(Geometric) Camera Calibration

CS635

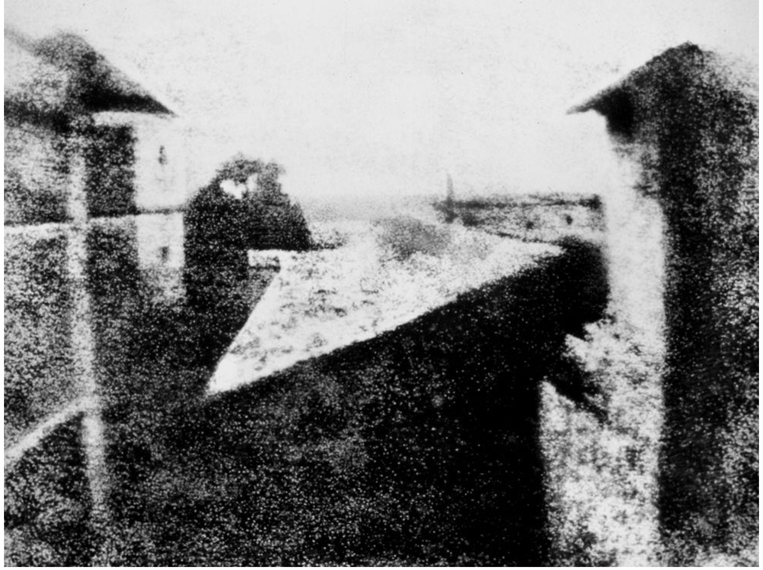
Daniel G. Aliaga
Department of Computer Science
Purdue University

Camera Calibration

- **Digital Cameras**
- Perspective Projection
- Aberrations
- Calibration

Cameras

- First photograph due to Nicéphore Niépce (1826):
  - Heliography: pewter covered with “Bitumen of Judea” (derived from crude oil) and then areas exposed to sunlight resisted dissolution in oil of lavender and petroleum



Digital Camera vs. Film Camera

- CCD: Charge-Coupled Device
- CMOS: Complementary Metal-oxide Semiconductor
  - (better for low light)

- CCD/CMOS:
  - Image plane is a CCD/CMOS array instead of film
  - Device is typically ¼ or ½ inch in size
  - What is sensitivity of film vs CCD/CMOS?
    - Approximately up to 40MP on consumer 35mm cameras
    - **But depends on focus, light and numerous other conditions**

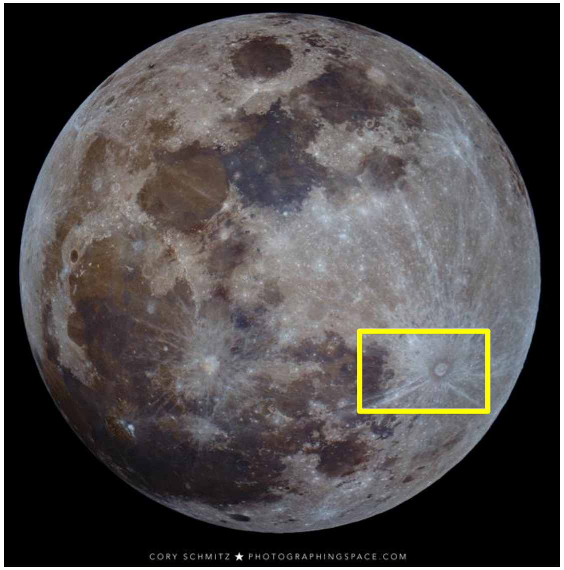
Digital Camera

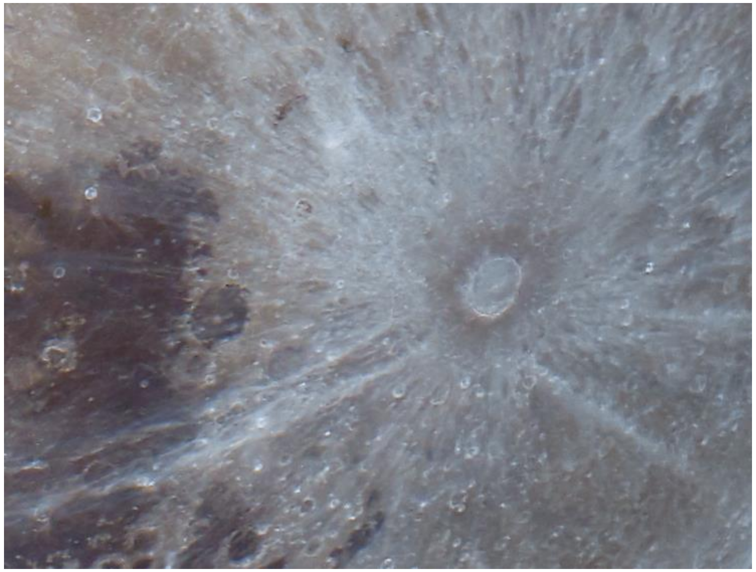
- Resolution
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)

0.25 MP

Digital Camera

- Resolution
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)
-





100 MP

(astrophotography)

Digital Camera

- Resolution
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)

100 MP

Digital Camera

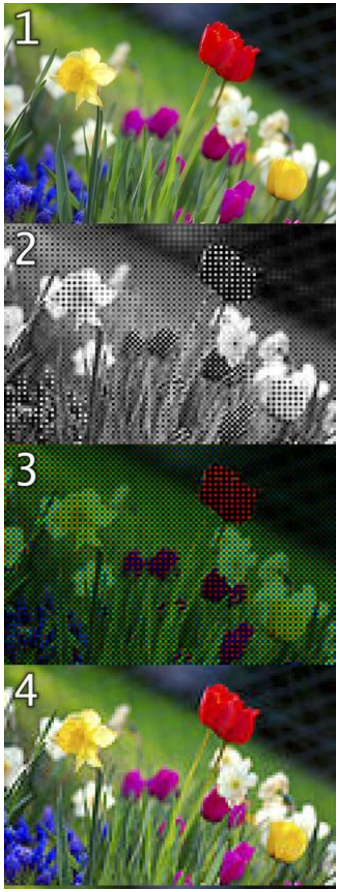
- Resolution
  - Lenses and light play a large role.
  - But, CCD/CMOS have various resolutions (e.g., 640x480, 100MP, 250GP)

272 GP

Used 12,000 images….Calibration is crucial!!!!

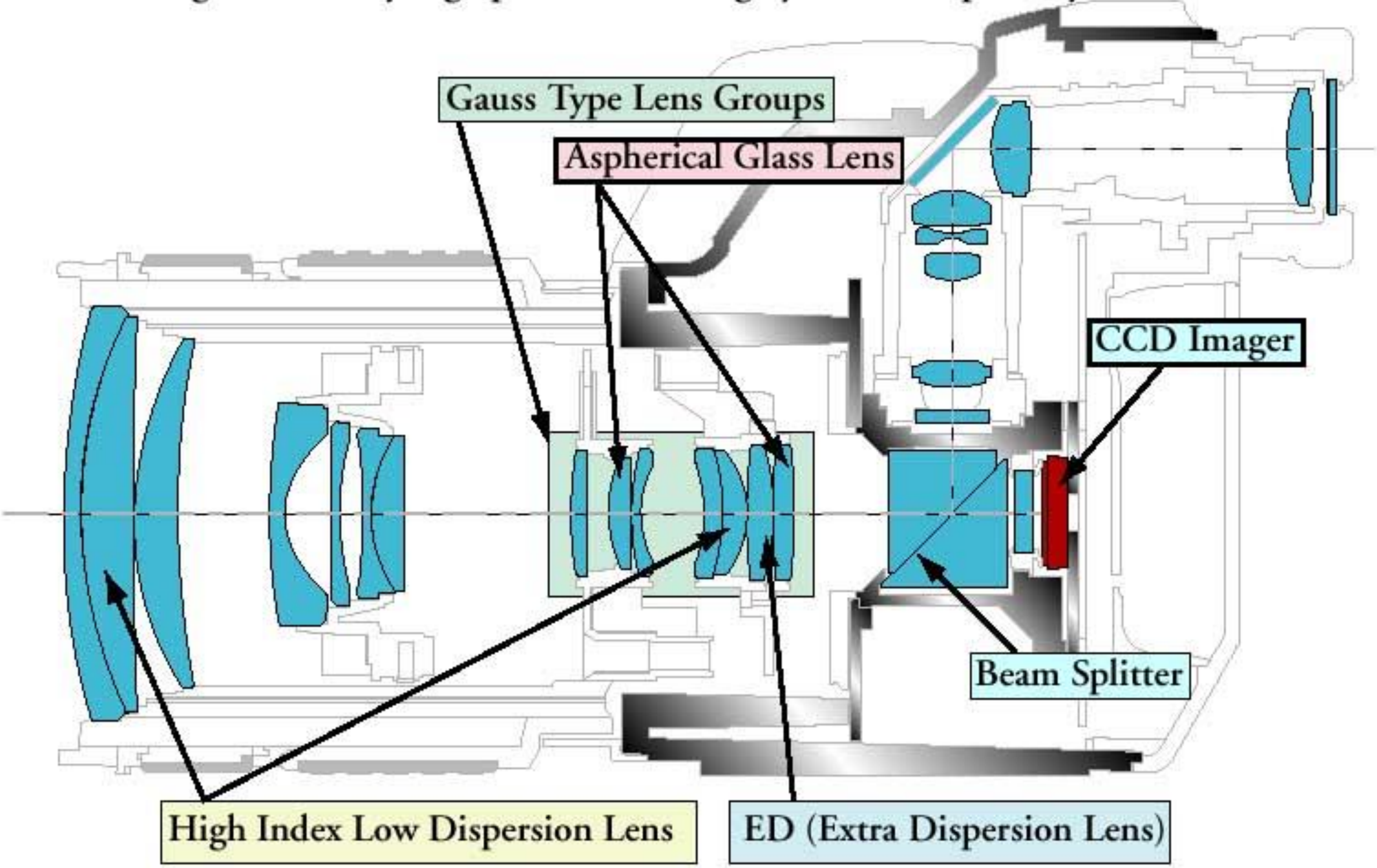
Digital Camera

- Number of CCD/CMOS
  - 1: captures RGB simultaneously, reducing the resolution by 1/3 (kinda) using Bayer color filter (e.g., RGGB)
  - 3: typical RGB
  - 200+ bands: hyperspectral cameras using multiple CCDs and filters
- Video
  - Digital cameras have a maximum “frame rate”, usually determined by the hardware and bandwidth
    - Interlaced: only “half” of the horizontal lines of pixels are present in each frame
    - Progressive scan: each frame has a full-set of pixels
  - Shutter:
    - Rolling: output line-by-line (e.g., CMOS, low-light)
    - Global: output all pixels at once



Digital Camera (D-SLR)

Exclusively developed 14 element, 11 group 4 X zoom for the E-10, designed as a very high precision and highly accurate optical system.

