Surface Triangulation

CS535

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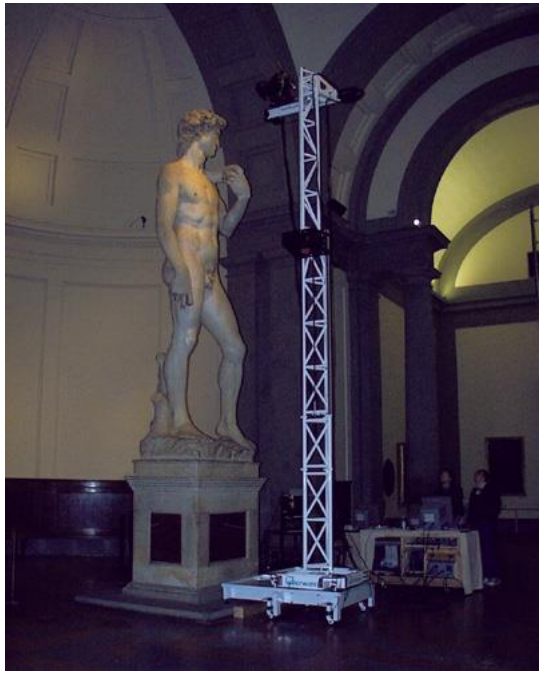
[help from Michael Kazhdan @ JHU and
Ioannis Stamos @ CUNY]

Motivation

- Time of flight
- Structured light
- Stereo images
- Shape from shading
- Etc.

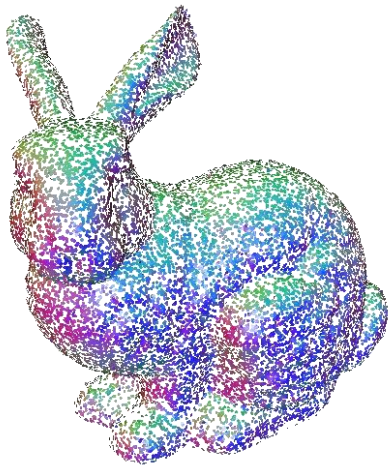
http://graphics.stanford.edu/projects/mich/

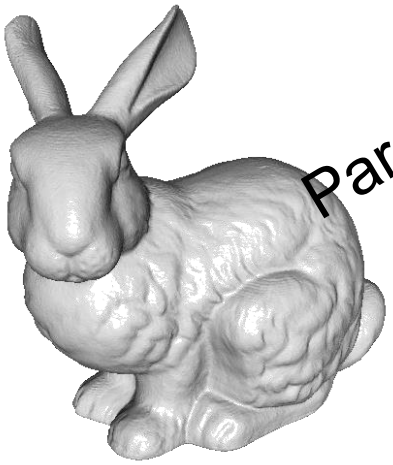


Motivation

Surface reconstruction

Geometry processing



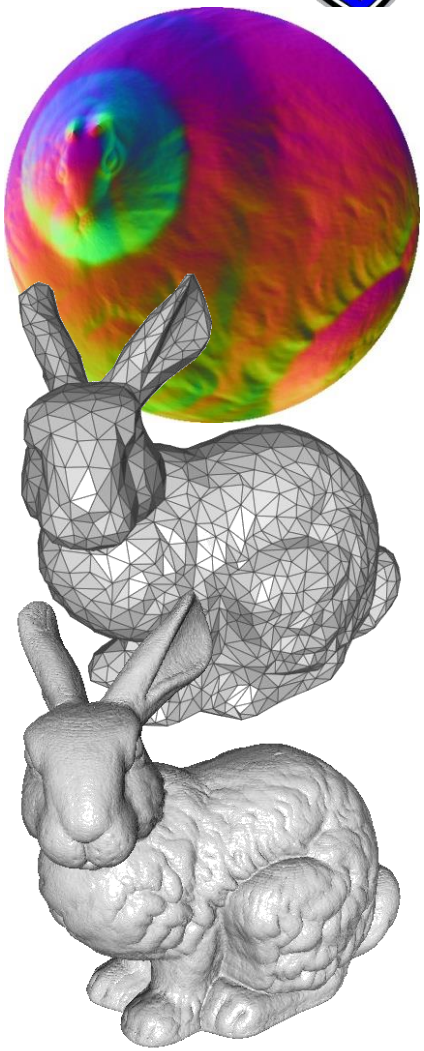


Parameterization

Decimation

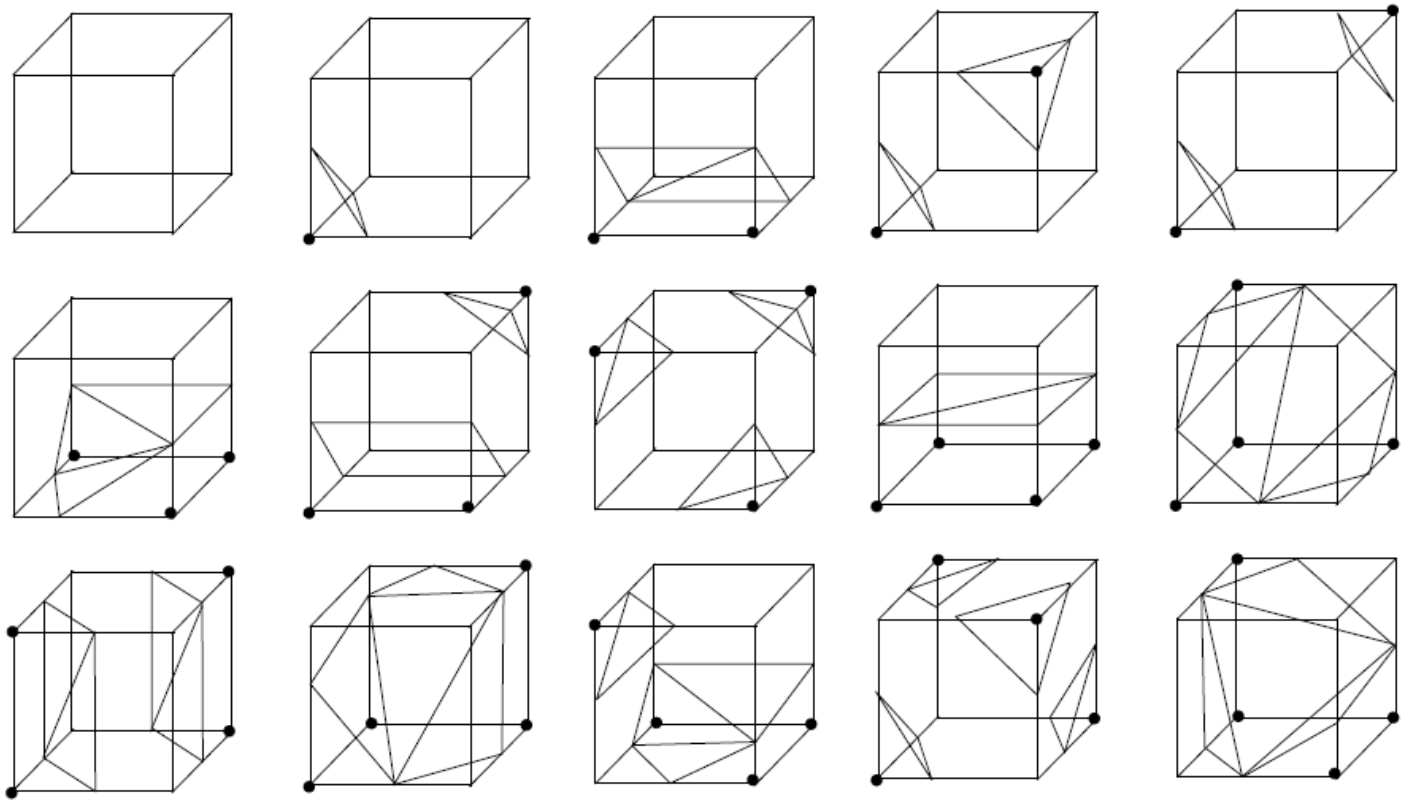
Filtering

etc.



One Option: Marching Cubes

If the function is sampled on a regular voxel grid, we can independently triangulate each voxel.

