CS635

Hough and Radon Transform

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Hough and Radon Transform

- Hough is more well known for shape detection
- Radon is more well known in medical tomography

- However, the two are very related and can help with 2D to 3D inference

Mappings

- Forward/Writing paradigm: a data point in the source space maps onto data points in the destination space

  -> Hough Transform although in a discrete setting;

  -> initialize function P(p) to zero. For each point x in the input image determine its contribution, weighted by K(x; p), to each of the points in P(p) and update P(p)"

  -> if the input data is sparse, the Hough transform is compact in computation time

- Reverse/Reading paradigm: a data point in the destination space is obtained from the data in the source space

  -> Radon Transform but in a continuous setting;

  -> for each p, collect all the values of I(x), apply the template weights K(x; p), and sum everything

  -> if we have only a viewpoints in parameter space, then the Radon paradigm is preferred

Hough Transform

- Patented in 1962 by Paul V.C. Hough
  - Compact 5 page patent
  - BTW, Hough was awarded another patent in 2006 (44 years later!)

Hough Transform

- A line in source image I becomes a (discrete) point in destination space

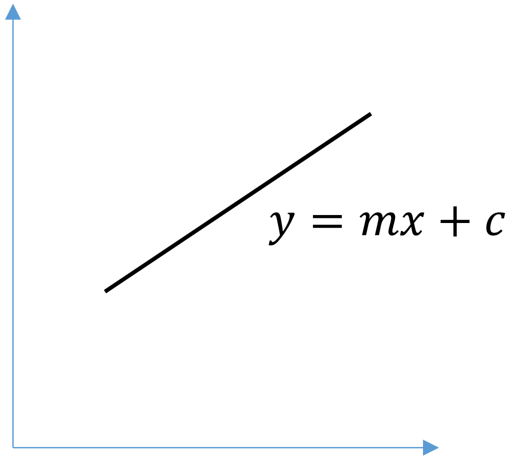
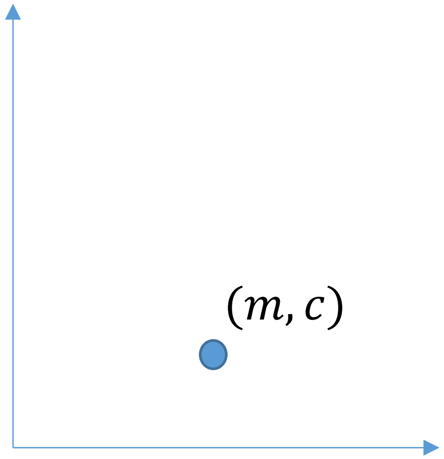


image space



Hough space

(bad for vertical lines)

Hough Transform

- A line in source image I becomes a (discrete) point in destination space

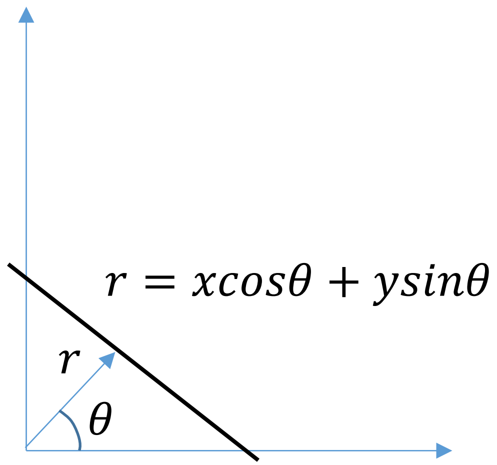
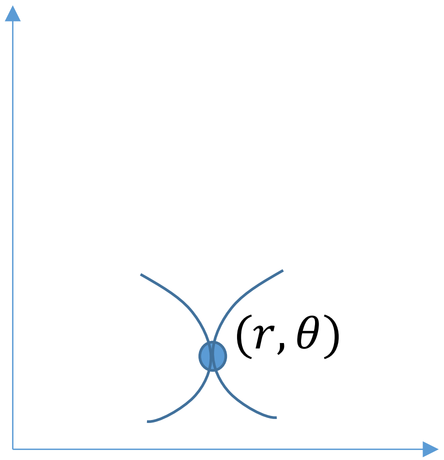
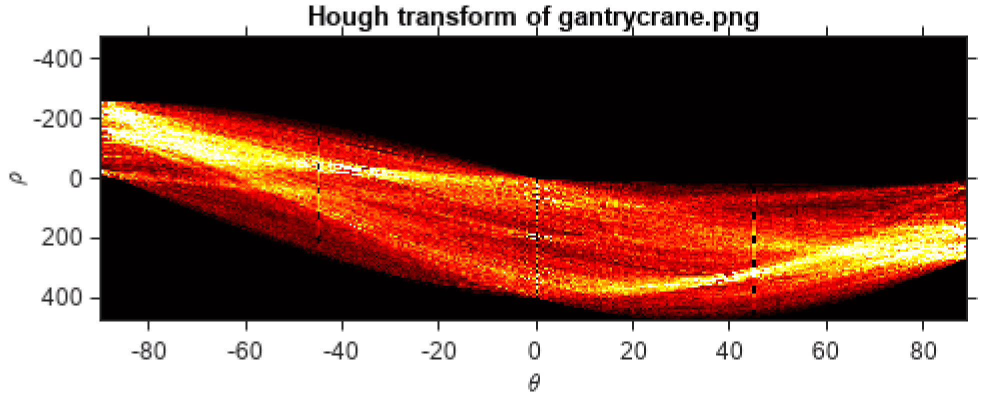


image space

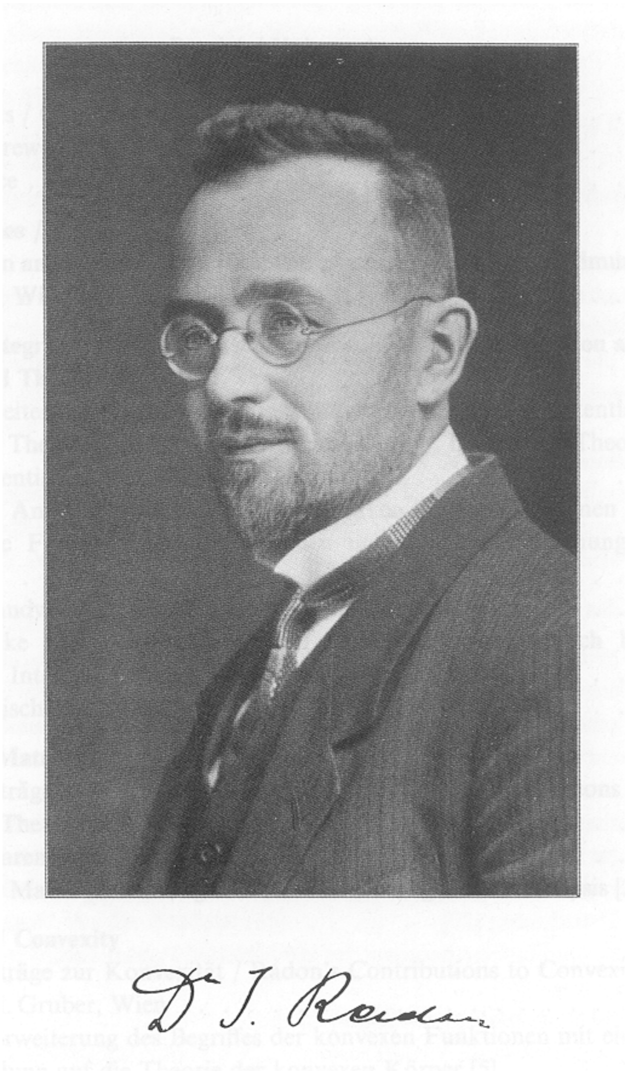


Hough space



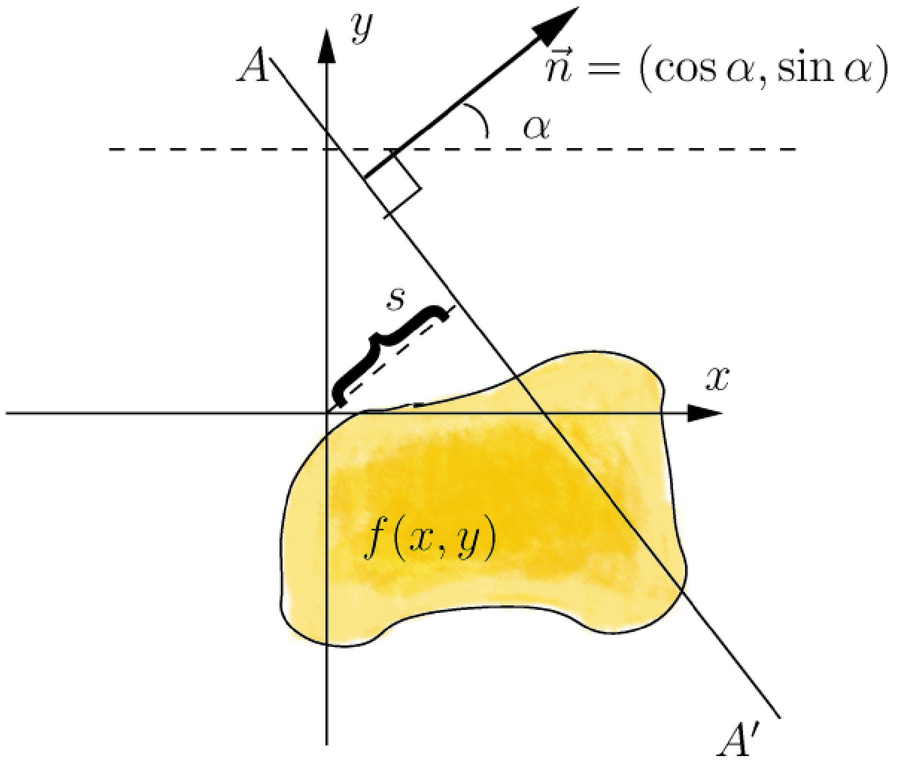
Radon Transform

- Johann Karl August Radon
- Born in Děčín (Austrian monarchy, now North Bohemia, CZ) in 1887
- Austrian mathematician living in Vienna
- Discovered the transform and its inversion in 1917 as pure theoretical result
- No practical applications during his life
- Died in 1956 in Vienna



