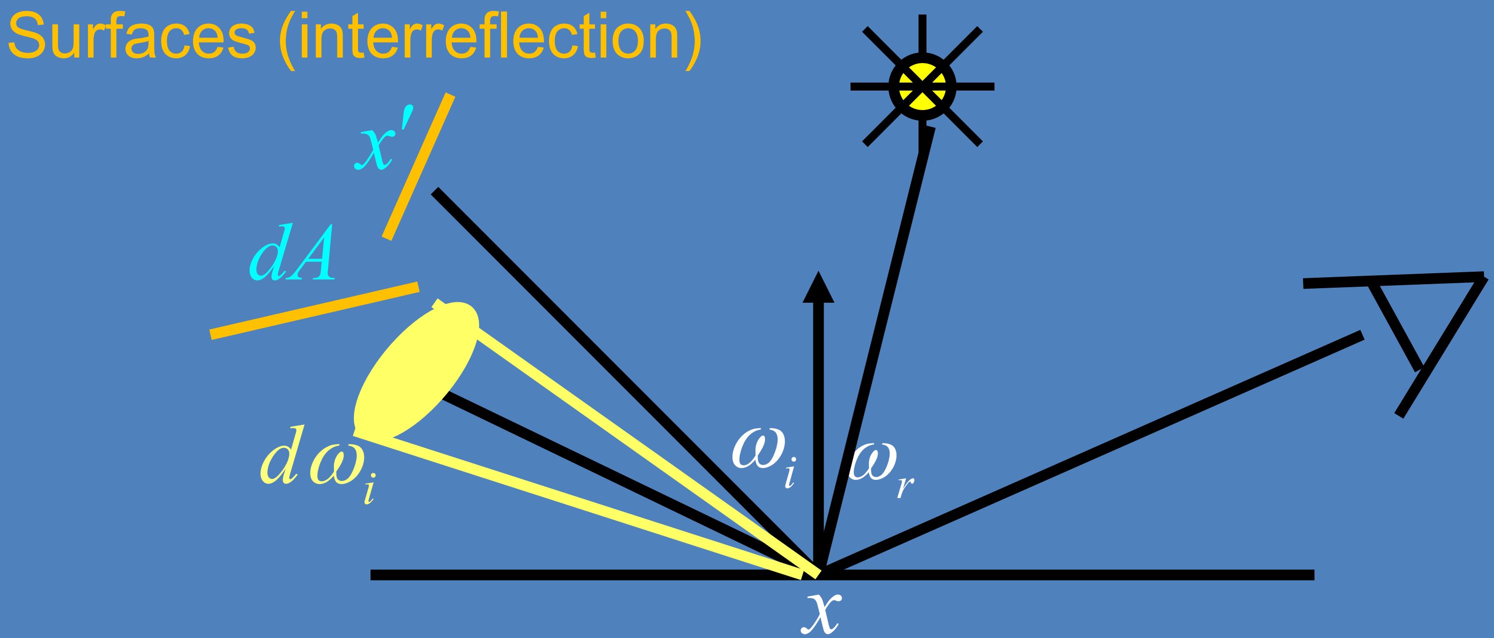
Surfaces (interreflection)



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light (from prev surface)	BRDF	Cosine of Incident angle

Rendering Equation



$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r) \cos\theta_i \, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Rendering Equation (Kajiya 1986)