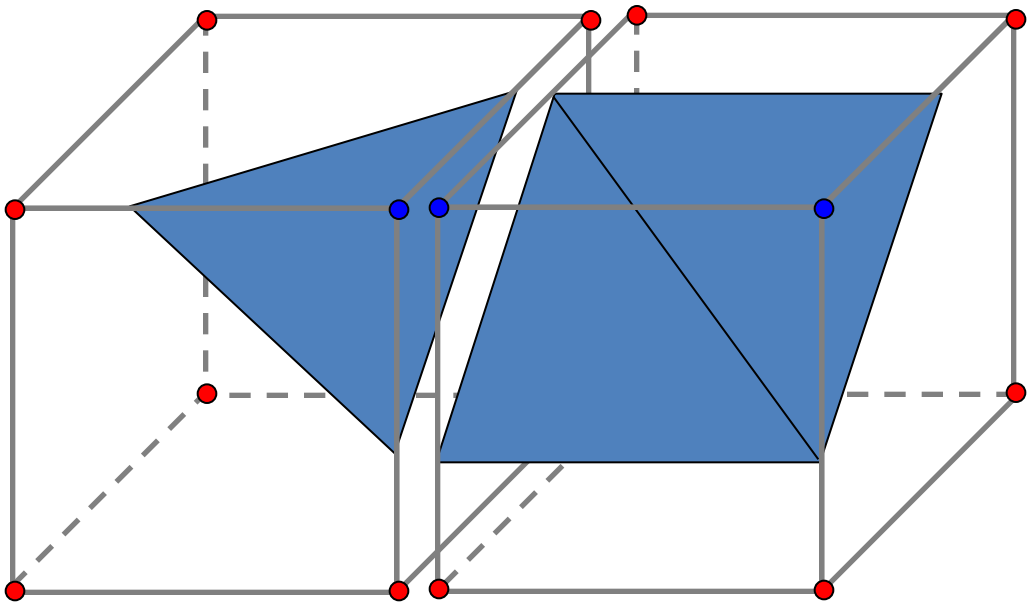


One Option: Marching Cubes

See blackboard…

Marching Cubes

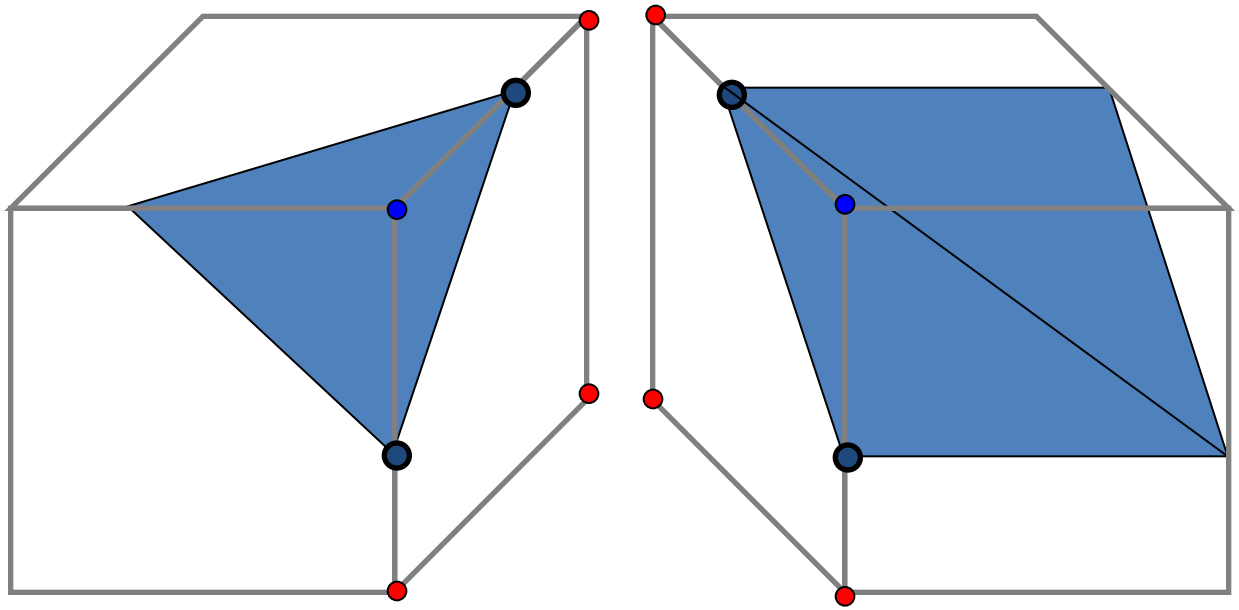
Although each of the voxels is triangulated independently, the mesh is usually water-tight.



Marching Cubes

Iso-vertices on an edge are only determined by the values on the corner of the edge:

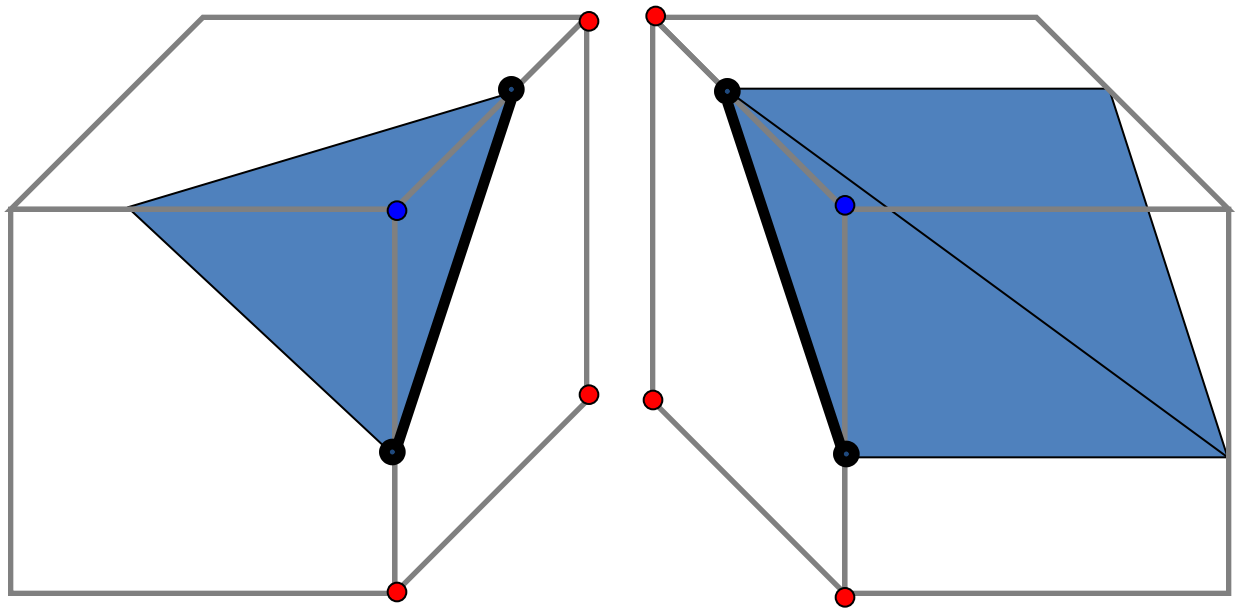
$\Rightarrow$ Iso-vertices are consistent across voxels.



Marching Cubes

Iso-edges on a face are only determined by the values on the face:

$\Rightarrow$ Each iso-edge is shared by two triangles so the mesh is water-tight.

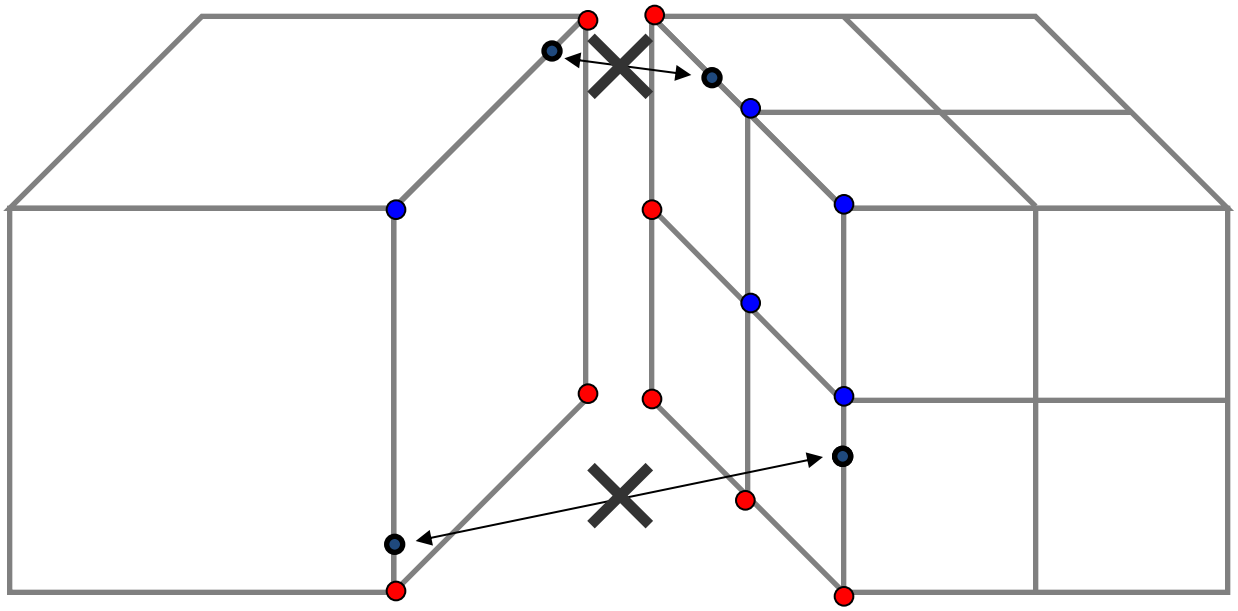


Challenges

Extracting a surface by independently triangulating the leaf octants, depth-disparities can cause:

- Inconsistent extrapolation to edges

  $\Rightarrow$ Inconsistent iso-vertex positions

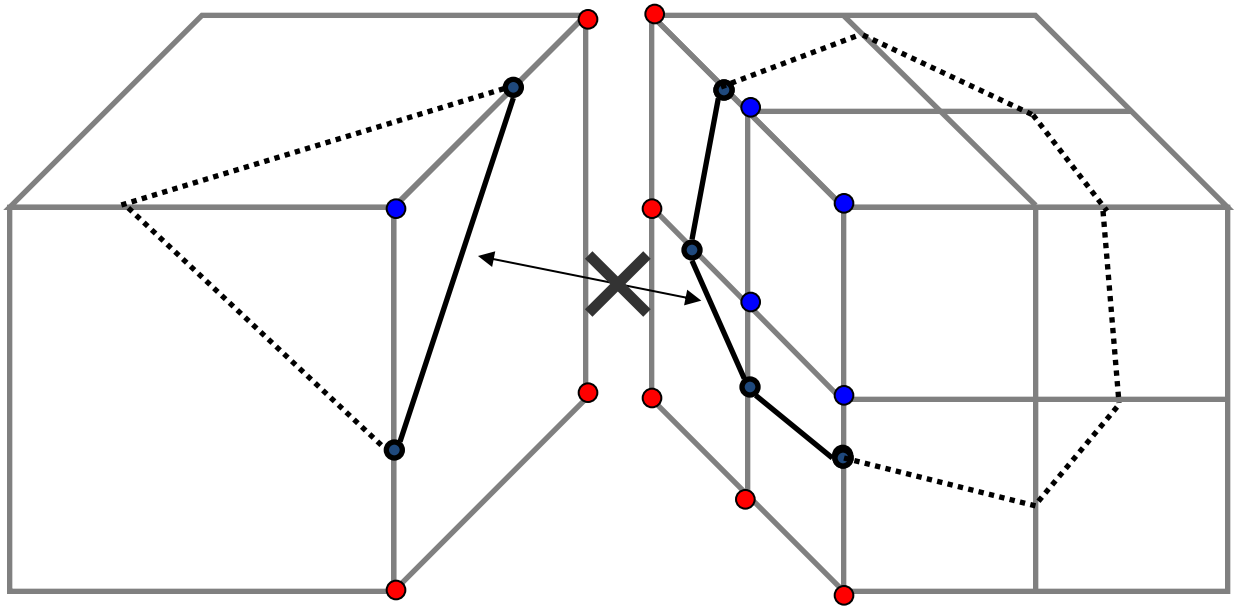


Challenges

Extracting a surface by independently triangulating the leaf octants, depth-disparities can cause:

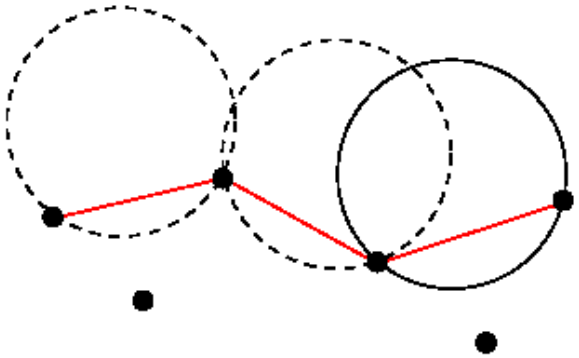
- Inconsistent extrapolation to faces

  ⇒ Inconsistent iso-edges



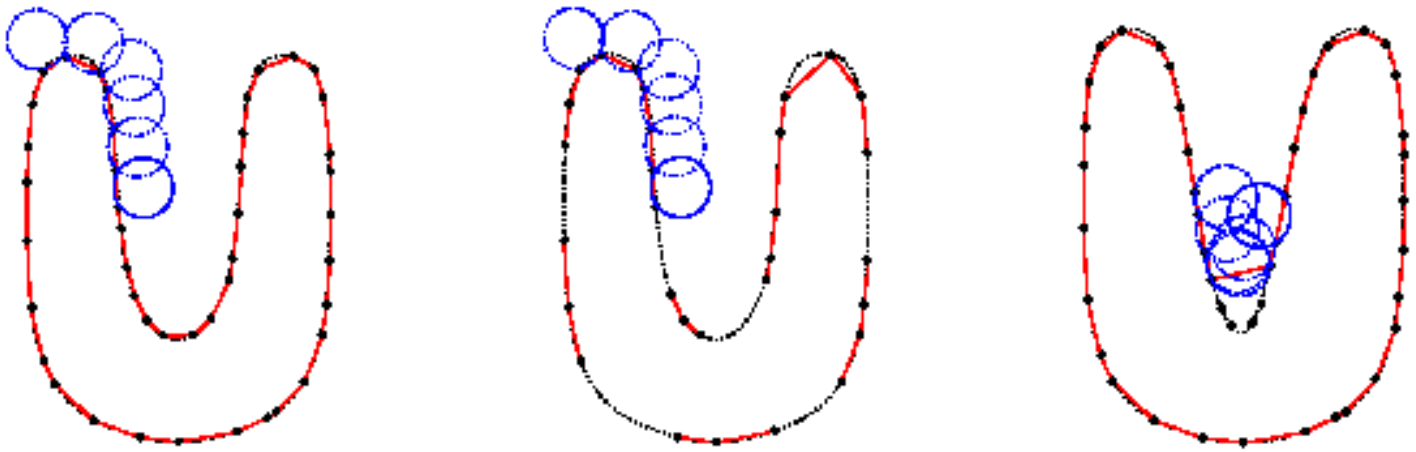
Ball-pivoting

Bernardini et al., IBM



Fixed-radius ball “rolling” over points selects subset of alpha-shape.

Pivoting in 2D



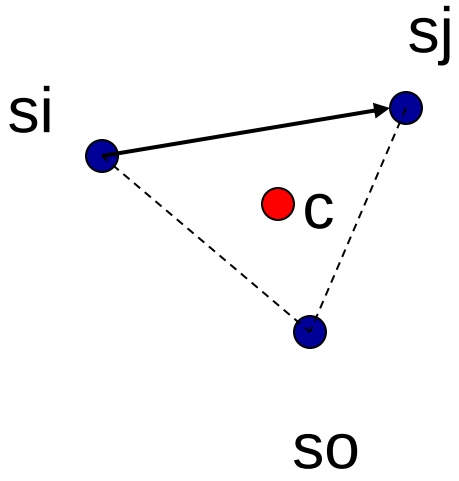
(a) *Circle of radius ρ pivots from point to point, connecting them with edges.*

(b) *When sampling density is low, some of the edges will not be created, leaving holes.*

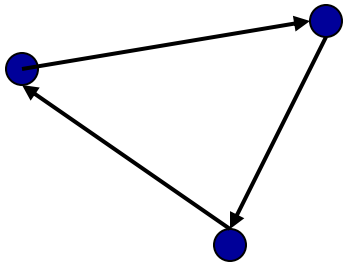
- *(c) When the curvature of the manifold is larger than 1/ρ, some of the points will not be reached by the pivoting ball, and features will be missed.*

The algorithm [Edge representation]

- Edge (si, sj)
  - Opposite point so, center of empty ball c
  - Edge: “Active”, “Boundary”, or “Frozen”



Pivoting example



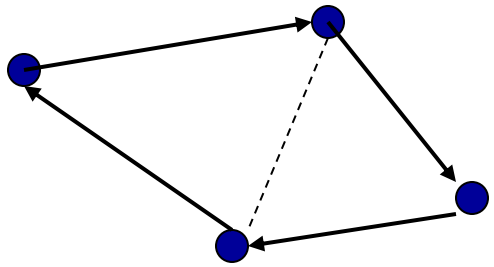
Initial seed triangle:

Empty ball of radius ρ passes through the three points

Active edge

Point on front

Pivoting example

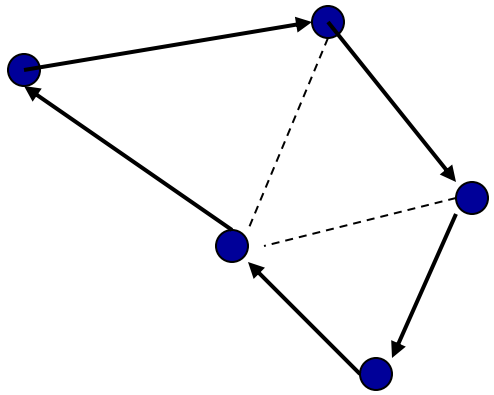


Ball pivoting around active edge

Active edge

Point on front

Pivoting example

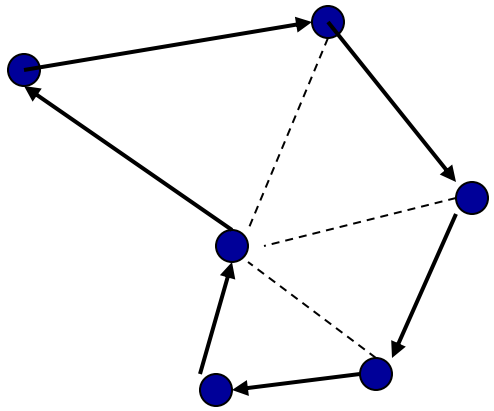


Ball pivoting around active edge

Active edge

Point on front

Pivoting example

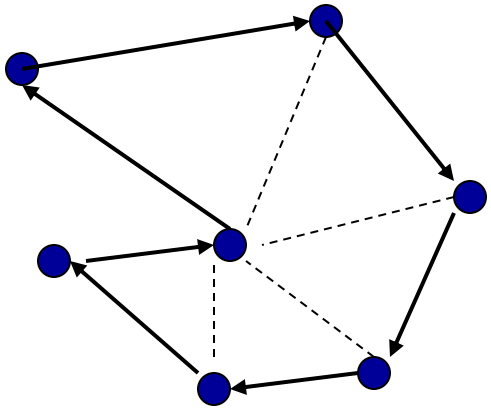


Ball pivoting around active edge

Active edge

Point on front

Pivoting example

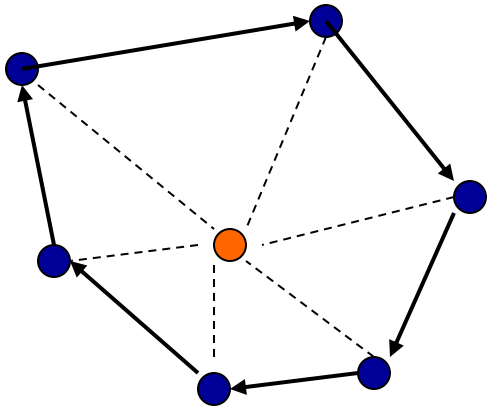


Ball pivoting around active edge

Active edge

Point on front

Pivoting example



Ball pivoting around active edge

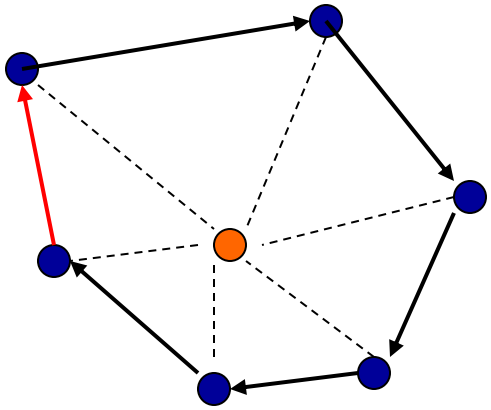
Active edge

Point on front

Internal point

Pivoting example

Boundary edge



Ball pivoting around active edge

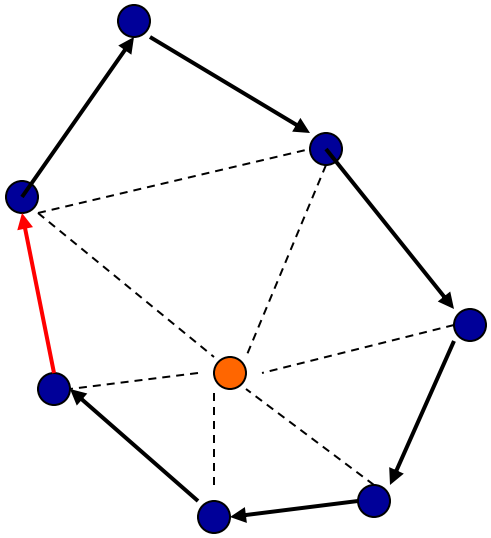
No pivot found

Active edge

- Point on front
- Internal point

Pivoting example

Boundary edge



Ball pivoting around active edge