Radon Transform

- Input space coordinates x, y
- Input function $\mathrm{f}(x, y)$
- Output space coordinates α, s
- Output function $\mathrm{F}(\alpha, s)$



[Images: J. Kukal, 2009]

RT and IRT

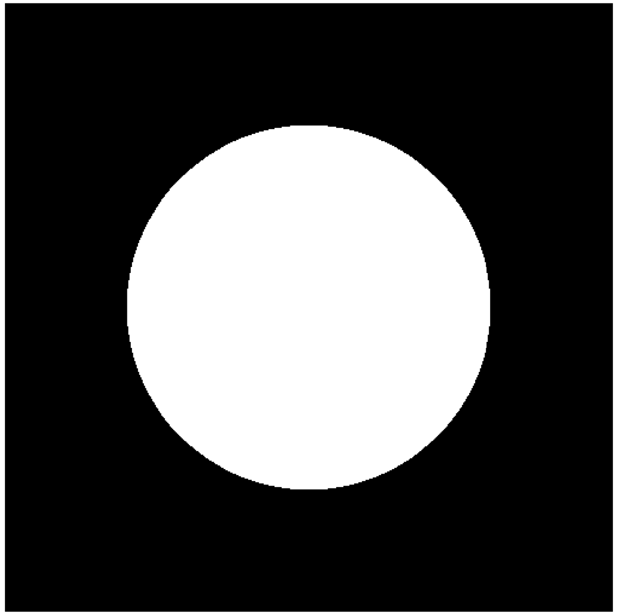
- Radon Transform

$$\mathrm{F}(\alpha, s) = \int_{-\infty}^{+\infty} \mathrm{f}\left(t \sin\alpha + s\cos\alpha, -t\cos\alpha + s\sin\alpha\right) \mathrm{d}t$$

- Inverse Radon Transform

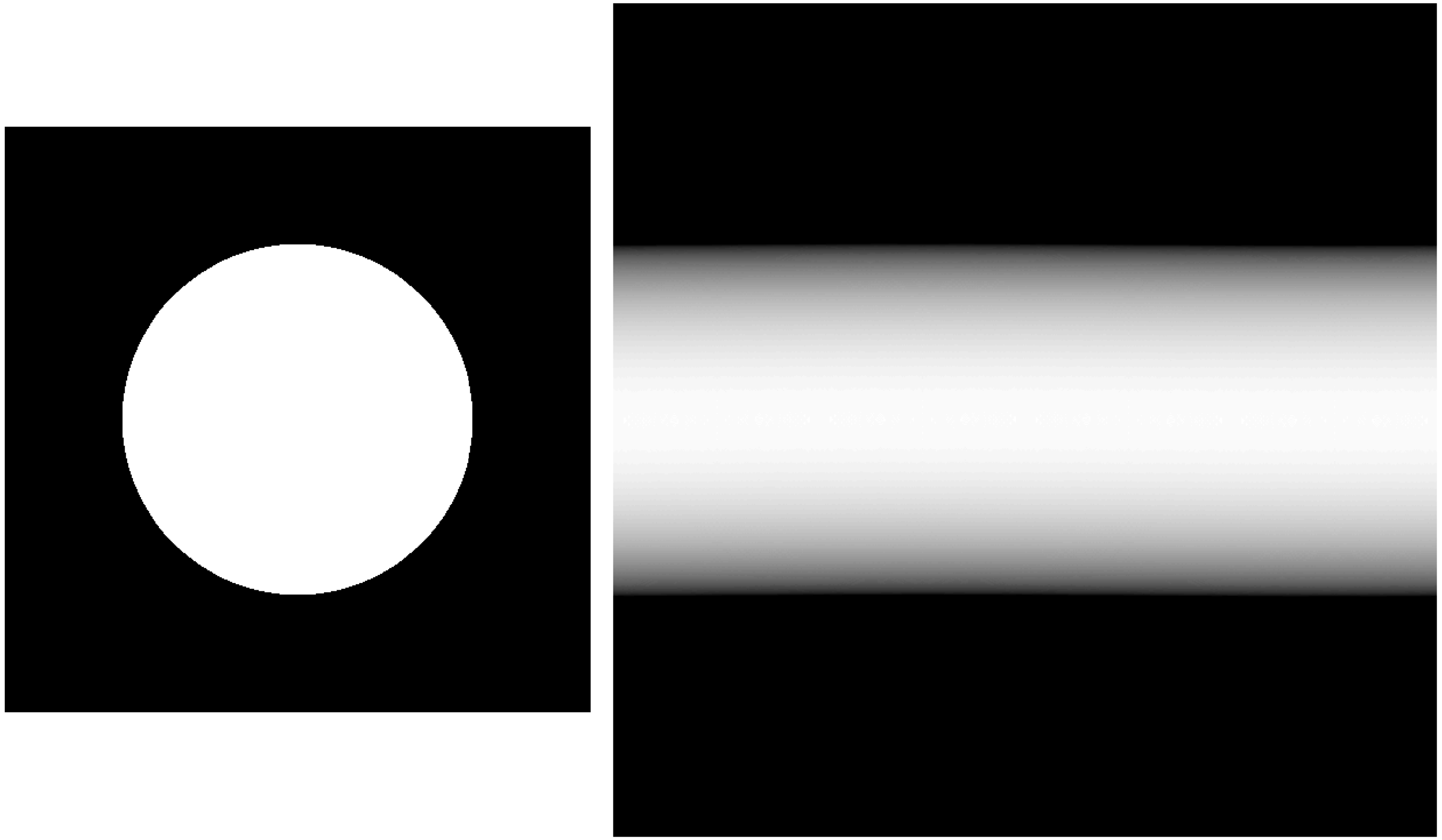
$$\mathrm{f}(x, y) = \int_{0}^{2\pi} \mathrm{F}\left(\alpha, x\cos\alpha + y\sin\alpha\right) \mathrm{d}\alpha$$

Full circle in RT



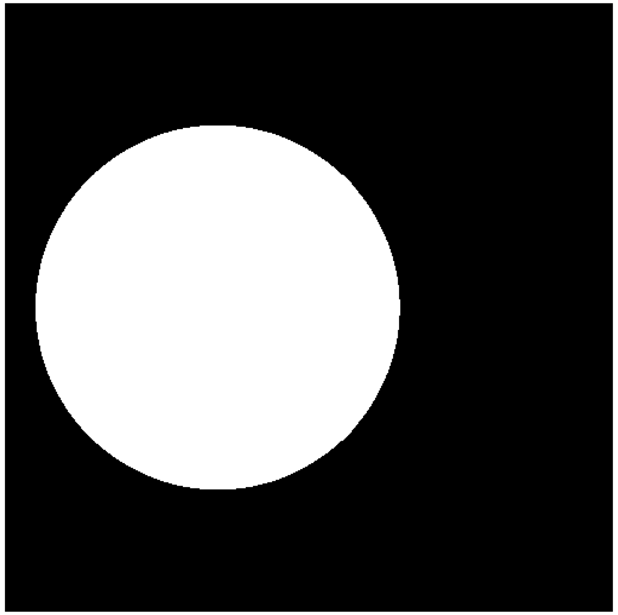
[Images: J. Kukal, 2009]

Full circle in RT



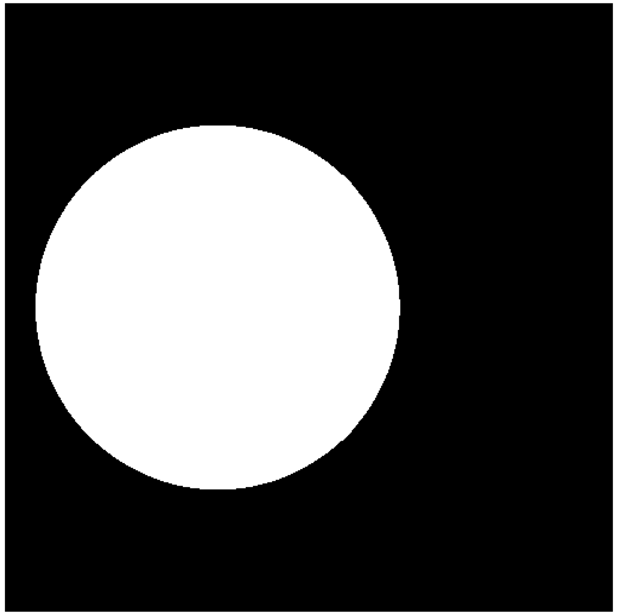
[Images: J. Kukal, 2009]

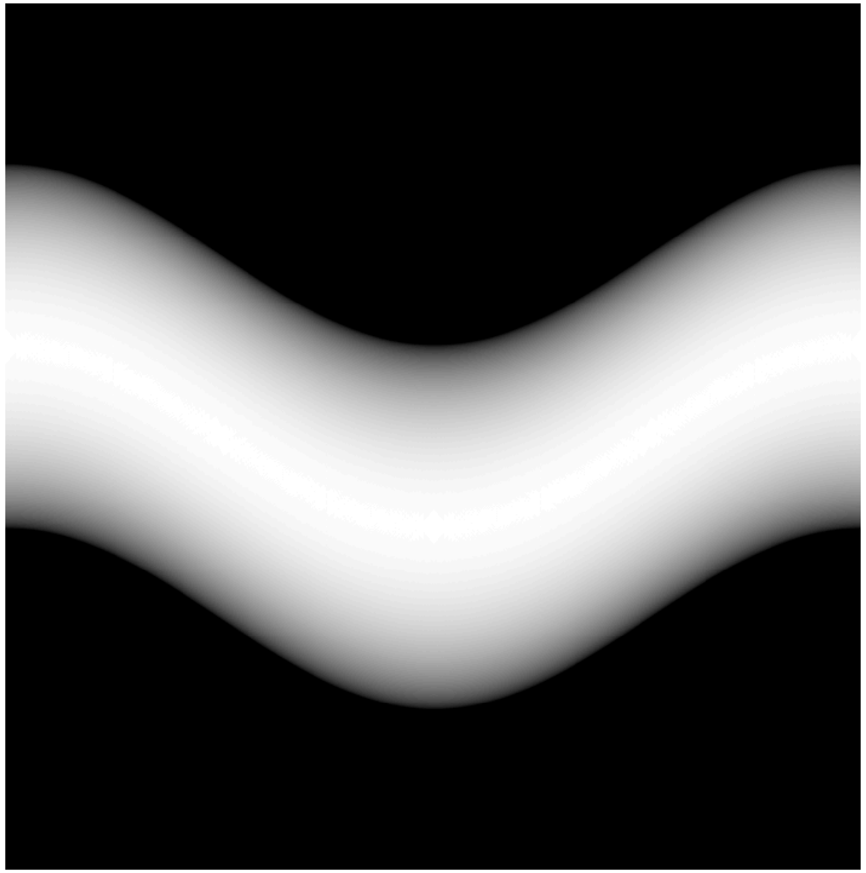
Shifted full circle in RT



[Images: J. Kukal, 2009]

