Linear Algebra for Computer Graphics

CS535

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Linear Algebra

- Why do we need it?
  - Geometric operations
    - Solving “mini geometric problems”…
  - Modeling transformation
    - Move “objects” into place relative to a world origin
  - Viewing transformation
    - Move “objects” into place relative to camera
  - Perspective transformation
    - Project “objects” onto image plane

Vector, Points, Matrices

- See white/black board…

Points, Vectors, and more

- A point $\mathbf{p} = (p_x, p_y, p_z)$ defines a location
- A vector $\mathbf{v} = (v_x, v_y, v_z)$ defines a direction
- A ray is a point with a direction: $\boldsymbol{r}(t) = \boldsymbol{p} + \boldsymbol{v}t$
- A line is an infinite line (e.g., $y = mx + b$)
- A line segment is a piece of line, say between a point a and a point b (e.g., $\boldsymbol{l}(t) = \boldsymbol{a}(1 - t) + \boldsymbol{b}t$

Basic geometric computations

- Distance of point to line? (2D or 3D)
- Is a point in a plane? (3D)
- Is a point inside a triangle? (2D)
- What is distance between to lines (3D)?

Transformations

- Most popular transformations in graphics
  - Translation
  - Rotation
  - Scale
  - Projection
- In order to use a single matrix for all, we use homogeneous coordinates…

Homogenous Coordinates

- A 4x4 matrix embeds a rotation, translation, as well as other matrix transformations including projection

- A vector is $[v_x \; v_y \; v_z \; 0]^T$

- A point is $[p_x \; p_y \; p_z \; 1]^T$

3D Transformations

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror over X axis

3D Transformations

Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 & 0 \\ \sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} \cos\Theta & 0 & -\sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

and many more…

Rotate around X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & -\sin\Theta & 0 \\ 0 & \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$