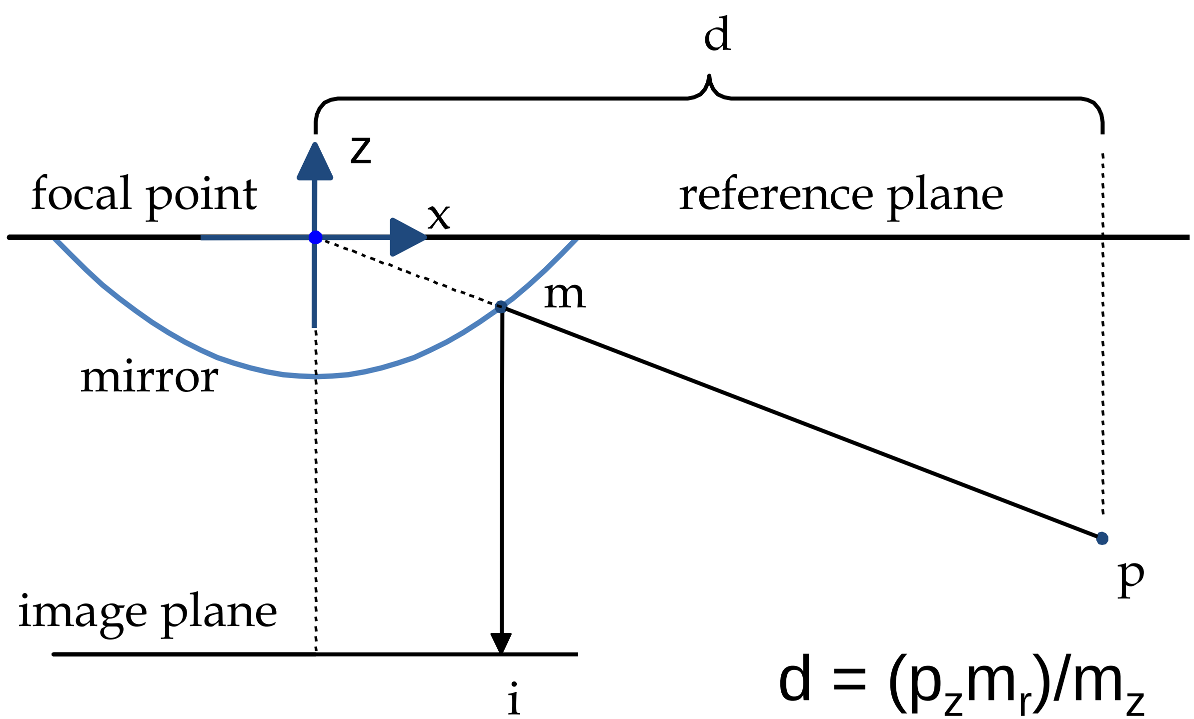


Paraboloidal Catadioptric Camera

A paraboloidal catadioptric camera

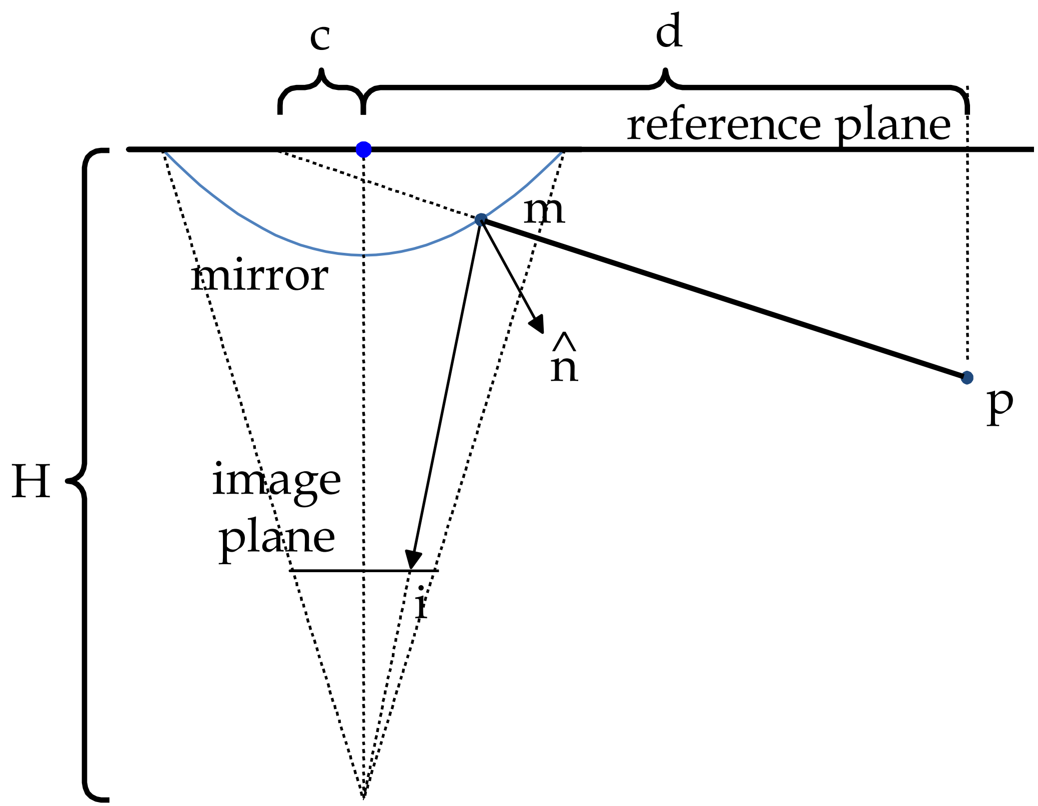
Motorized cart with camera, computer, battery, radio remote control

Ideal Camera Model



An ideal paraboloidal catadioptric setup for computing distance between the mirror's focal point and a 3D point

Our Camera Model



A paraboloidal catadioptric setup that accounts for perspective projection occurring in a practical system

Our Camera Model

- Assuming incident equals reflected angle:

$$\frac{i-m}{\|i-m\|}\cdot\frac{\hat{n}}{\|\hat{n}\|}=\frac{p-m}{\|p-m\|}\cdot\frac{\hat{n}}{\|\hat{n}\|}$$

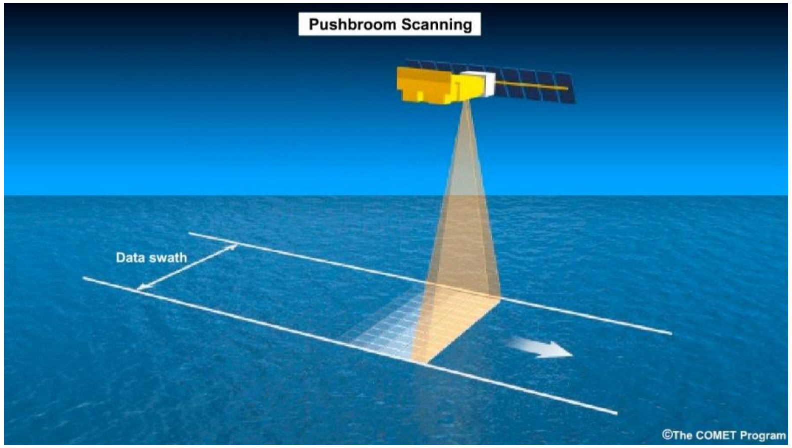
- And given a 3D point *p*, mirror radius *r*, convergence distance *H*, we group and rewrite in terms of m_r:

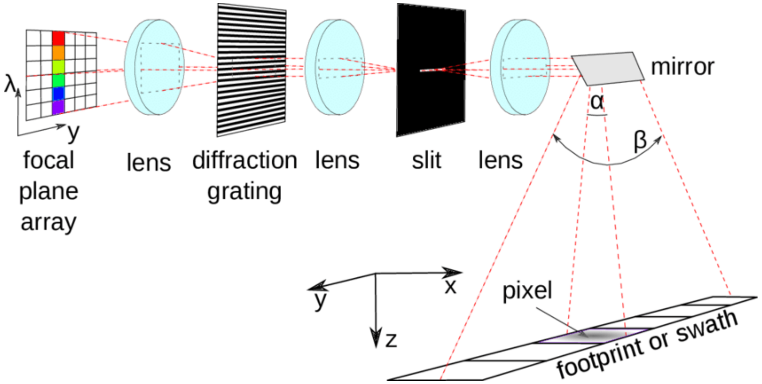
$$m_r^5-p_r m_r^4+2r^2 m_r^3+(2p_r rH-2r^2 p_r)m_r^2+ (r^4-4r^2 p_z H)m_r-(r^4 p_r+2r^3 H p_r) = 0$$