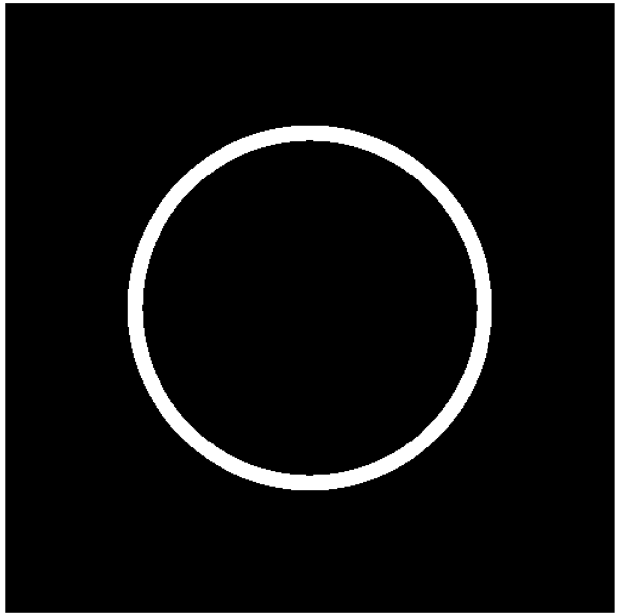
Shifted full circle in RT





[Images: J. Kukal, 2009]

Empty circle in RT



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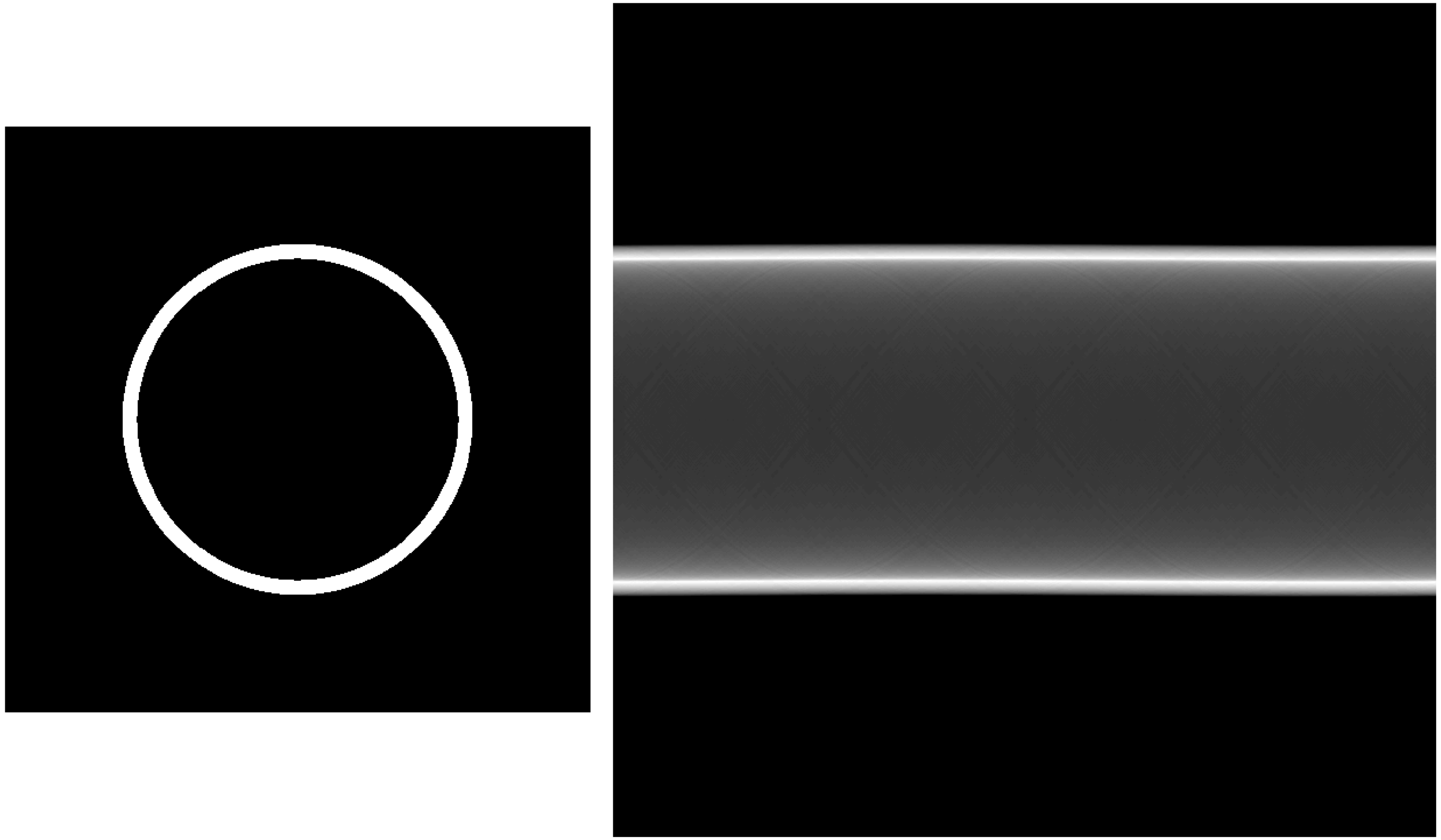
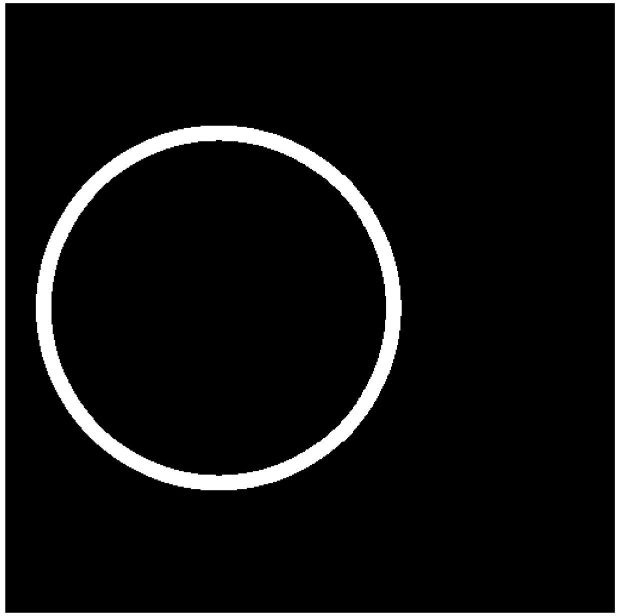
Empty circle in RT



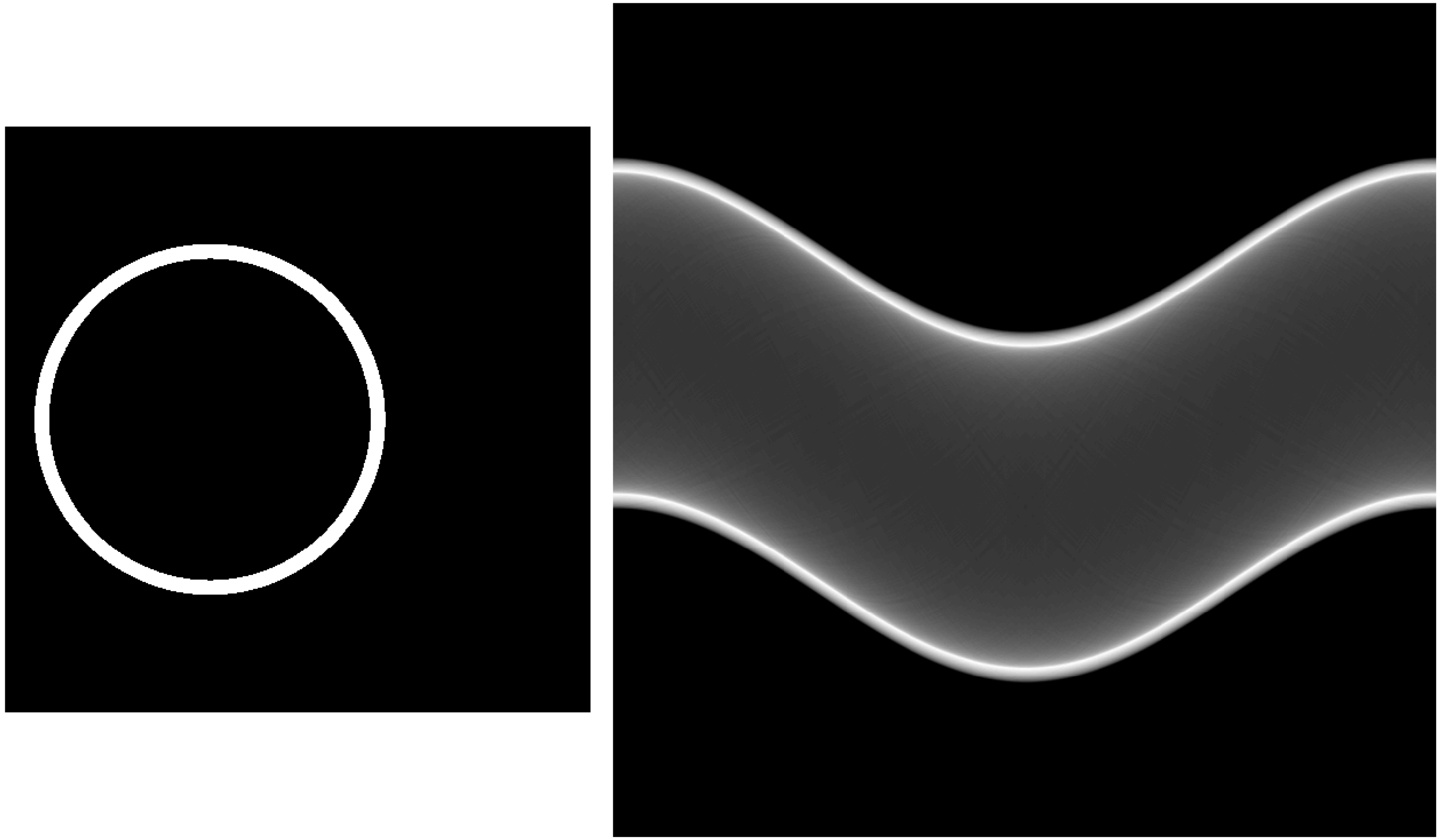
[Images: J. Kukal, 2009]