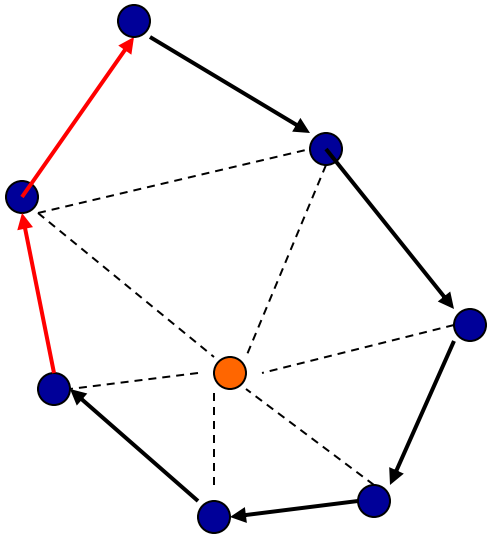
Active edge

Point on front

Internal point

Pivoting example

Boundary edge



Ball pivoting around active edge

No pivot found

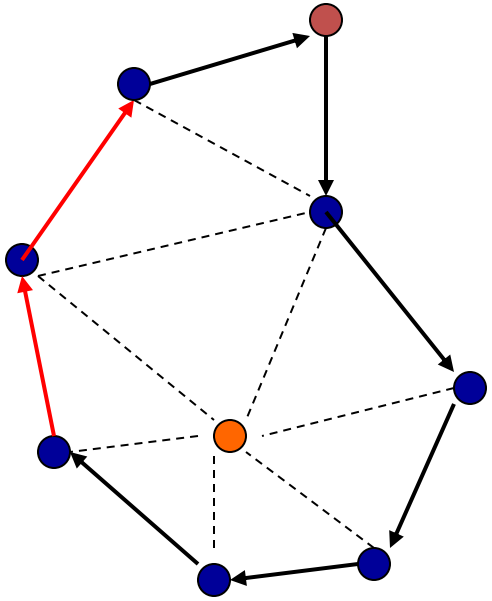
Active edge

Point on front

Internal point

Pivoting example

Boundary edge



Ball pivoting around active edge

Active edge

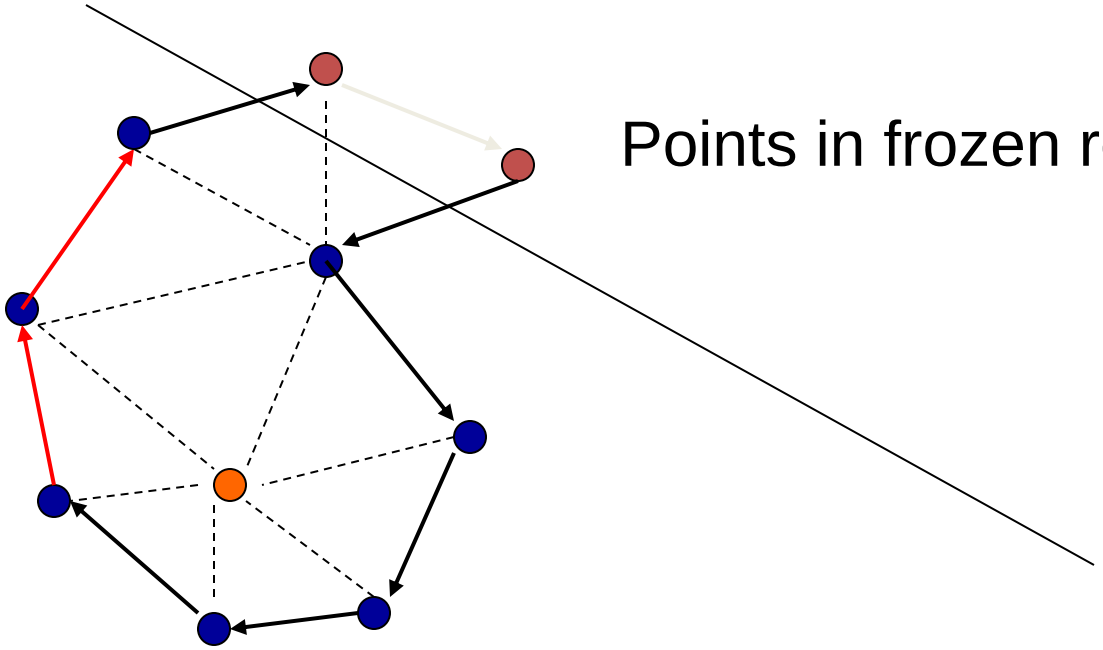
Point on front

Internal point

Pivoting example

Boundary edge

Frozen edge



Points in frozen region

Ball pivoting around active edge

Active edge

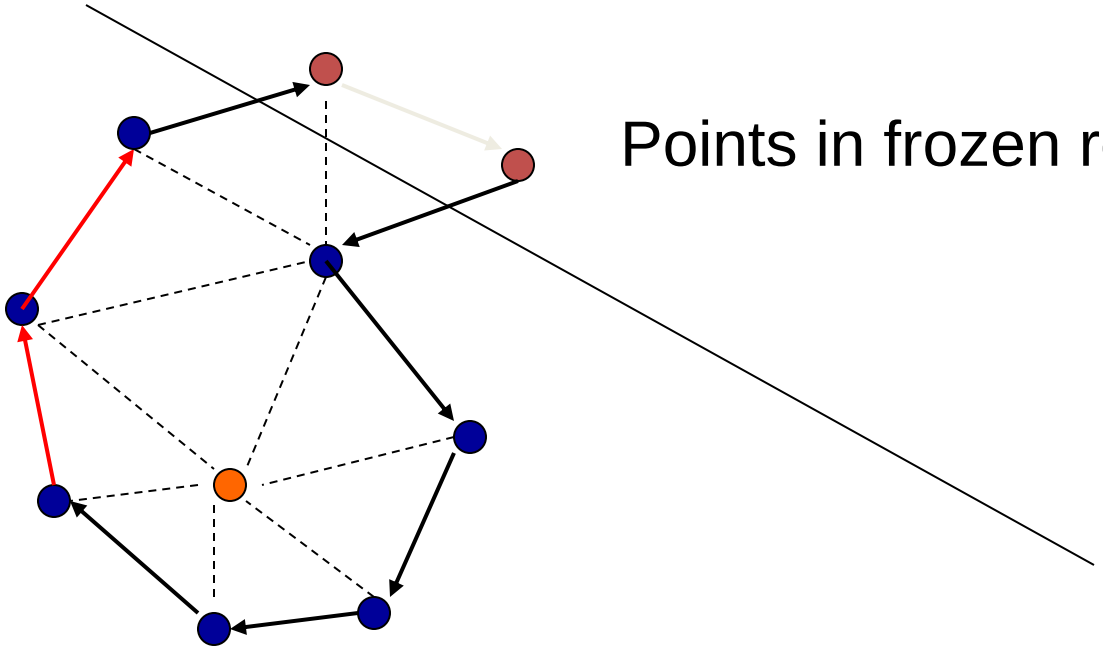
Point on front

Internal point

Pivoting example

Boundary edge

Frozen edge



Points in frozen region

Ball pivoting around active edge

Active edge

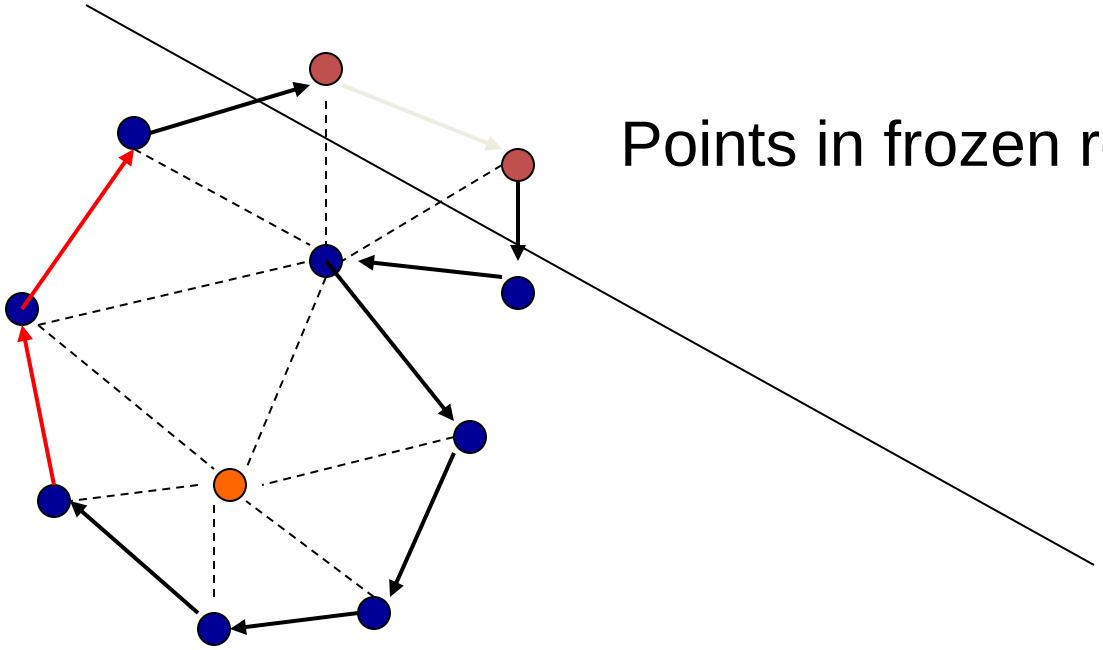
Point on front

Internal point

Pivoting example

Boundary edge

Frozen edge



Points in frozen region

Ball pivoting around active edge

Active edge

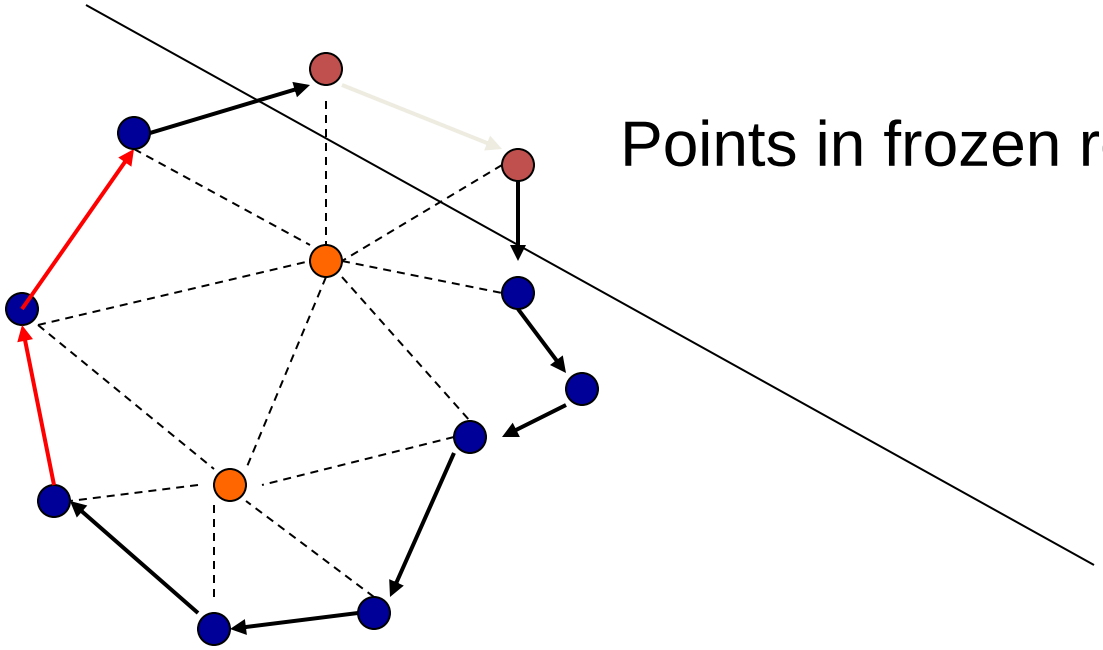