- To obtain a new expression for distance *d*:

$$\mathbf{d = (p_z m_p)/m_z - m_z/\tan(\alpha) + m_r}$$

Digital Camera (Satellite)

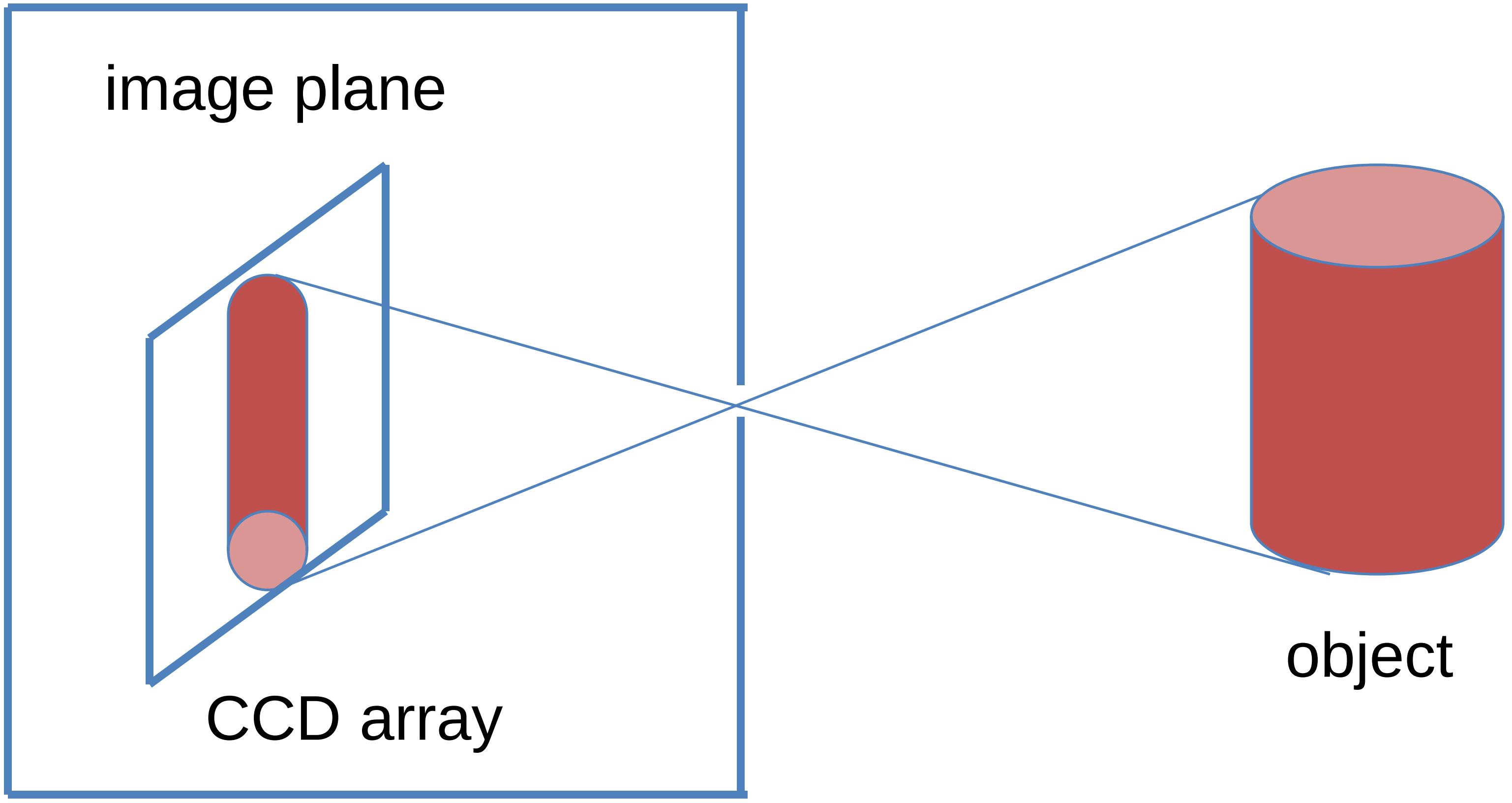




Digital Camera

- Thus not such a simple device…
- So what do we do?

The simplest 1-CCD camera in town



Exposures

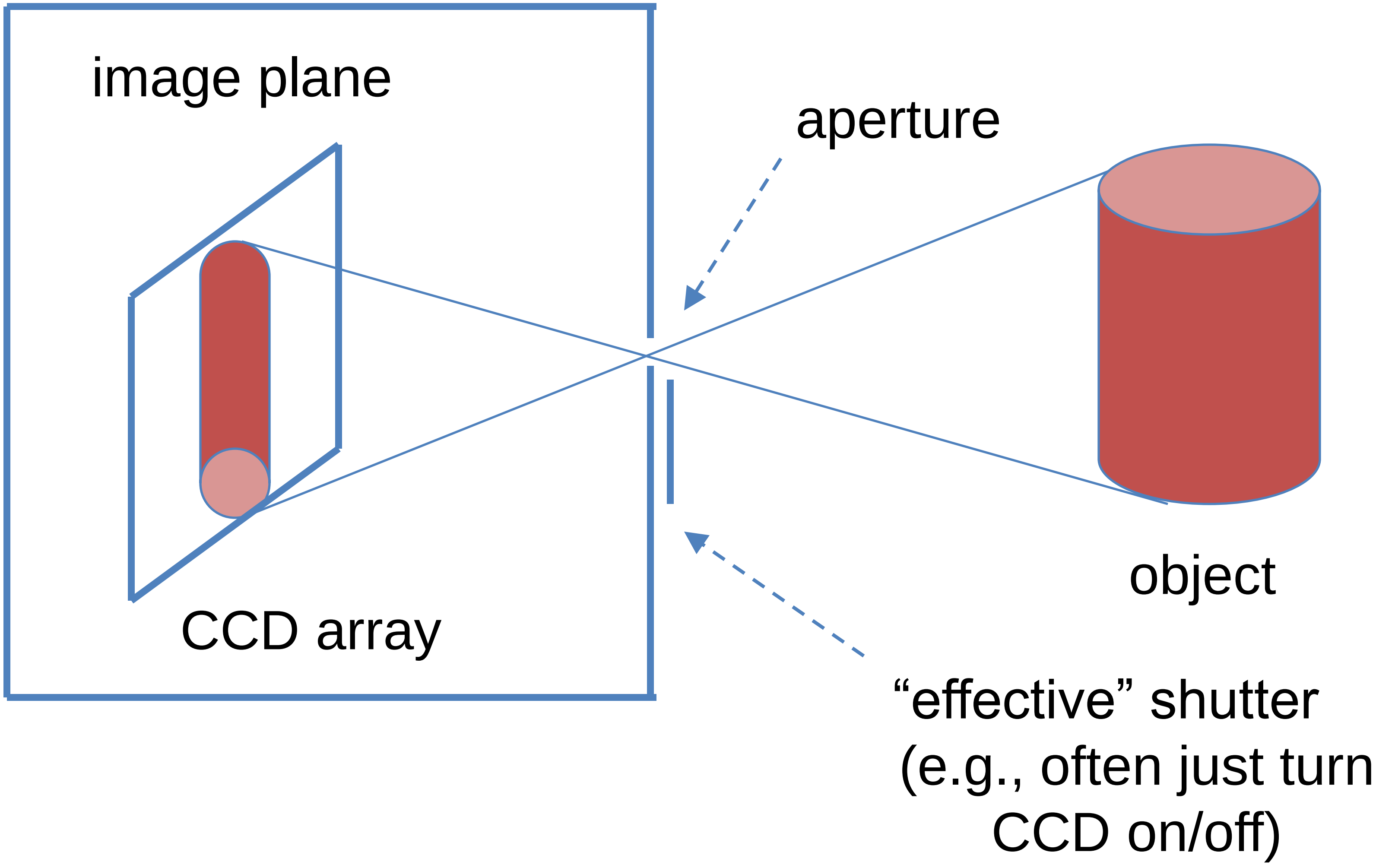


image plane
aperture
CCD array
object
"effective" shutter
(e.g., often just turn
CCD on/off)