Shifted empty circle in RT



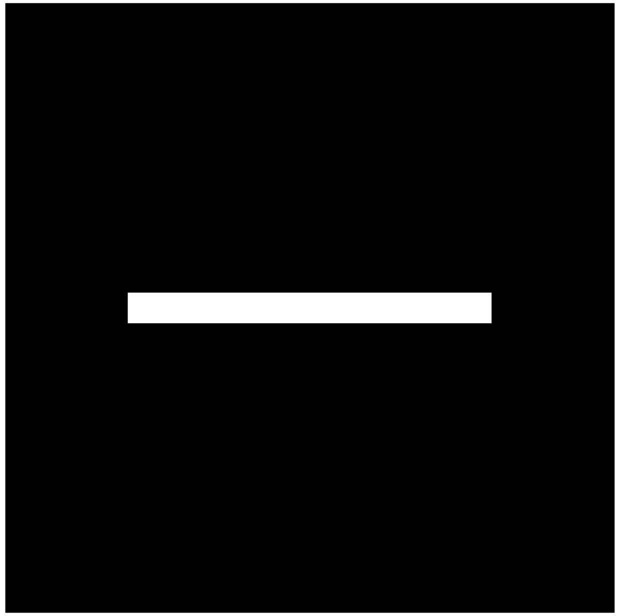
[Images: J. Kukal, 2009]

Shifted empty circle in RT



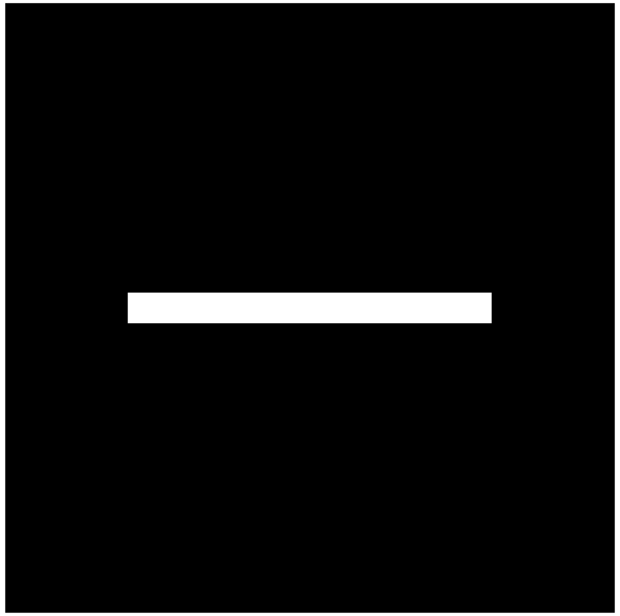
[Images: J. Kukal, 2009]

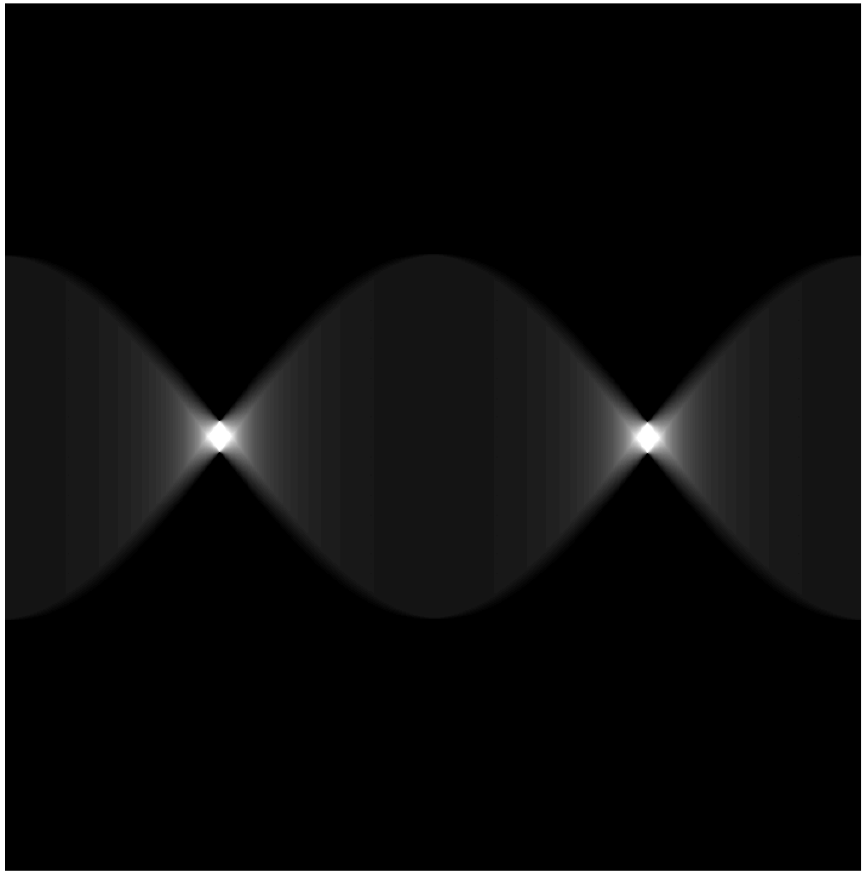
Thin stick in RT



[Images: J. Kukal, 2009]

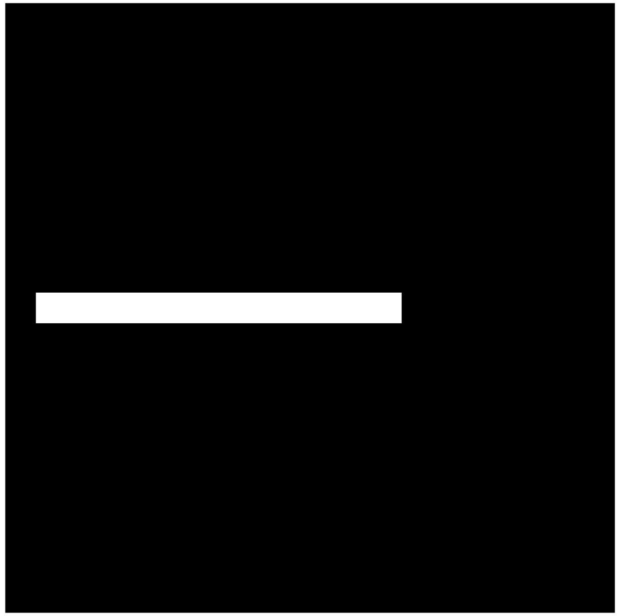
Thin stick in RT





[Images: J. Kukal, 2009]

Shifted thin stick in RT



[Images: J. Kukal, 2009]