Point on front

Internal point

Pivoting example

Boundary edge

Frozen edge



Points in frozen region

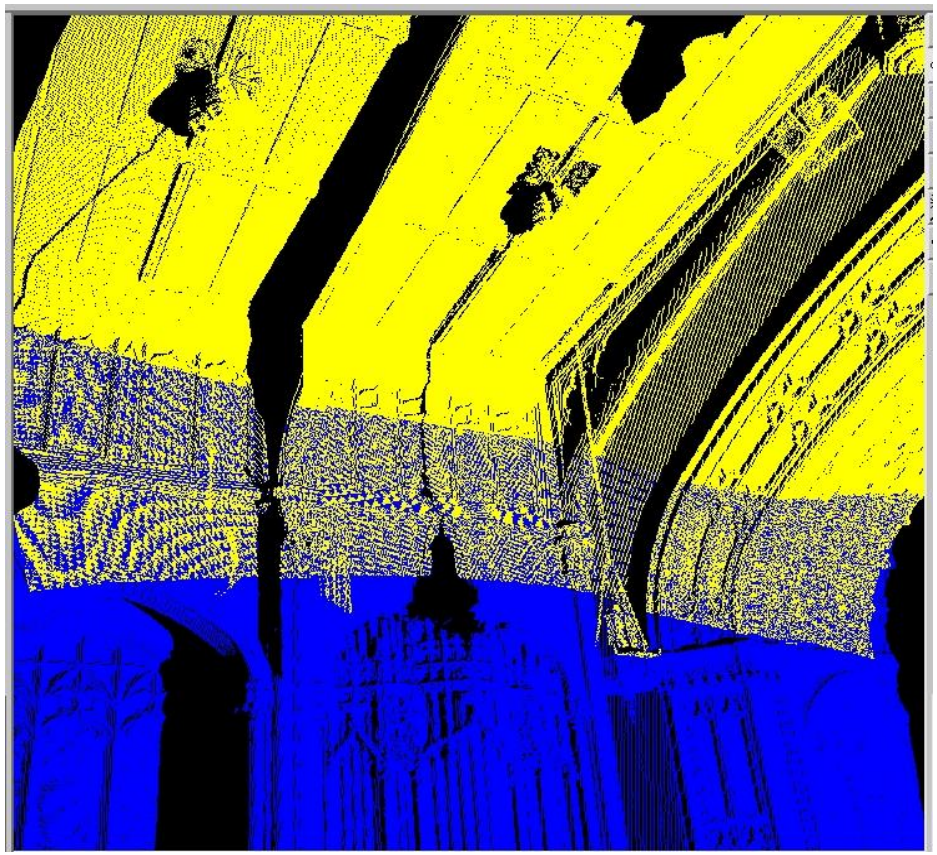
Ball pivoting around active edge

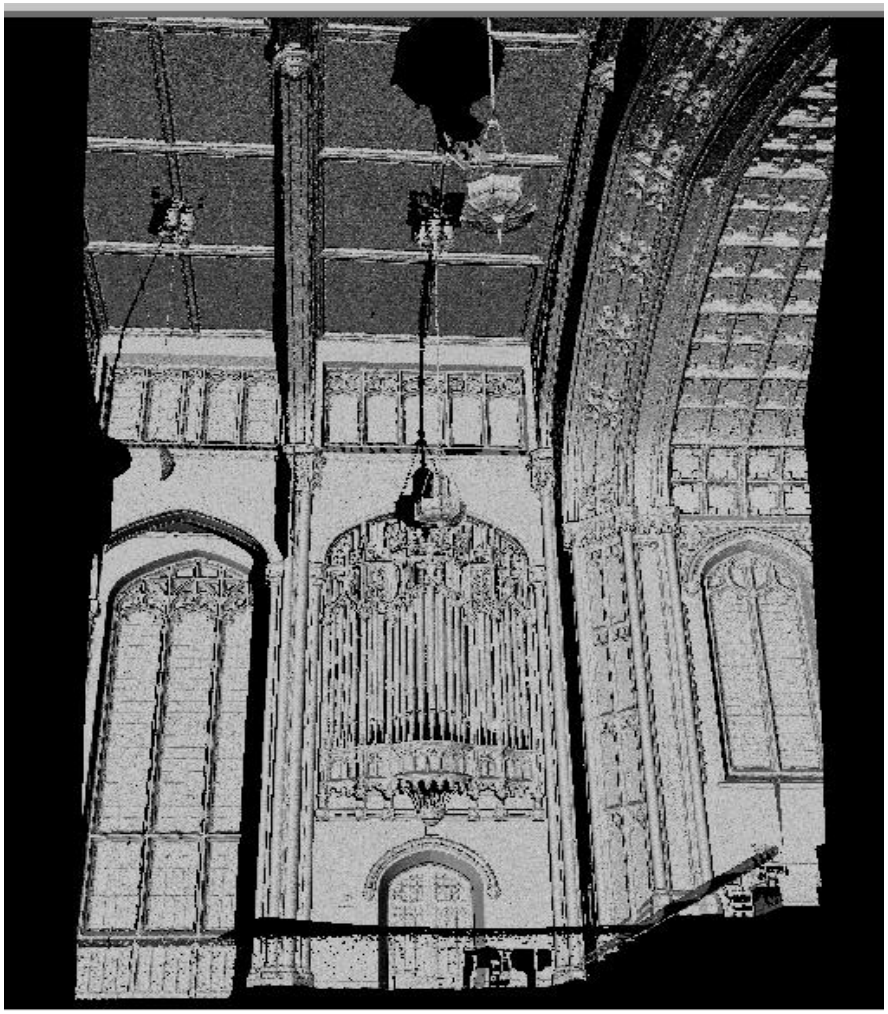
Active edge

Point on front

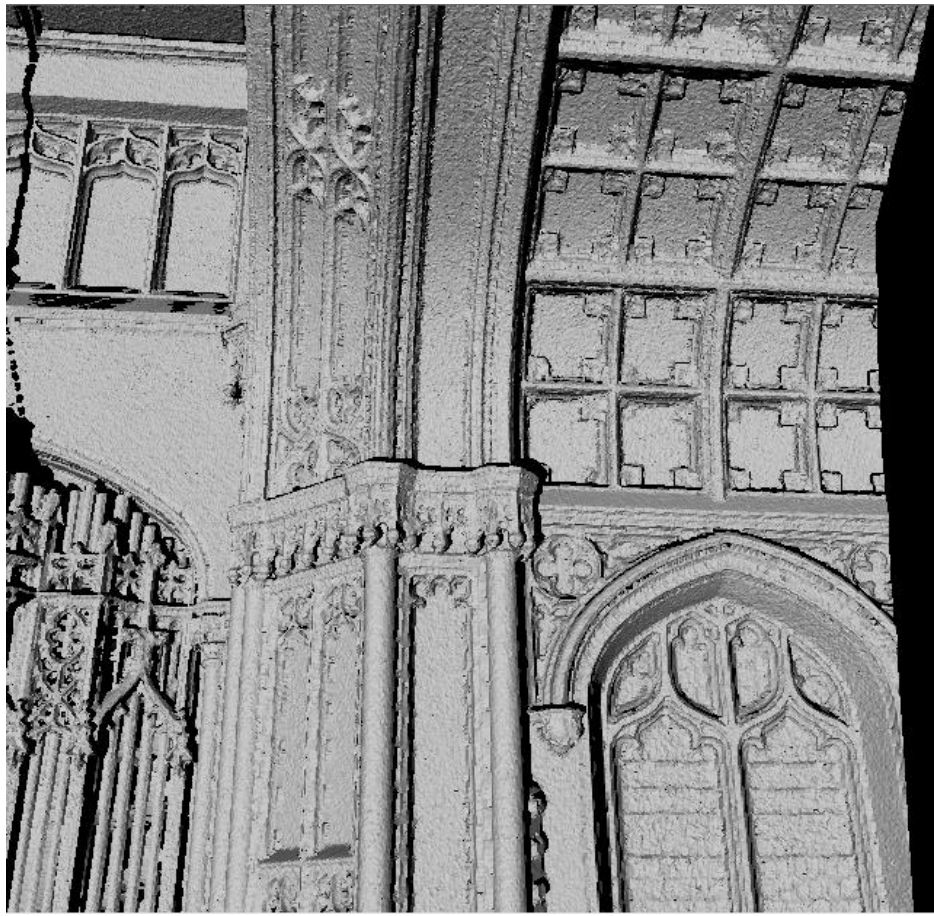
Internal point

Ball Pivoting Algorithm





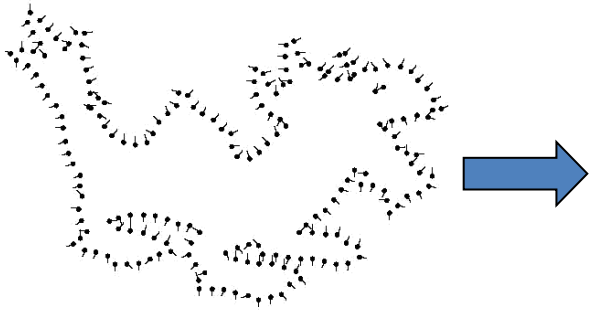
Ball Pivoting Algorithm



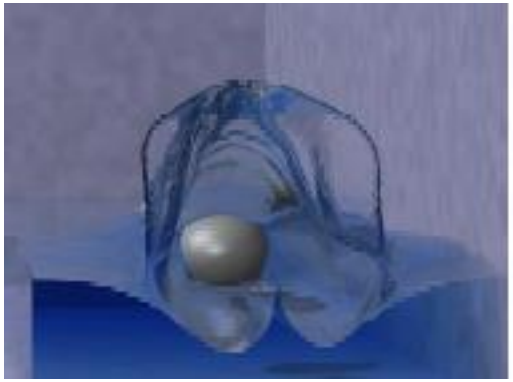
Implicit Representation

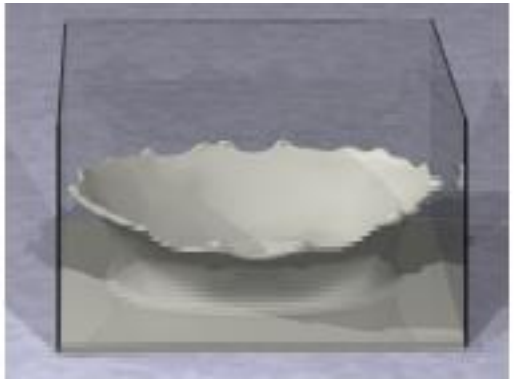
Another option is representing a 3D model by an implicit function for:

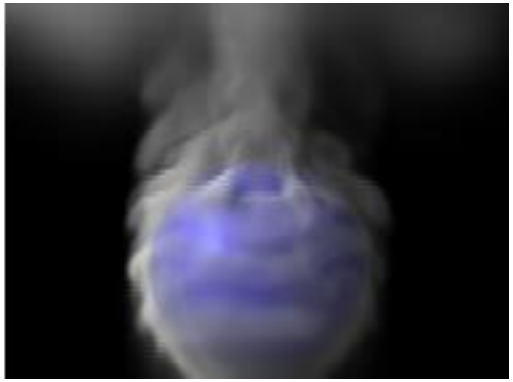
- Reconstruction
- Fluid Dynamics
- 3D Texturing



Kazhdan 2005







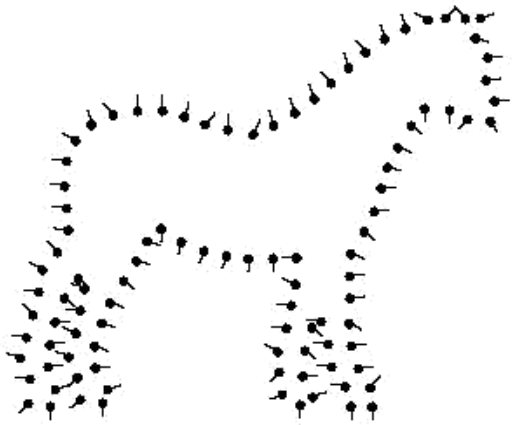
Losasso *et al.* 2004

Implicit Function Fitting

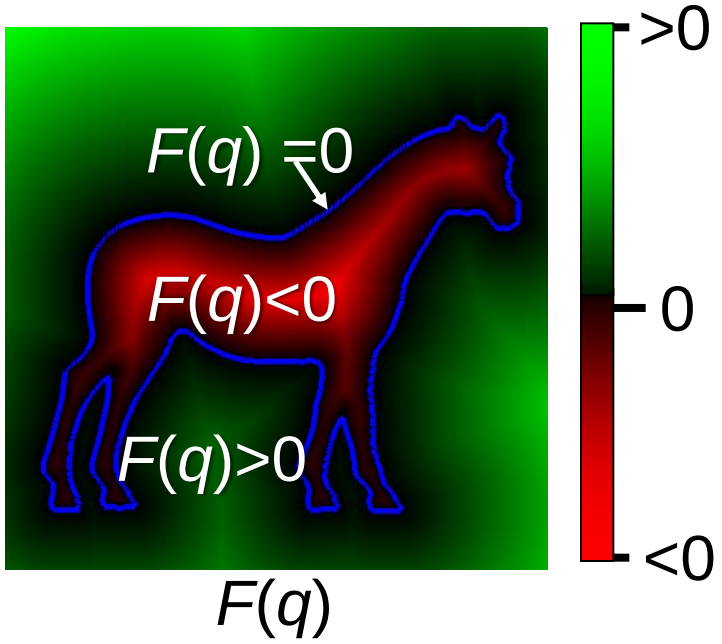
Given point samples:

– Define a function with value zero at the points.

– Extract the zero isosurface.