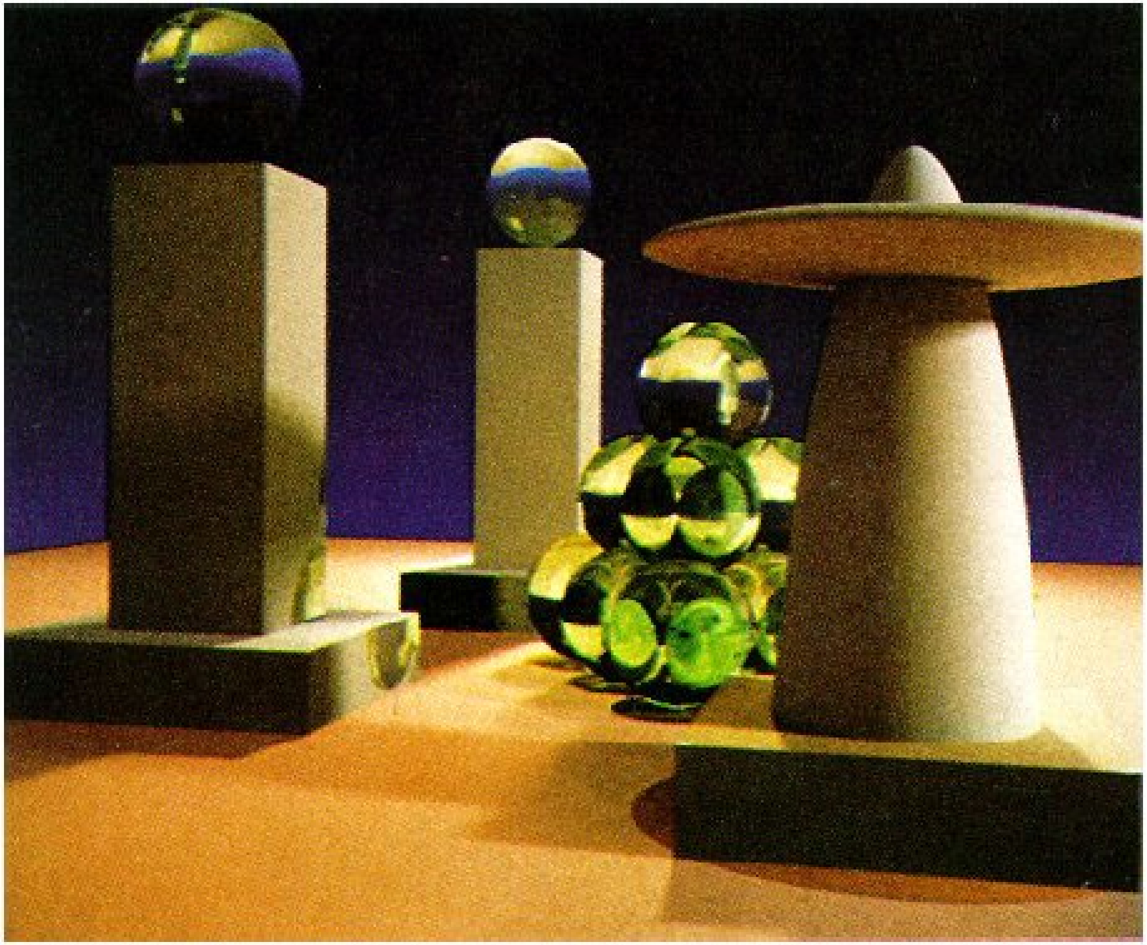


Figure 6. A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

Rendering Equation as Integral Equation

$$L_r(x,\omega_r) = L_e(x,\omega_r) + \int_{\Omega} L_r(x',-\omega_i)\, f(x,\omega_i,\omega_r)\cos\theta_i\, d\omega_i$$

Reflected Light (Output Image)	Emission	Reflected Light	BRDF	Cosine of Incident angle
UNKNOWN	KNOWN	UNKNOWN	KNOWN	KNOWN

Is a Fredholm Integral Equation of second kind [extensively studied numerically] with canonical form

$$l(u) = e(u) + \int l(v)\, K(u,v)\, dv$$

Kernel of equation

Linear Operator Equation

$$l(u) = e(u) + \int l(v) \boxed{K(u,v)dv}$$

Kernel of equation

$$L = E + KL$$

which is effectively a simple matrix equation (or system of simultaneous linear equations) where

L, E are vectors,

K is the light transport matrix (more on this later!)

Solving the Rendering Equation (=how to compute L?)