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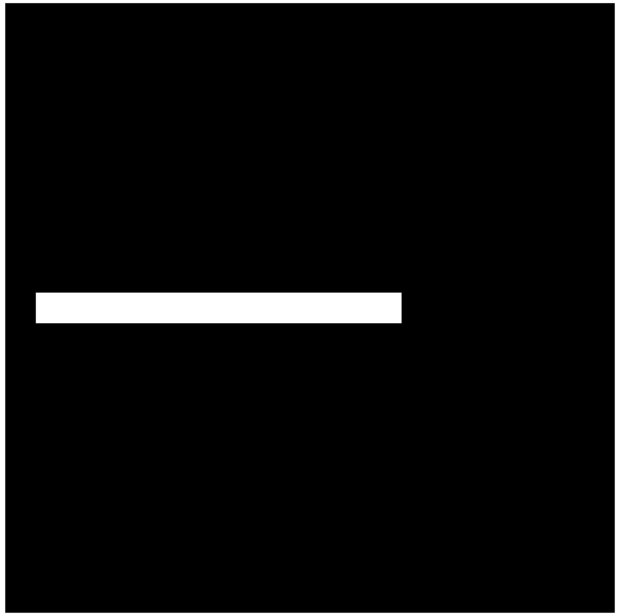
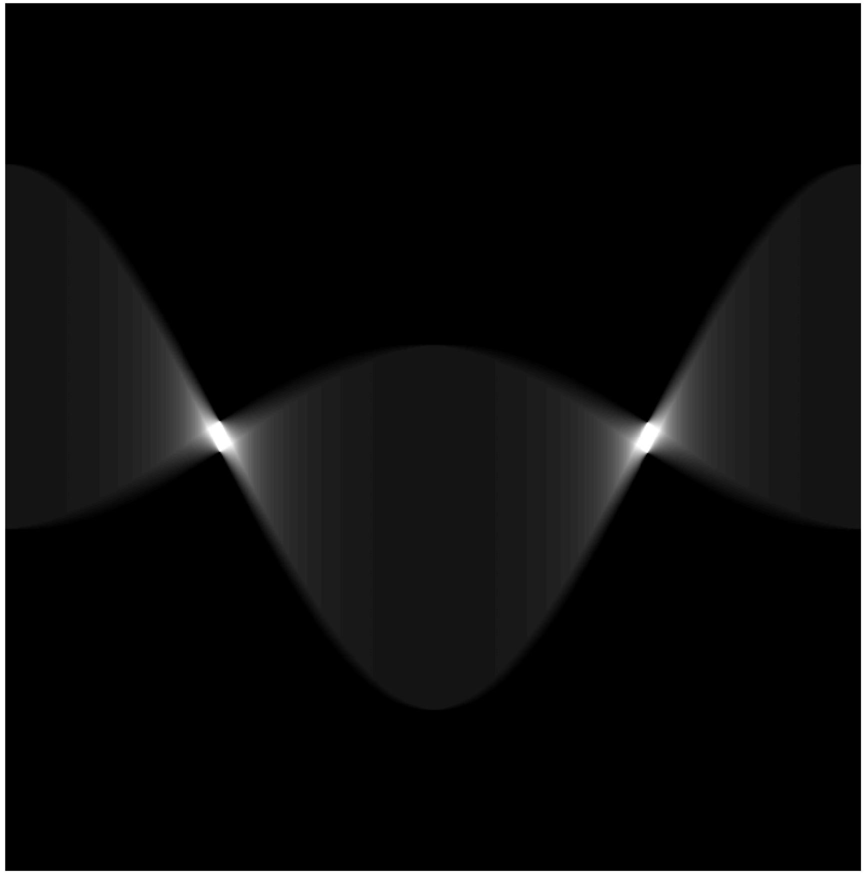
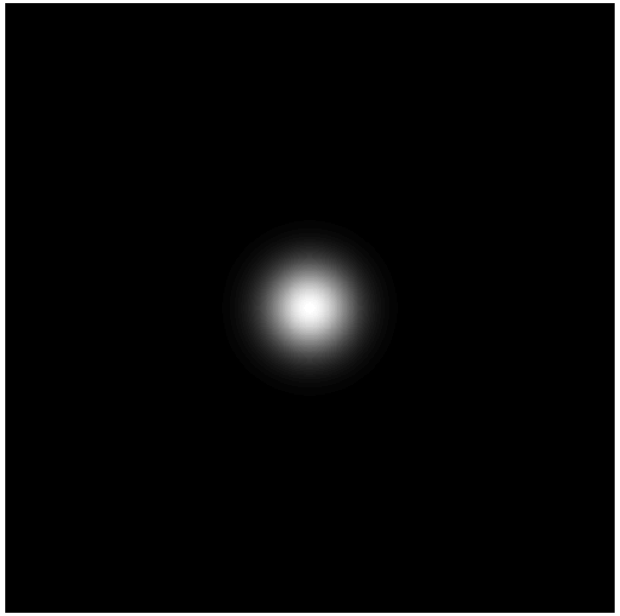




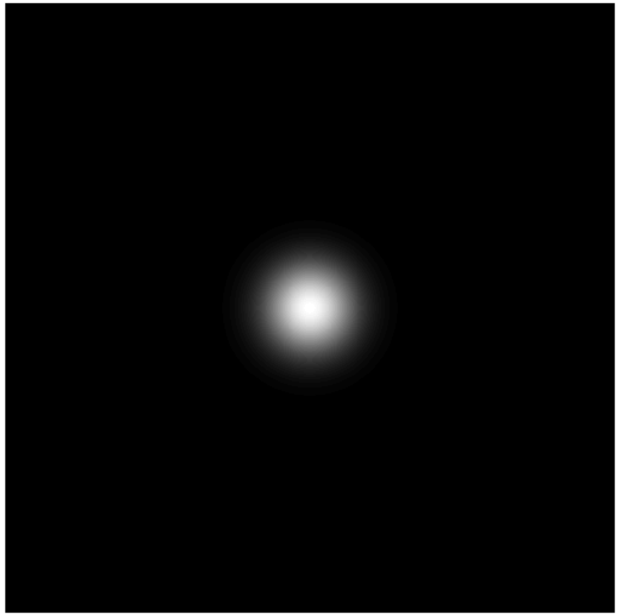
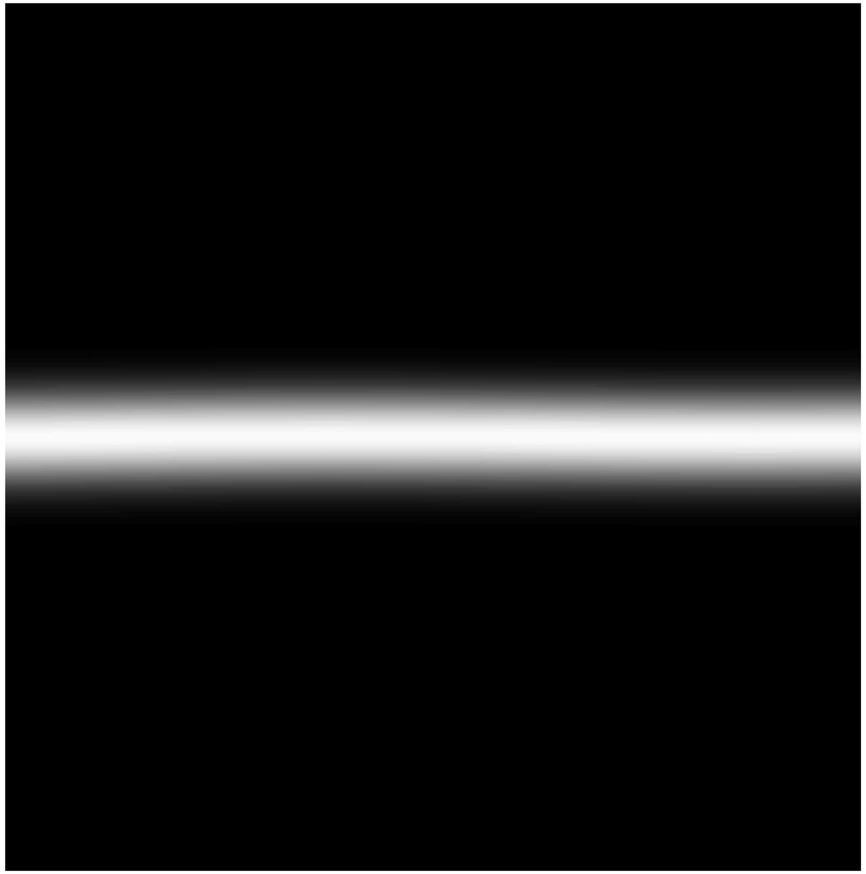
[Images: J. Kukal, 2009]

2D Gaussian in RT



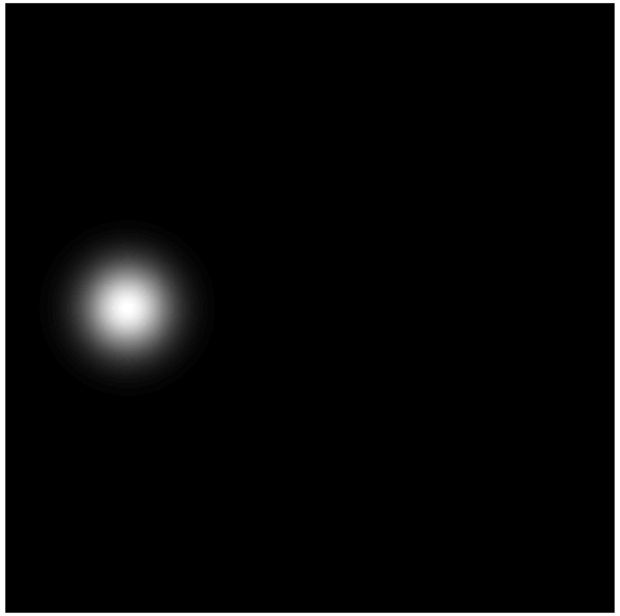
[Images: J. Kukal, 2009]