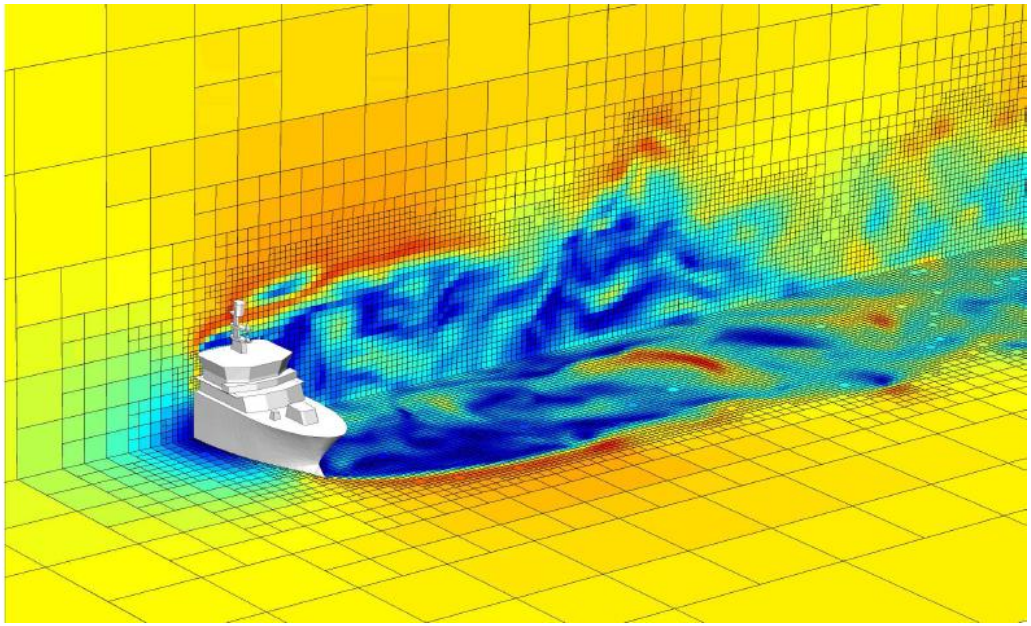
Sample points



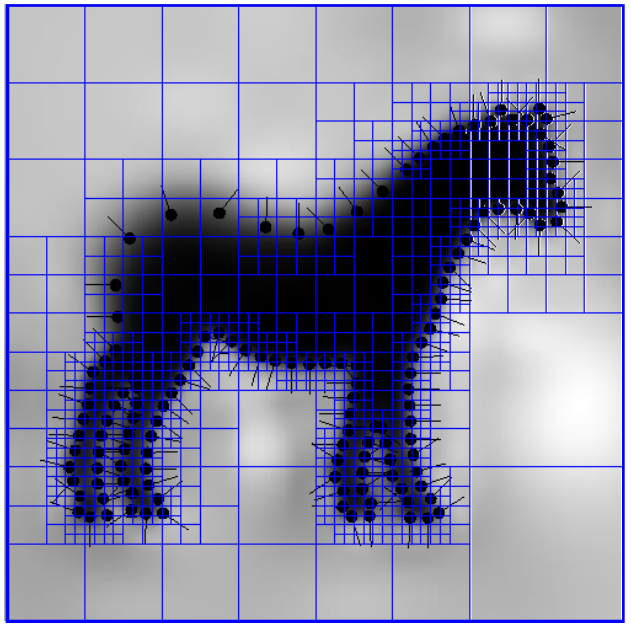
F(q)

Octree Representation

An option is to sample a function on an adaptive grid



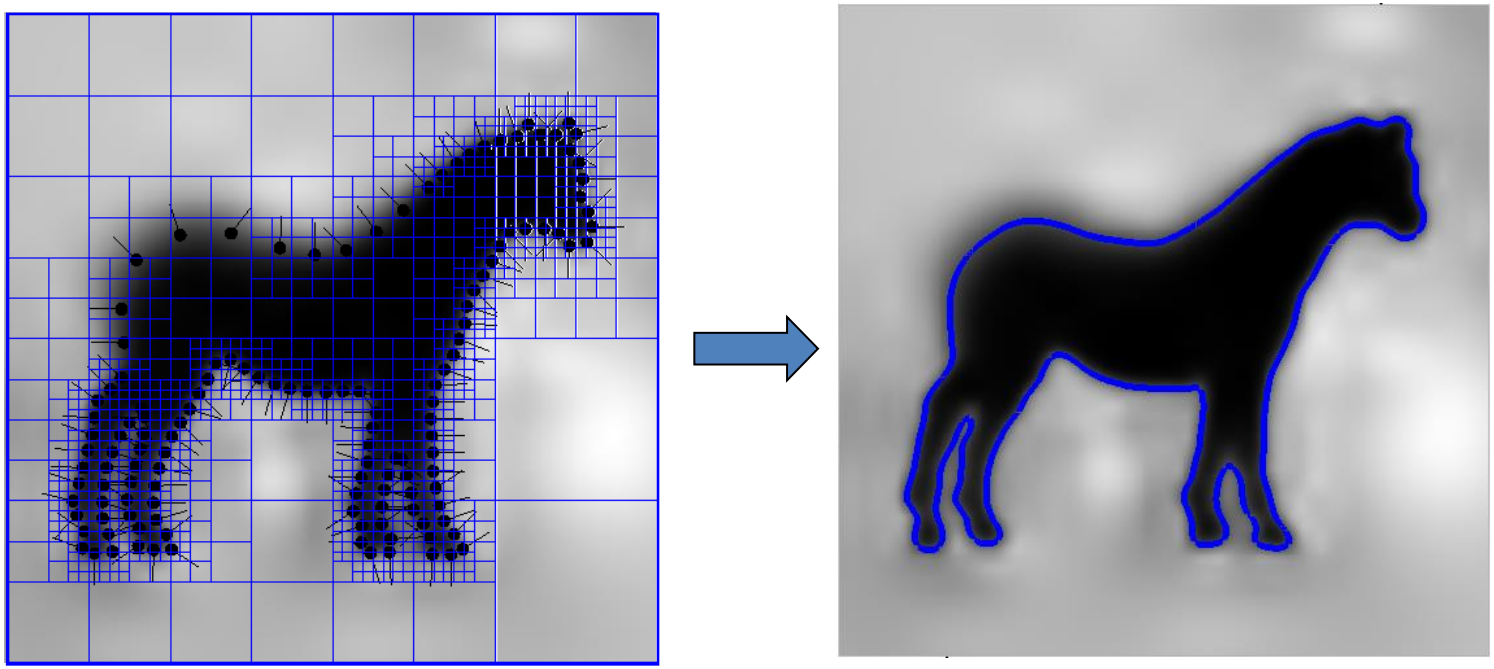
Popinet 2003



Kazhdan *et al.* 2006

Octree Extraction

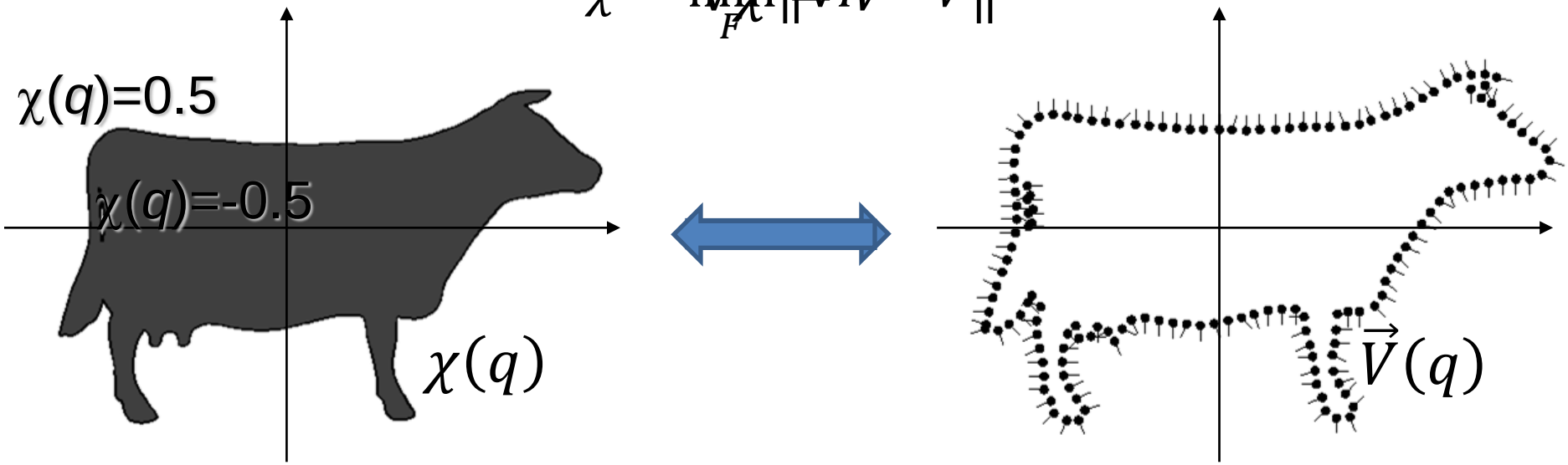
From the sampled function, we would like to extract an implicit surface.