- In general, too hard for analytic solution
- But there are approximations and some nice observations…

Solving the Rendering Equation (=how to compute L?)

$$L = E + KL$$

$$IL - KL = E$$

$$(I - K)L = E$$

$$L = (I - K)^{-1} E$$

(using Binomial Theorem)

$$L = (I + K + K^2 + K^3 + \ldots)E$$

$$L = E + KE + K^2 E + K^3 E + \ldots$$

where term n corresponds to n-th bounces of light

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

(Two bounce indirect)
[Caustics, etc…]

Ray Tracing

$$L = E + KE + K^2E + K^3E + \ldots$$

Emission directly
From light sources

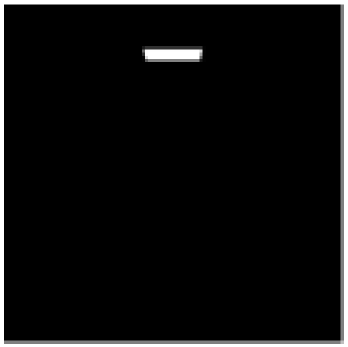
Direct Illumination
on surfaces

Global Illumination
(One bounce indirect)
[Mirrors, Refraction]

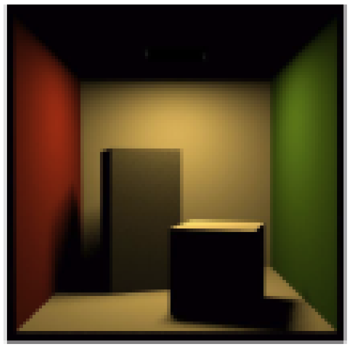
(Two bounce indirect)
[Caustics, etc…]

OpenGL
Shading

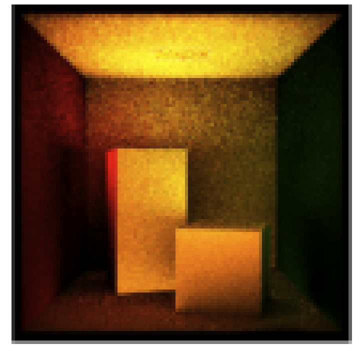
Successive Approximation



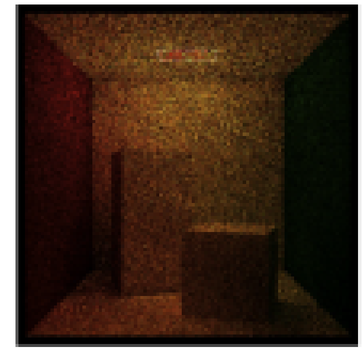
L_e



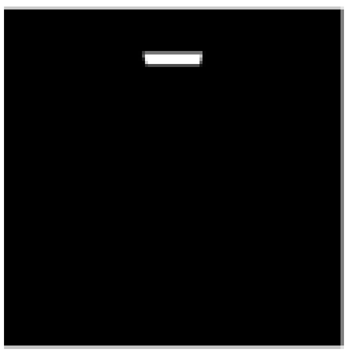
$K \circ L_e$



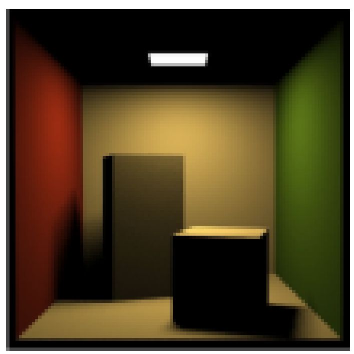
$K \circ K \circ L_e$



$K \circ K \circ K \circ L_e$

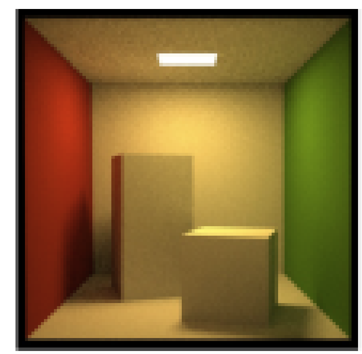


L_e



$L_e + K \circ L_e$

$L_e + \cdots K^2 \circ L_e$



$L_e + \cdots K^3 \circ L_e$

Global Illumination and Related Concepts

- Colors and Perception
  - Color models
- Example based:
  - BRDFs
- Making it faster:
  - Ambient occlusion
  - (Path tracing)
- Analytical:
  - Light Transport
  - Radiosity