2D Gaussian in RT

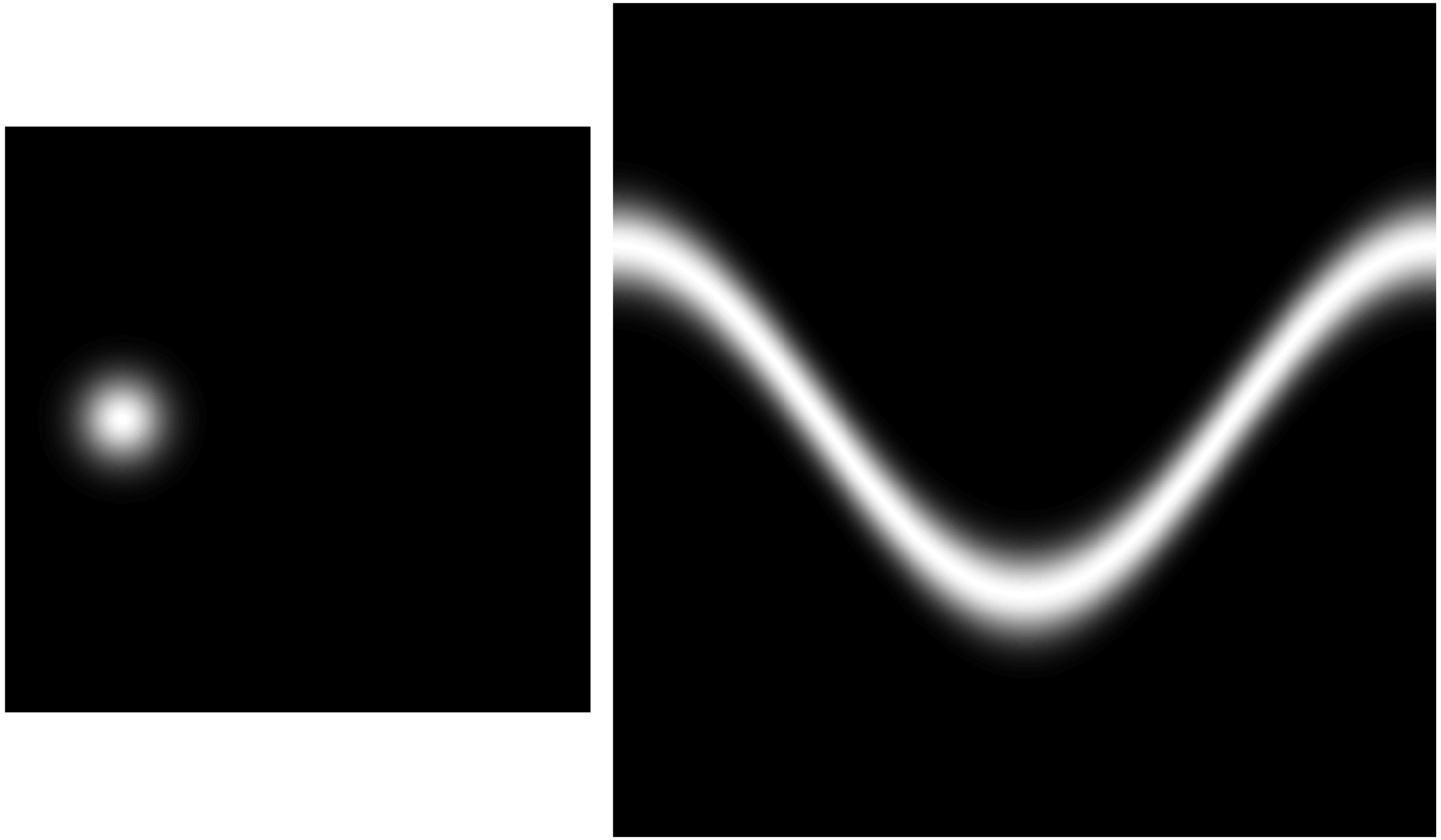
[Images: J. Kukal, 2009]

Shifted 2D Gaussian in RT



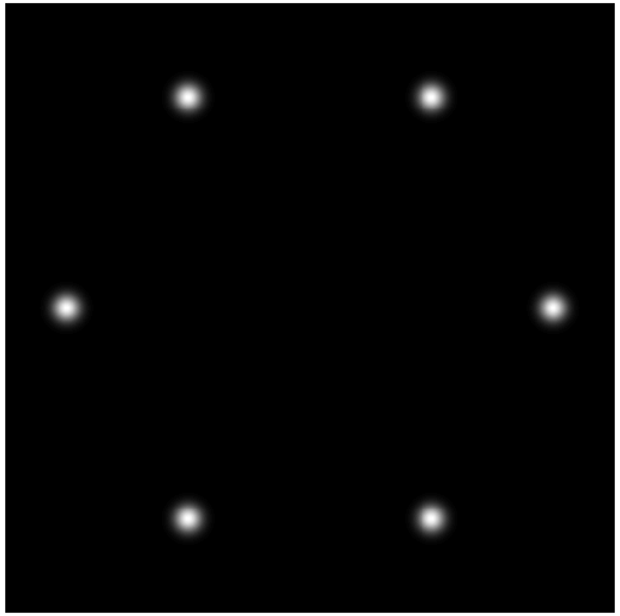
[Images: J. Kukal, 2009]

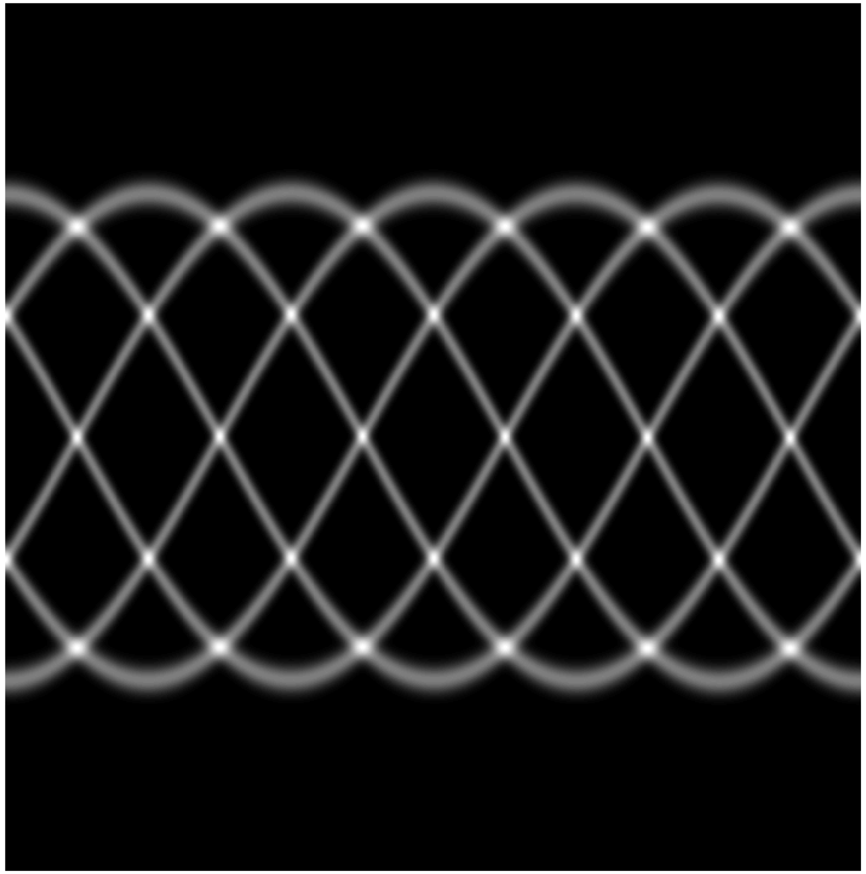
Shifted 2D Gaussian in RT



[Images: J. Kukal, 2009]

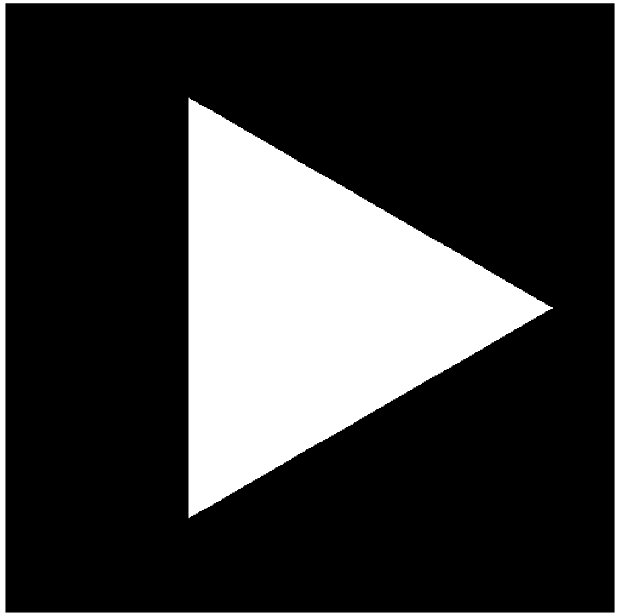
Six 2D Gaussians in RT

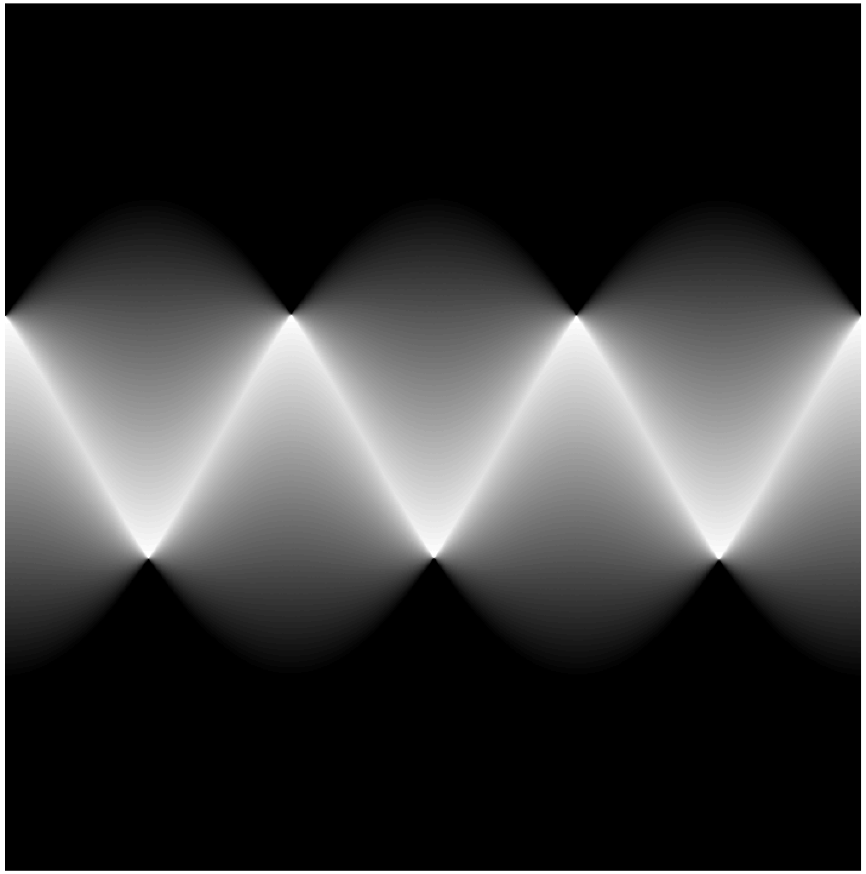




[Images: J. Kukal, 2009]