Exposures

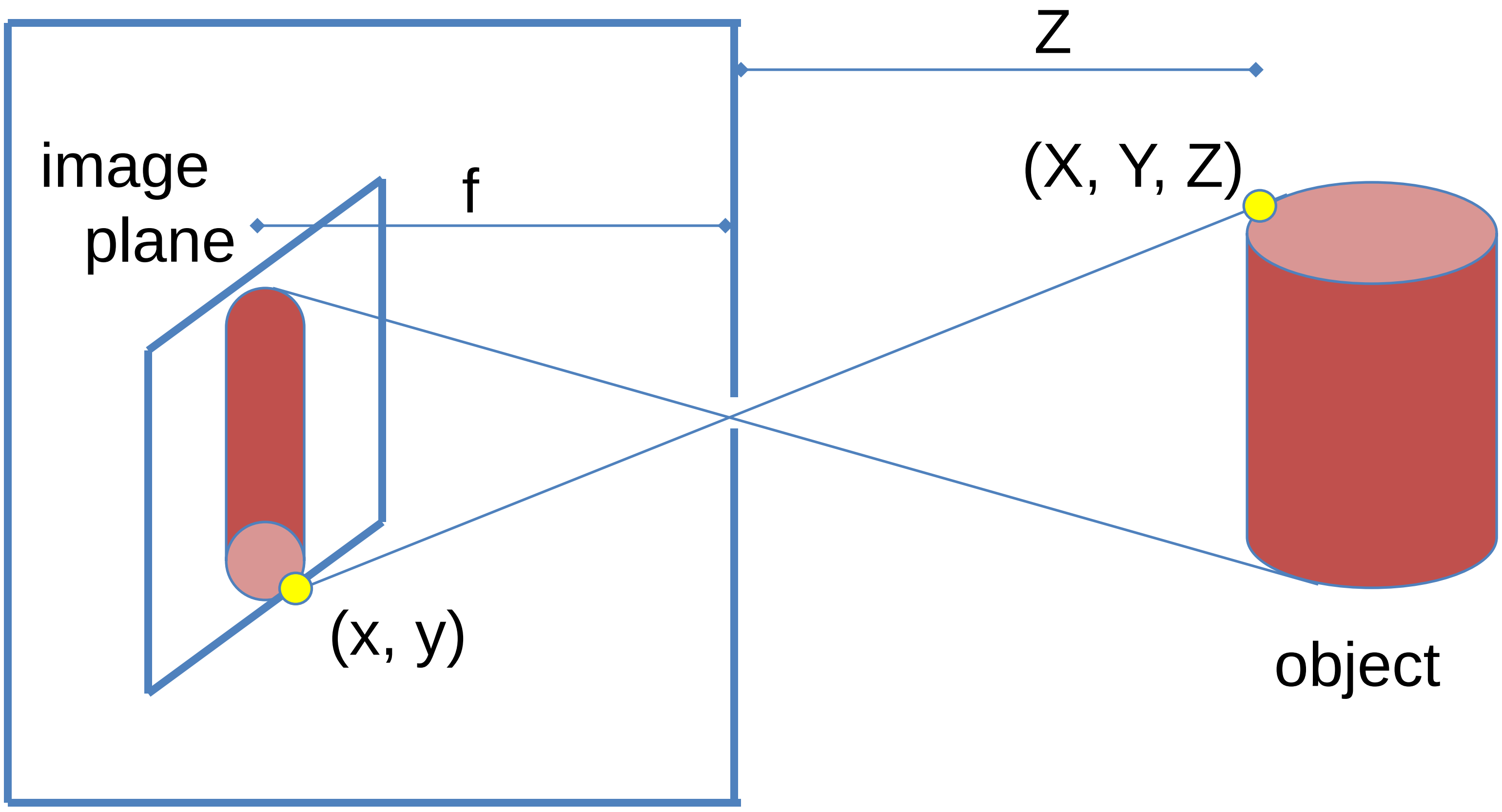
- An "exposure" is when the CCD is exposed to the scene, typically for a brief amount of time and with a particular set of camera parameters

- The characteristics of an "exposure" are determined by multiple factors, in particular:
  - Camera aperture
    - Determines amount of light that shines onto CCD
  - Camera shutter speed
    - Determines time during which aperture is "open" and light shines on CCD

Camera Calibration

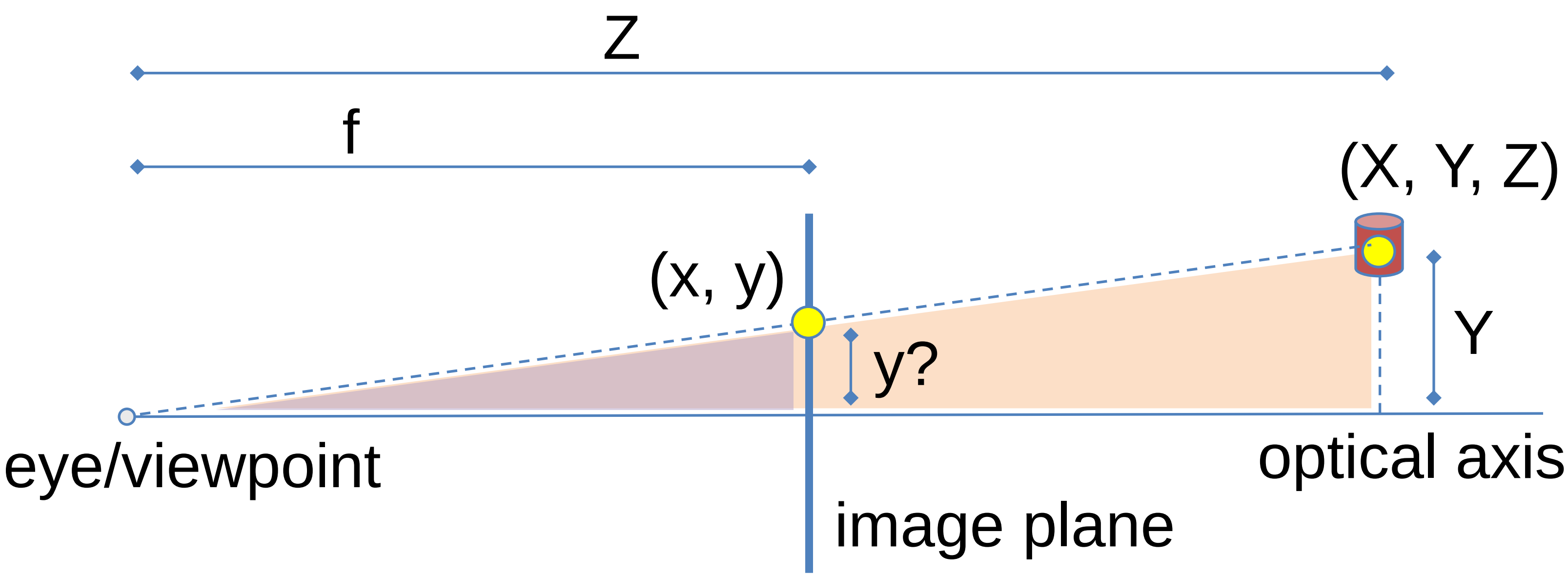
- Digital Cameras
- **Perspective Projection**
- Aberrations
- Calibration

Perspective Projection



$$x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z}$$

Perspective Projection



$$\frac{y}{f} = \frac{Y}{Z} \quad \Rightarrow \quad y = f\,\frac{Y}{Z} \quad \& \quad x = f\,\frac{X}{Z}$$

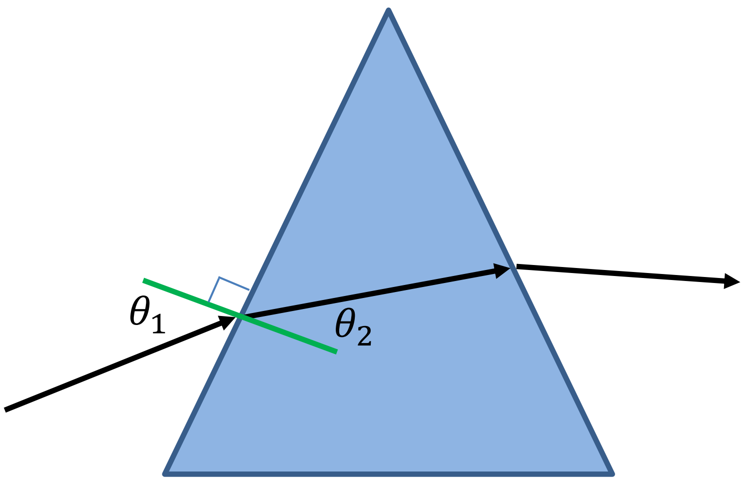
Camera Calibration

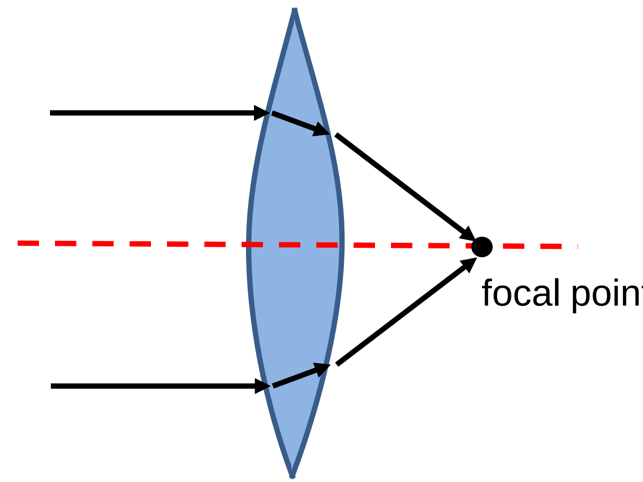
- Digital Cameras
- Perspective Projection
- **Aberrations**
- Calibration

Thins Lens Assumption

- A "real" lens system does not produce a perfect image
- Aberrations are caused by i) imperfect manufacturing and ii) our approximate but practical models
  - Lenses typically have a spherical surface (easy manufacture)
    - However, aspherical lenses would better compensate for refraction but are more difficult to manufacture
  - Typically 1st order approximations are used
    - Remember $\sin \Omega = \Omega - \Omega^3/3! + \Omega^5/5! - \ldots$
    - Thus, thin-lens equations only valid iff $\sin \Omega \approx \Omega$
    - Thin-lens means the lens is thin compared to distances

Thins Lens Assumption





Light bends through a prism

- Recall Snell's Law: $n = \frac{\sin(\theta_1)}{\sin(\theta_2)}$
- Expression to compute refraction much simpler if $\theta = \sin(\theta)$ approximation used:

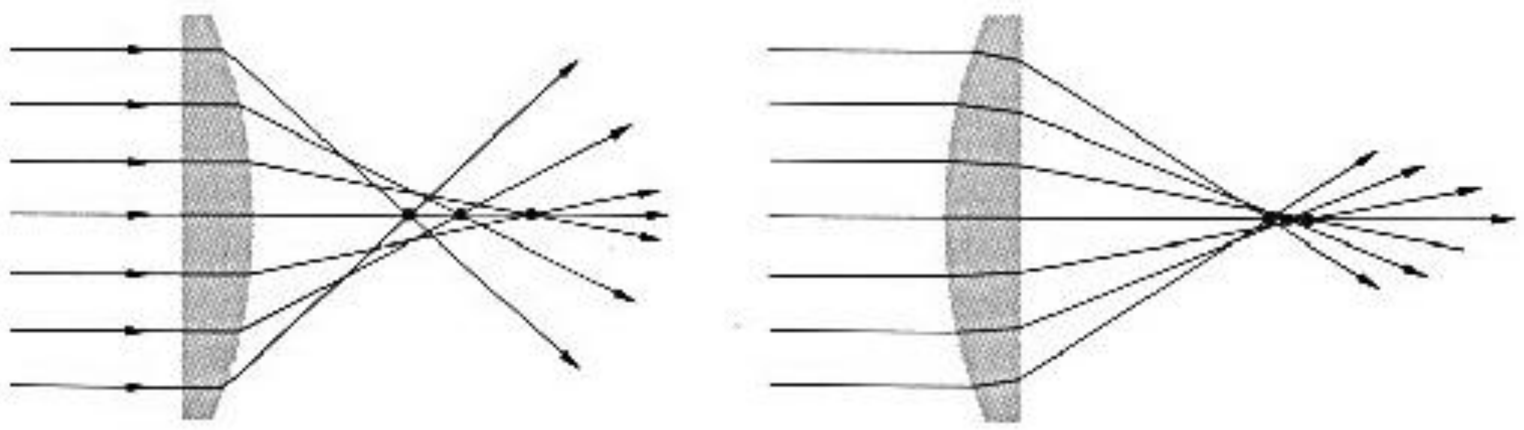
$$\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2}$$

Aberrations

- Most common aberrations:
  – Spherical aberration
  – Coma
  – Astigmatism
  – Curvature of field
  – Chromatic aberration
  – **Distortion**

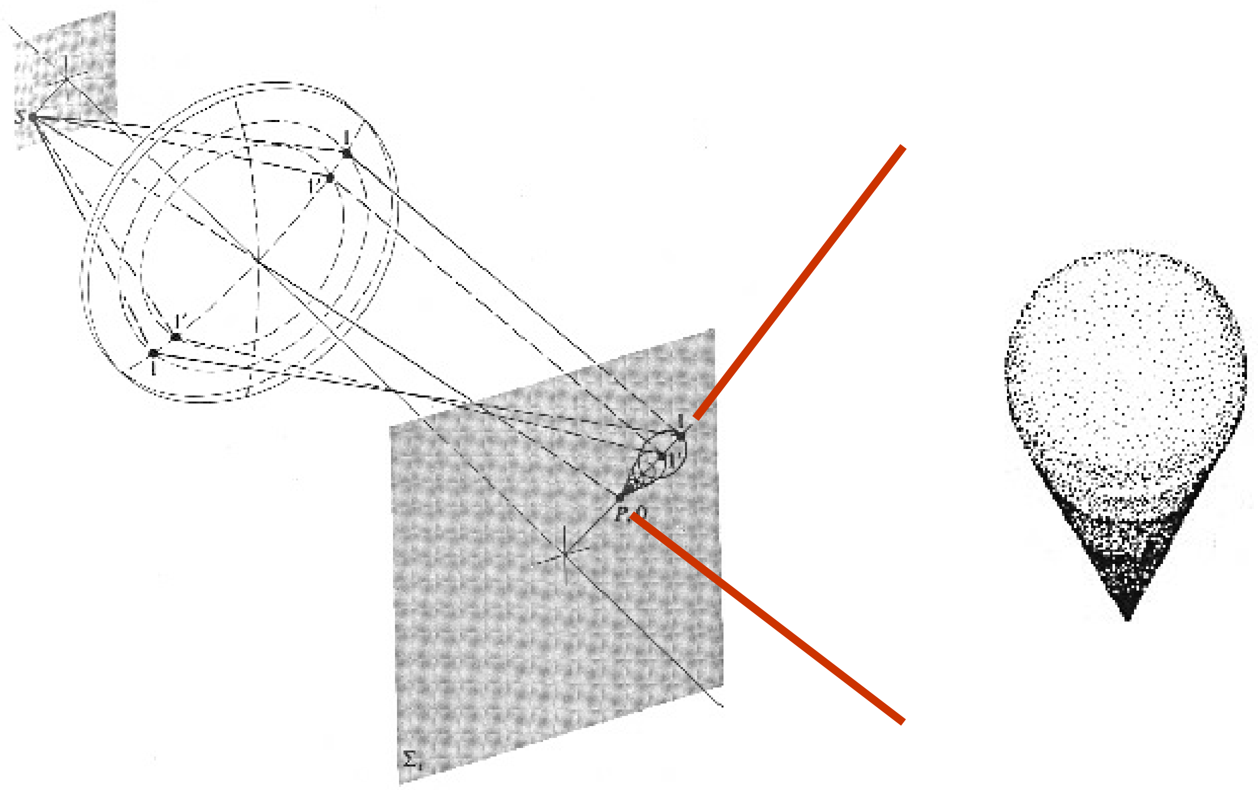
Spherical Aberration

- Deteriorates axial image



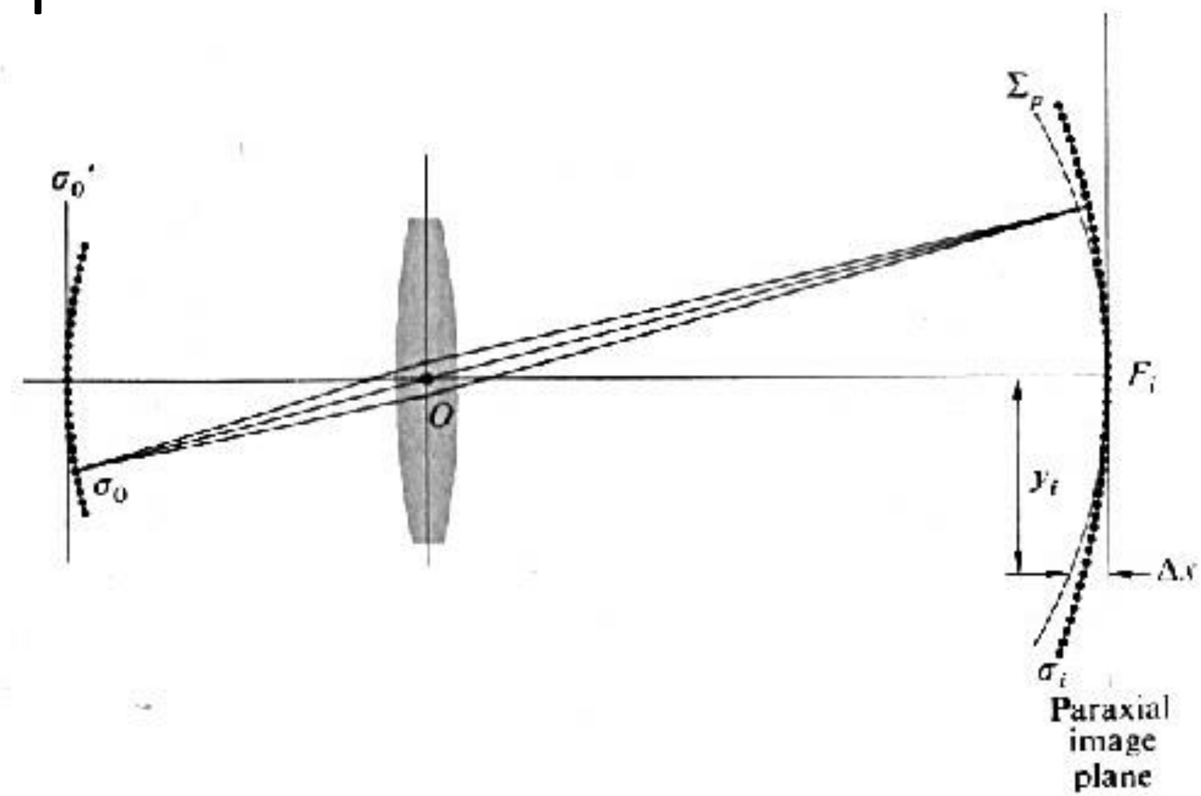
Coma

- Deteriorates off-axial bundles of rays



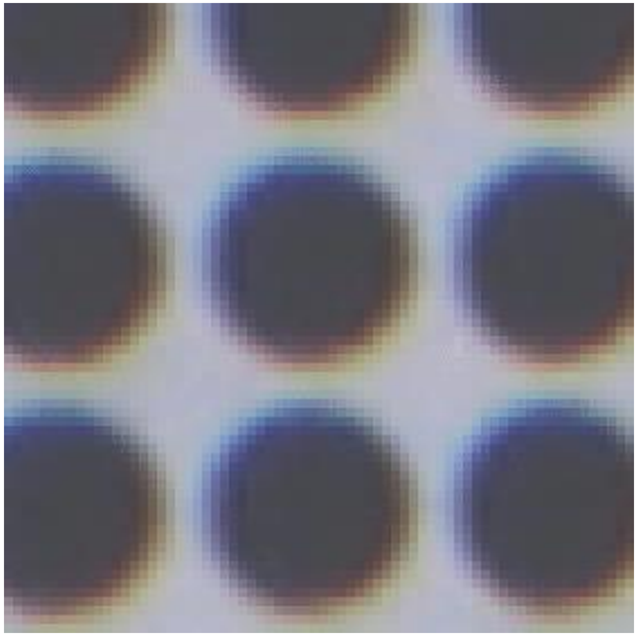
Astigmatism and Curvature of Field

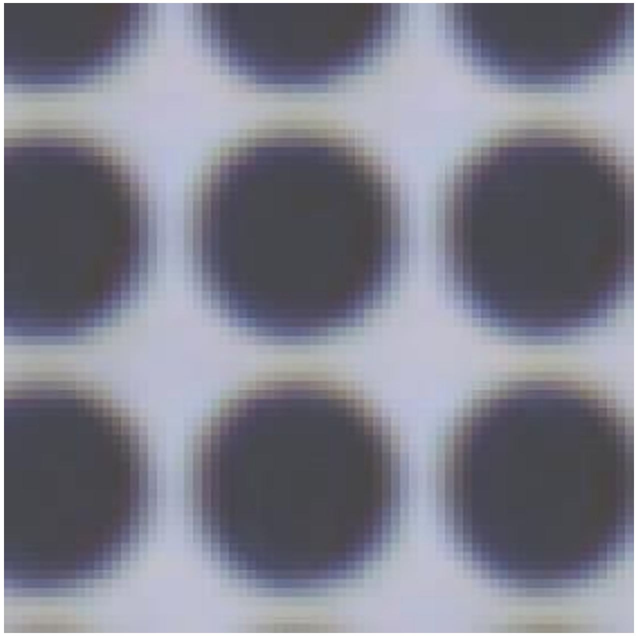
- Produces multiple (two) images of a single object point