Poisson Surface Reconstruction

- Oriented points $\approx$ samples of indicator gradient.
- Fit a scalar field to the gradients.

$$\chi = \min_F \left\| \nabla F - \vec{V} \right\|^2$$

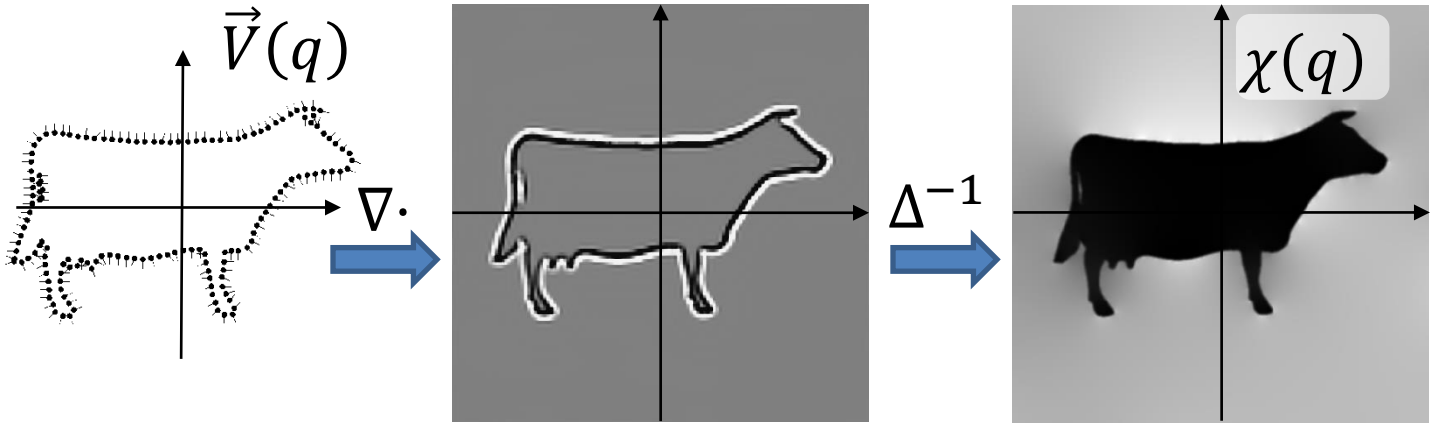


Poisson Surface Reconstruction

- But in general $\vec{V}$ is not integrable (e.g., is a set of samples of an unknown function)
- So we apply a divergence operator so that can solve in best least squares way…

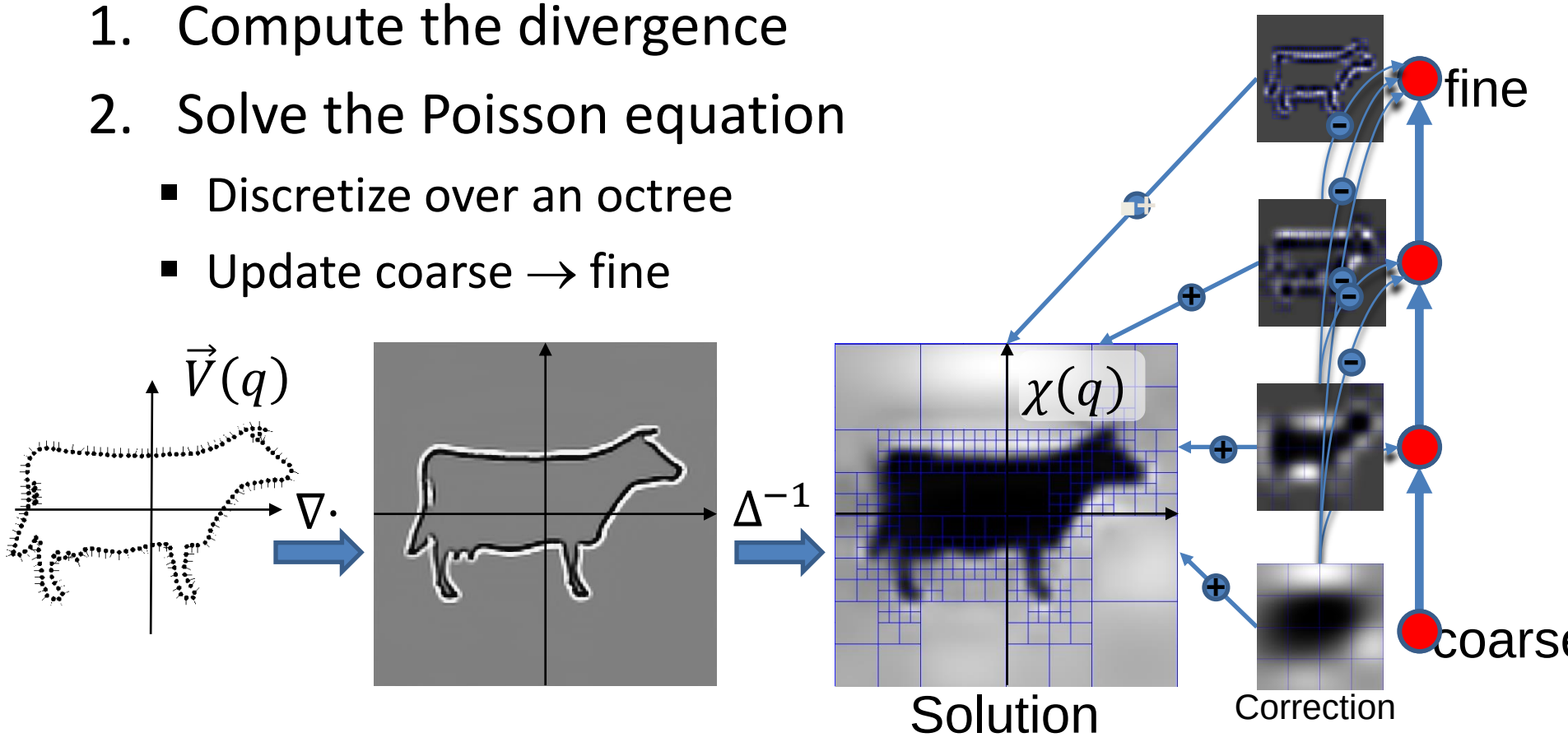
Poisson Surface Reconstruction

1. Compute the divergence
2. Solve the Poisson equation $\Delta\chi = \nabla \vec{V}$
   or written as $\nabla^2\chi = \nabla \vec{V}$



Poisson Surface Reconstruction

1. Compute the divergence
2. Solve the Poisson equation
   - Discretize over an octree
   - Update coarse → fine



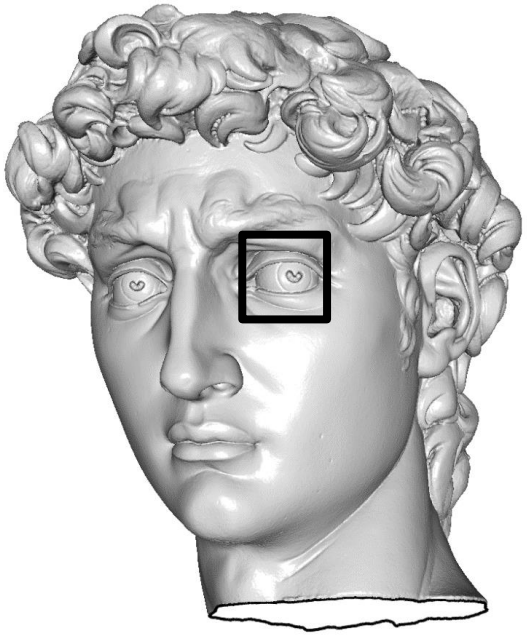
Poisson Surface Reconstruction

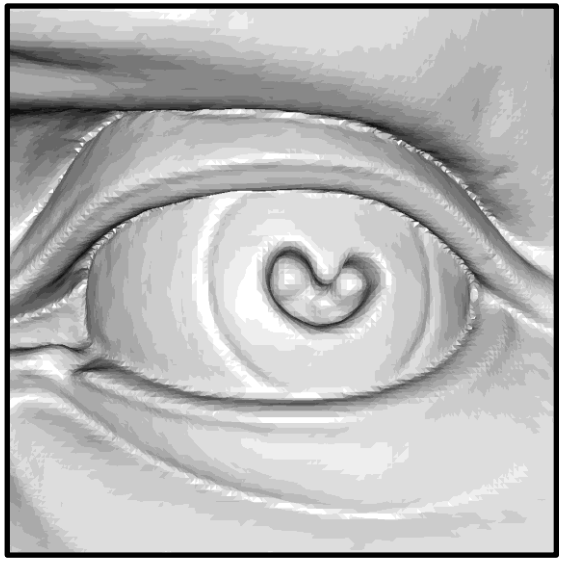
Properties:

- ✓ Supports noisy, non-uniform data
- ✗ Over-smoothes
- ✗ Solver time is super-linear

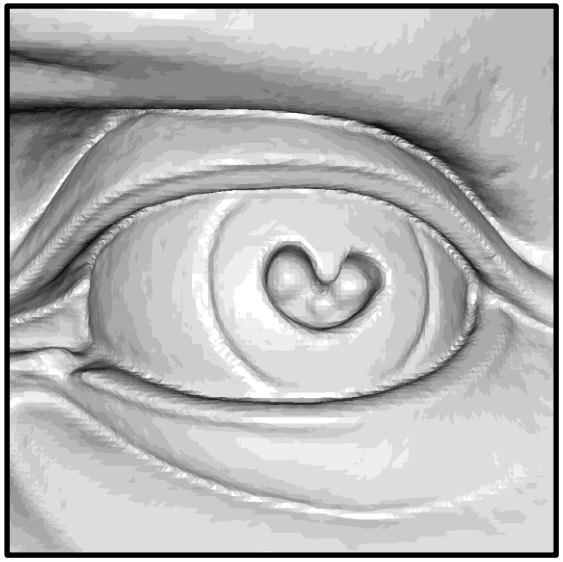
Screened Poisson Reconstruction

- Higher fidelity – at same triangle count
- Faster – solver time is linear





Poisson



Screened Poisson