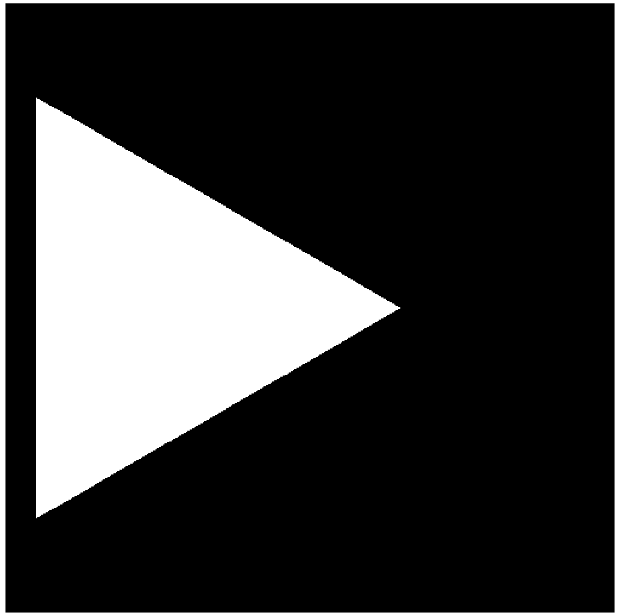
Full triangle in RT

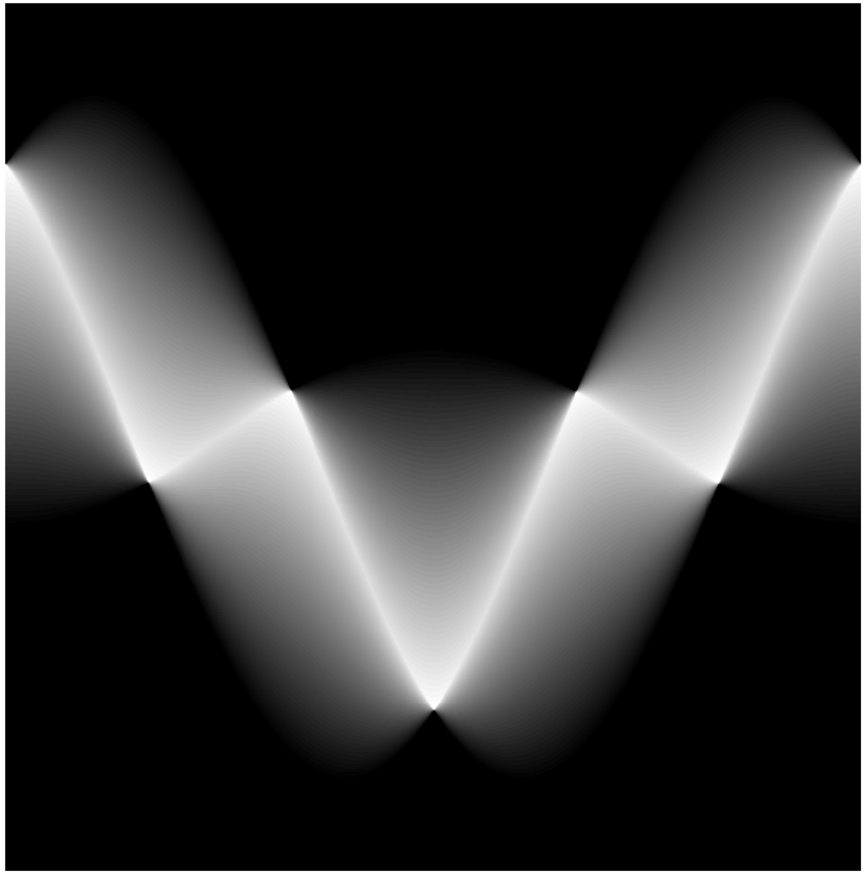




[Images: J. Kukal, 2009]

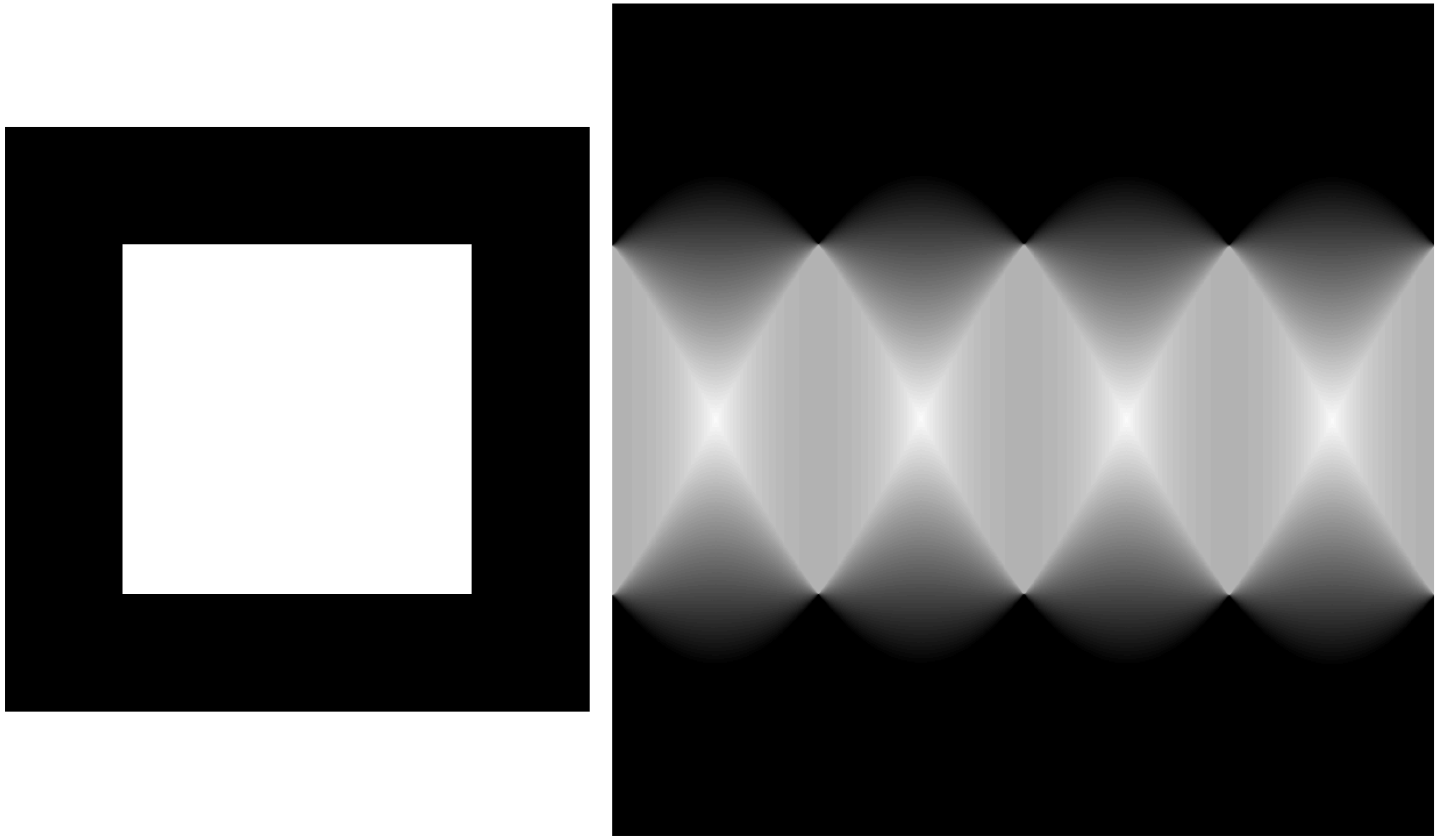
Shifted full triangle in RT





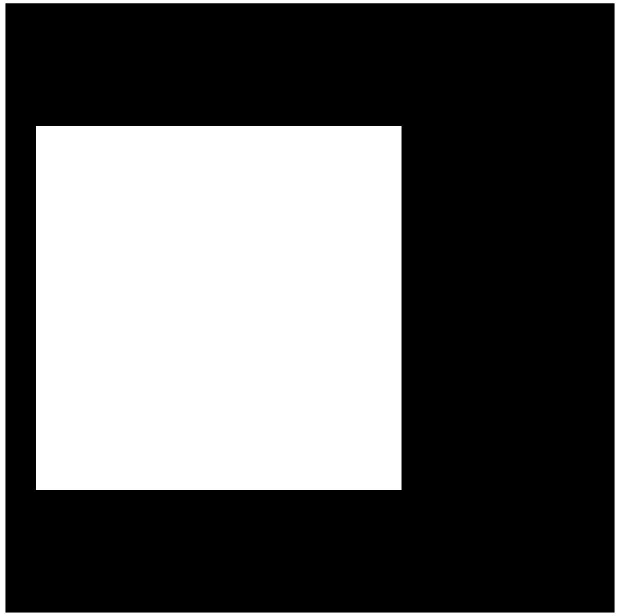
[Images: J. Kukal, 2009]

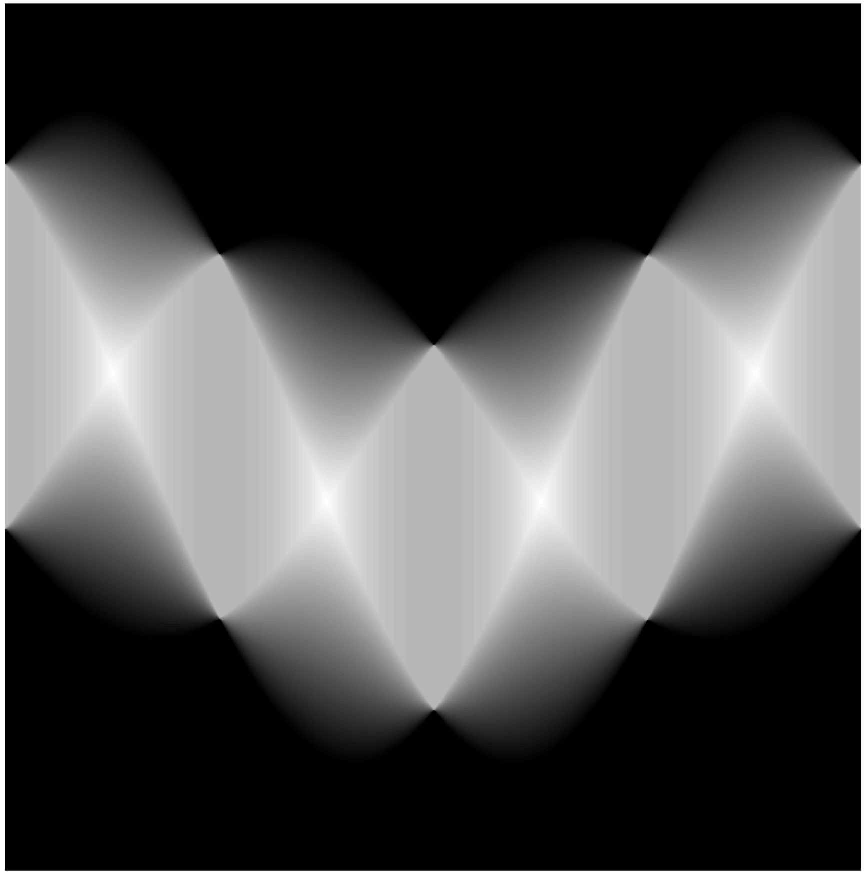
Full square in RT



[Images: J. Kukal, 2009]