Chromatic Aberration

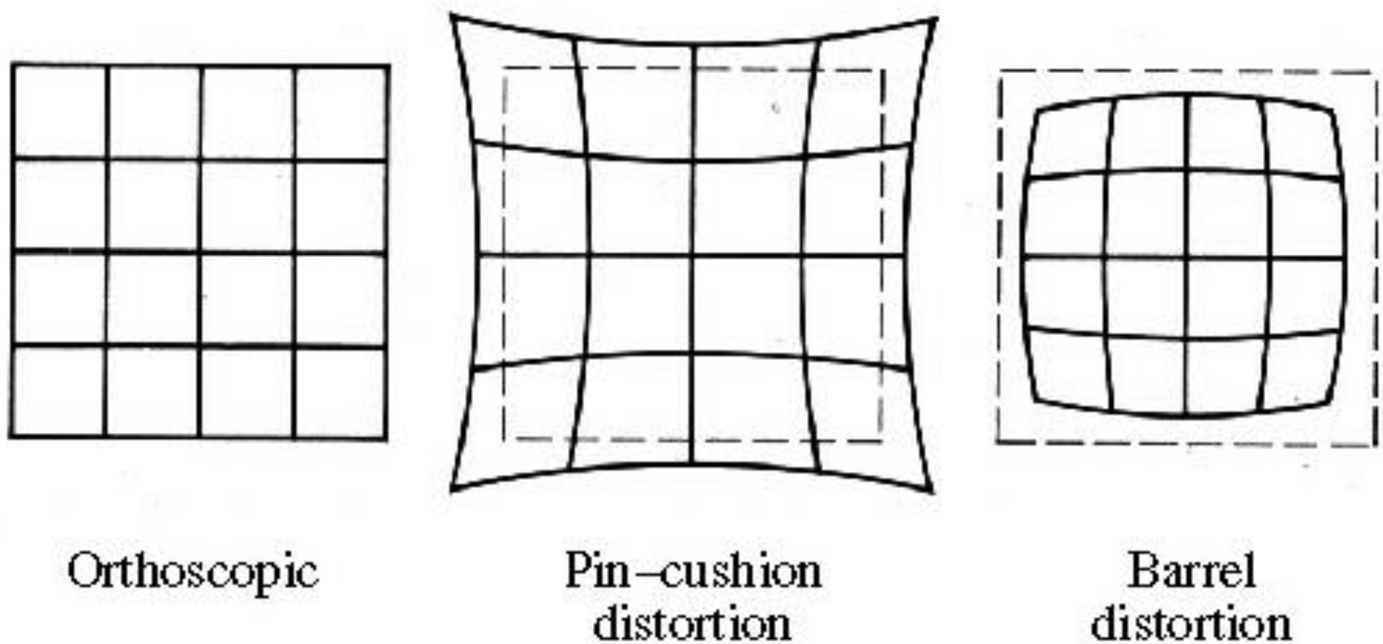
- Caused by wavelength dependent refraction
  - Apochromatic lenses (e.g., RGB) can help



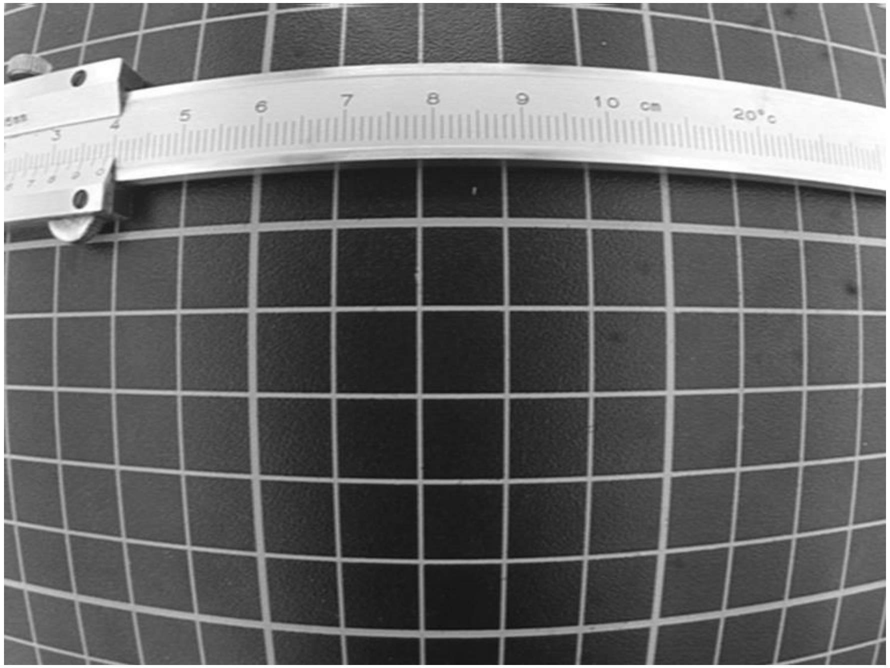


Distortion

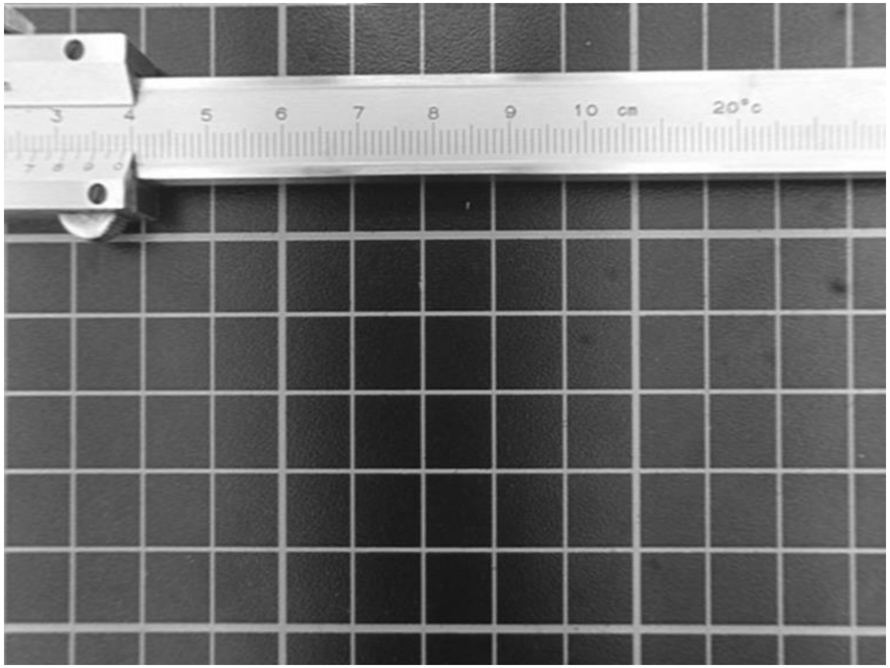
- Radial (and tangential) image distortions



Radial Distortion



before



after

Radial Distortion

- (x, y) pixel before distortion correction
- (x', y') pixel after distortion correction
- Let $r = (x^2 + y^2)^{-1}$

- Then
  - $x' = x(1 - \Delta r/r)$
  - $y' = y(1 - \Delta r/r)$
  - where $\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + \dots$

- Finally,
  - $x' = x(1 - k_0 - k_1 r^2 - k_2 r^4 - \dots)$
  - $y' = y(1 - k_0 - k_1 r^2 - k_2 r^4 - \dots)$

Camera Calibration

- Digital Cameras
- Perspective Projection
- Aberrations
- **Calibration**

Tsai's Camera Calibration

- A widely used camera model to calibrate conventional cameras based on a pinhole camera

- Reference
  - "A Versatile Camera Calibration Technique for High-Accuracy 3D Machine Vision Metrology Using Off-the-Shelf TV Cameras and Lenses", Roger Y. Tsai, IEEE Journal of Robotics and Automation, Vol. 3, No. 4, August 1987

Zhang's Camera Calibration

- Another widely used camera model to calibrate conventional cameras based on a pinhole camera
- Many implementations are floating around!
- Reference
  - "A Flexible New Technique for Camera Calibration", Zhengyou Zhang, IEEE Trans. on PAMI, 22(11):1330-1334, 2000

Bouguet's Camera Calibration

- Another widely used camera model to calibrate conventional cameras based on a pinhole camera
- Many implementations are floating around!
- Reference: http://robots.stanford.edu/cs223b04/JeanYvesCalib/

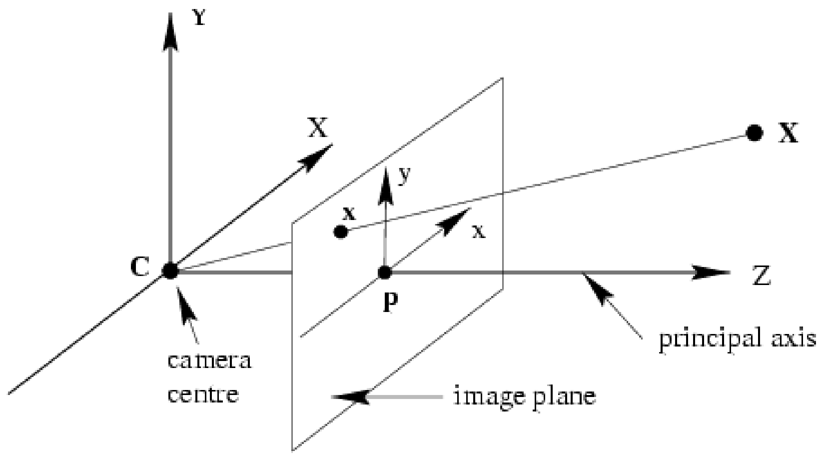
Calibration Goal

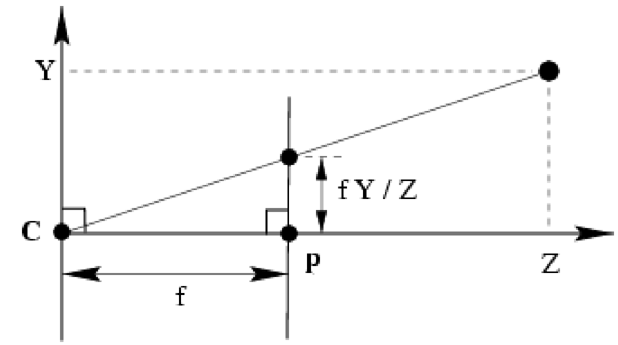
- Determine the intrinsic and extrinsic parameters of a camera (with lens)

Camera Parameters

- Intrinsic/Internal
  - Focal length f
  - Principal point (center) p_x, p_y
  - Pixel size s_x, s_y
  - (Distortion coefficients) $k_1, \ldots$
- Extrinsic/External
  - Rotation ϕ, φ, ψ
  - Translation t_x, t_y, t_z

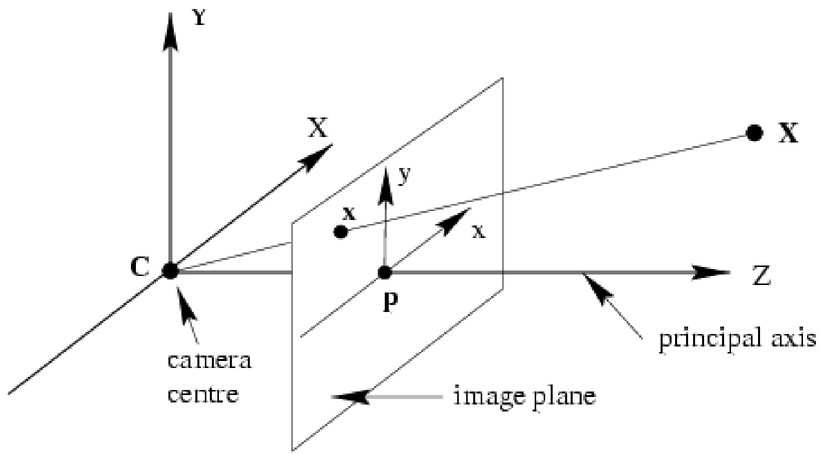
Focal Length

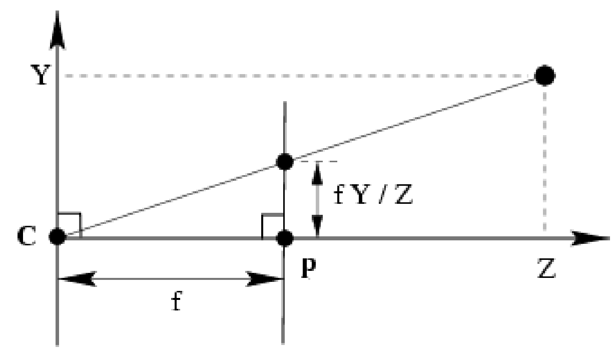




$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX / Z \\ fY / Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

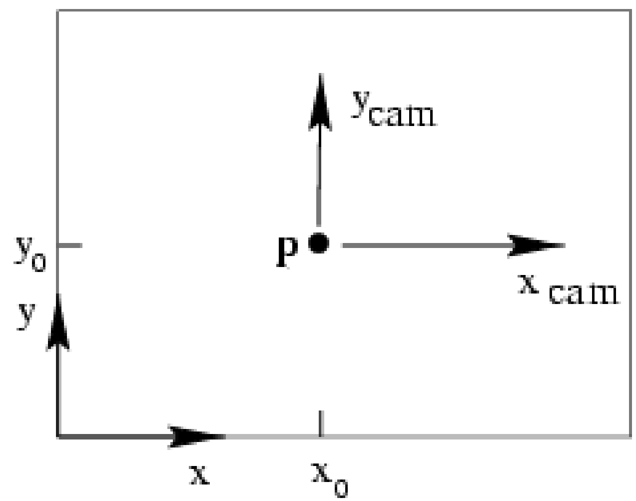
Focal Length





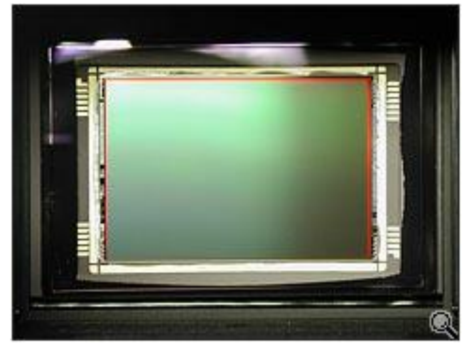
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \end{pmatrix} \Leftarrow \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point



$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \Leftarrow \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

CCD Camera: Pixel Size

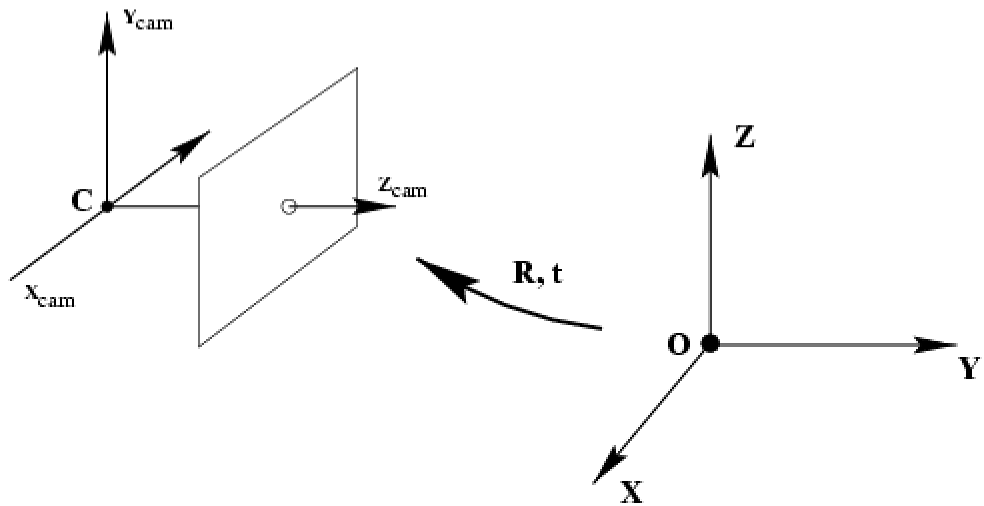


$$K = \begin{bmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{bmatrix} \begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha_x & 0 & p_x & 0 \\ 0 & \alpha_y & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(intrinsic) calibration matrix

Translation & Rotation



$$\tilde{x}_{cam} = R(\tilde{X} - C)$$
$$\tilde{x}_{cam} = R\tilde{X} - RC$$
$$-t$$

$$\tilde{x}_{cam} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

(extrinsic) calibration matrix P

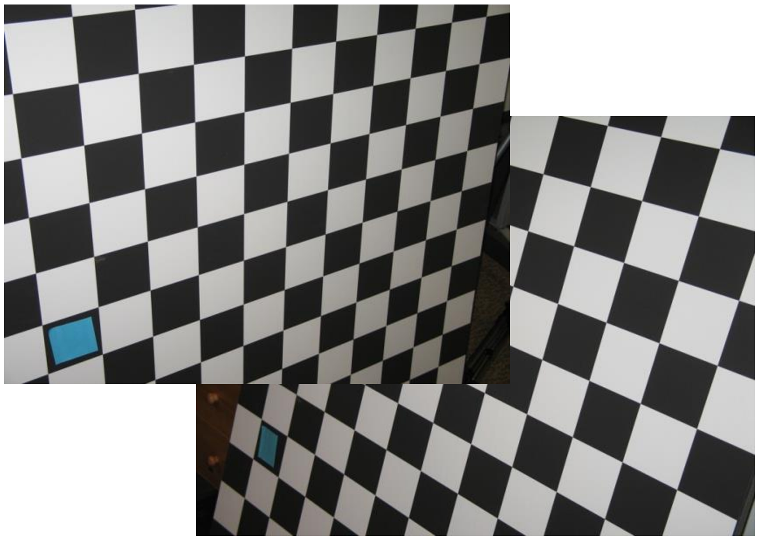
$$R = R_\phi R_\varphi R_\psi$$

3x3 rotation matrices

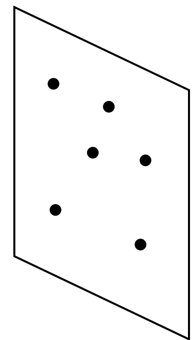
$$t = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T$$

translation vector

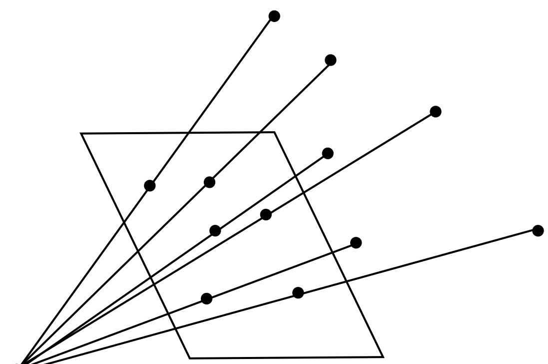
Calibration Task



physical arrangement
(calibration pad)



observation
(camera with initial parameters)



calibration result
(camera with calibrated parameters)

Given $\tilde{X}_i \leftrightarrow \tilde{x}_i$ What is K? P ?

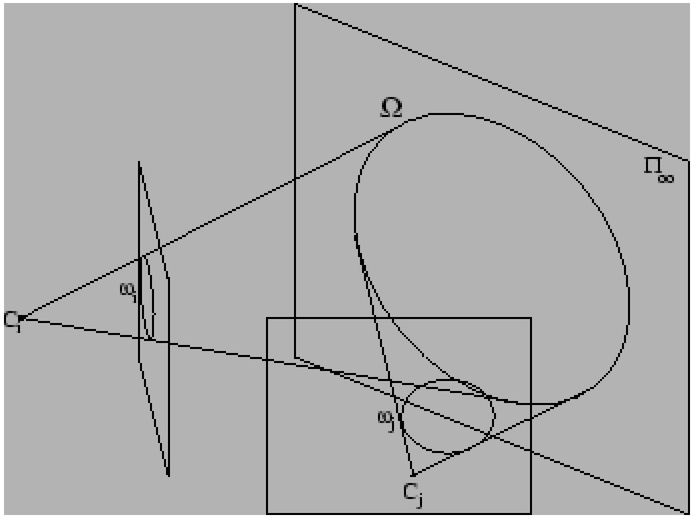
Absolute Conic

- A conic section, conic or a quadratic curve is a curve obtained from a cone's surface intersecting a plane
- The absolute conic is a conic on the plane at infinity consisting of points X such that
  - $X = [x\ y\ z\ t]$
  - Where $x^2 + y^2 + z^2 = 0$ and $t = 0$
  - Often $d = [x\ y\ z]^T$
  - Hence $\mathrm{d}^{\mathrm{T}} \cdot d = 0$

Absolute Conic

- The Absolute Conic Ω is invariant under Euclidean transformations (e.g., rotation/ translation) and critical to camera calibration

(e.g., like moon following you on straight road)

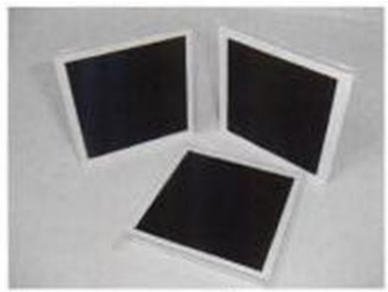


Absolute Conic

- Given point on Ω called $x_\infty = [d^T \; 0]^T$, its image on a general camera is
$$v = KRd$$
- Combining the above with $d^T \cdot d = 0$
$$v^T \cdot (KK^T)^{-1} \cdot v = 0$$
- Thus the image of conic is represented by
$$\omega = (KK^T)^{-1}$$
- Since we know the general form of K (i.e., 5 unknowns: $\alpha, \beta, u_0, v_0, \gamma$), ω is symmetric and defined up to a scale

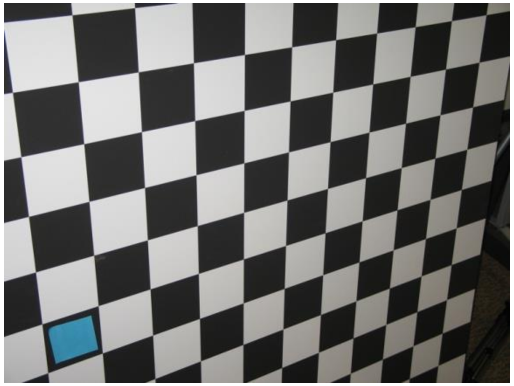
Simple Internal Parameters Calibration Device

- Observe these 3 planes, forming 3 homographies



- Each $H = [h_1 \; h_2 \; h_3]$ gives constraints
  $h_1^T \omega h_2 = 0$ and $h_1^T w h_1 = h_2^T h_2$
  Conic ω is determined from 5 or more such equations, up to a scale (5 orthogonal line pairs)
- Compute K from $\omega = (KK^T)^{-1}$ using Cholesky factorization, for example

Zhang's Camera Calibration



- **1. Detect corners**
- **2. Estimate matrix P**
- **3. Recover instrinsic/extrinsic parameters**
- 4. Refine: bundle adjustment

Typical Formulation

Let $M = KP$

$$\tilde{x}_{cam} = M\tilde{X}$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x'/w' \\ y'/w' \end{pmatrix}$$

$$x = (m_1 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X})/(m_3 \cdot \tilde{X})$$

A Linear Formulation

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$
$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

for $i = 1 \ldots n$ observations

$$(m_1 - x_i m_3) \cdot \tilde{X}_i = 0$$
$$(m_2 - y_i m_3) \cdot \tilde{X}_i = 0$$

$2n$ homogeneous linear equations and 12 unknowns (coefficients of M)

Thus, given $n \geq 6$ can solve for M; namely $Qm = 0$

$$Q = \begin{bmatrix} \tilde{X}_1^T & 0^T & -x_1 \tilde{X}_1^T \\ 0^T & \tilde{X}_1^T & -y_1 \tilde{X}_1^T \\ \ldots & \ldots & \\ \tilde{X}_n^T & 0^T & -x_n \tilde{X}_n^T \\ 0^T & \tilde{X}n^T & -y_n \tilde{X}_n^T \end{bmatrix} \qquad m = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

A Linear Formulation

- Goal: $\min\|Qm\|$ subject to $\|m\| = 1$
- Recall: normal equation $Ax = b \rightarrow A^T Ax = A^T b$
- Solution: so solve $Q^T Qm = 0$, e.g., use eigenvector of $Q^T Q$ associated with the smallest eigenvalue. Use m to make matrix M.

Decomposing M into Camera Parameters

$$M = \rho[A\ b] = K[R\ t]$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

(often $\gamma = \pi/2$ which means no skew))

Decomposing M into Camera Parameters

$$M = \rho[A\ b] = K[R\ t]$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$ (often $\gamma = 0$ which means no skew))

$B = KR$ and $b = Kt$ (so B is first 3x3 of M)

Let $A = BB^T = (KR)(KR)^T = KK^T$

$$A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & \beta\gamma + u_0 v_0 & u_0 \\ u_0 v_0 + \beta\gamma & \beta^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

Decomposing M into Camera Parameters

$$A = \begin{bmatrix} \alpha^2 + \gamma^2 + u_0^2 & \beta\gamma + u_0 v_0 & u_0 \\ u_0 v_0 + \beta\alpha & \beta^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

(assumes square pixels and equal focal length in x and y)

How to solve for the original K?

Decomposing M into Camera Parameters

$$A = \begin{bmatrix} k_u & k_c & u_0 \\ k_c & k_v & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}$$

$$u_0 = A_{13}$$

$$v_0 = A_{23}$$

$$\beta = \sqrt{k_v - v_0^2}$$

$$\gamma = \frac{k_c - u_0 v_0}{\beta}$$

$$\alpha = \sqrt{k_u - u_0^2 - \gamma^2}$$

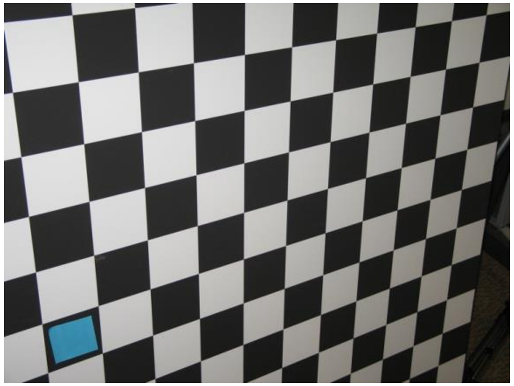
$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Recall: $B = KR$ and $b = Kt$

$$R = K^{-1}B$$

$$t = K^{-1}b$$

Zhang's Camera Calibration



- 1. Detect corners
- 2. Estimate matrix P
- 3. Recover instrinsic/extrinsic parameters
- **4. Refine: bundle adjustment**

Bundle Adjustment

- Given initial guesses, use nonlinear least squares to refine/compute the calibration parameters
- Simple but good convergence depends on accuracy of initial guess

Bundle Adjustment

Recall

$$x = (m_1 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$y = (m_2 \cdot \tilde{X}) / (m_3 \cdot \tilde{X})$$

$$E = \frac{1}{mn} \sum_{ij} \left[(x_{ij} - \frac{m_{i1} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 + (y_{ij} - \frac{m_{i2} \cdot \tilde{X}_j}{m_{i3} \cdot \tilde{X}_j})^2 \right]$$

Goal is $E \rightarrow 0$

Bundle Adjustment

Option A:

Define M as a matrix of 11 unknowns (i.e., $m_{34} = 1$)

And solve for m_{ij}

Can be made very efficient, especially for sparse matrices

Option B:

Define M as function of intrinsic and extrinsic parameters so that it is “recomputed” during each loop of the optimization