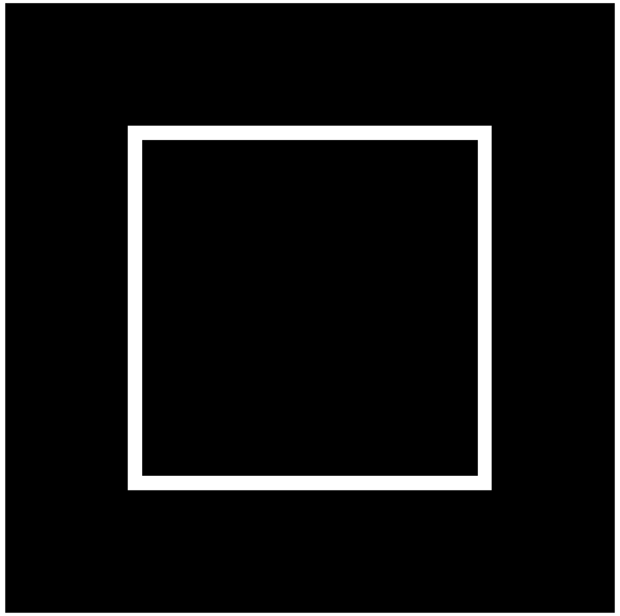
Shifted full square in RT

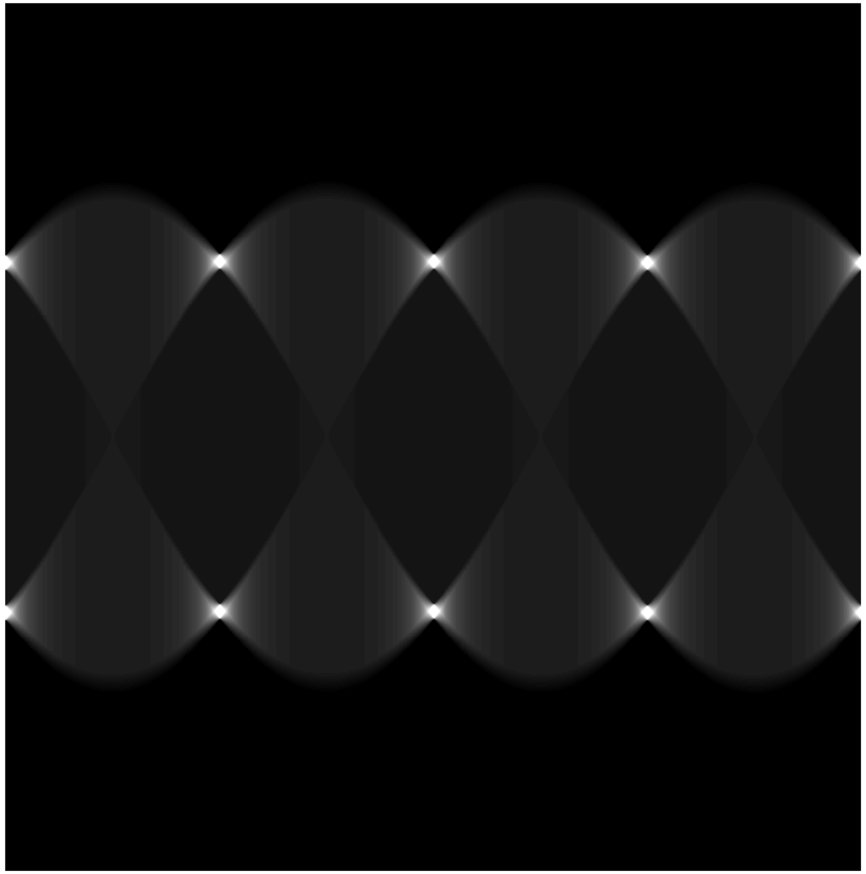




[Images: J. Kukal, 2009]

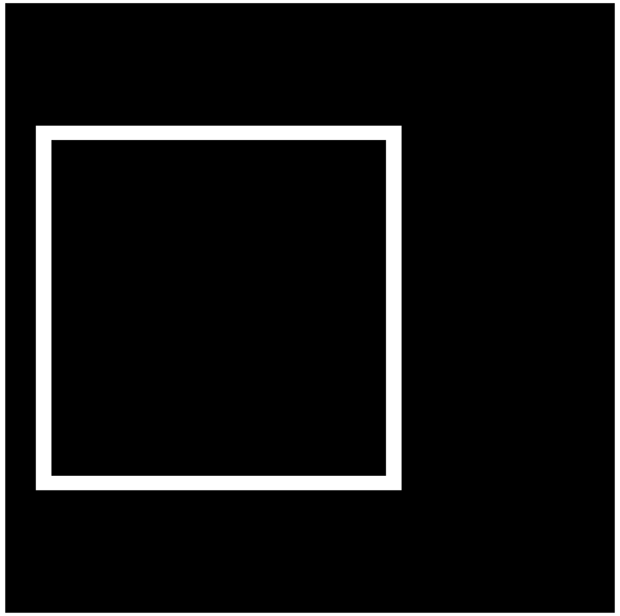
Empty square in RT

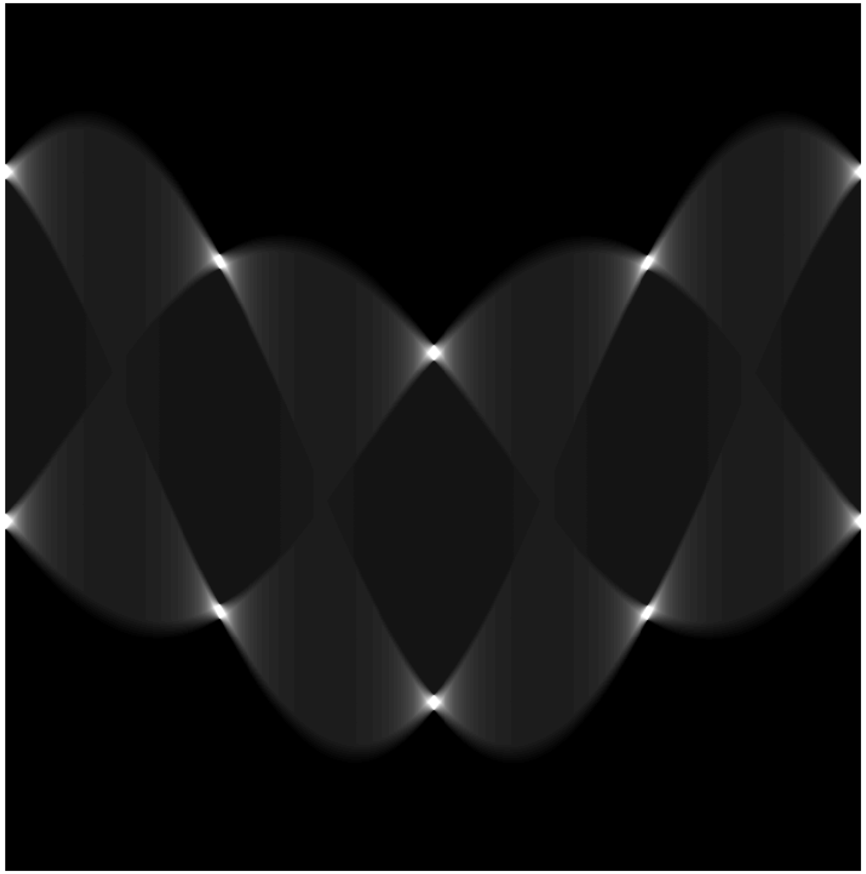




[Images: J. Kukal, 2009]

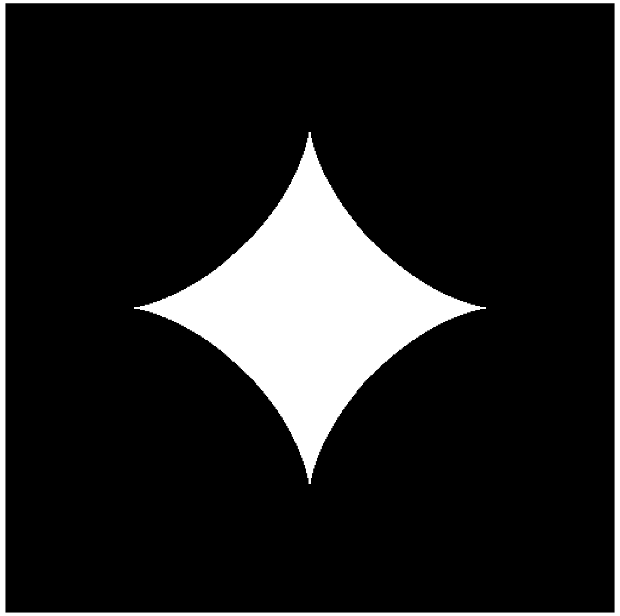
Shifted empty square in RT

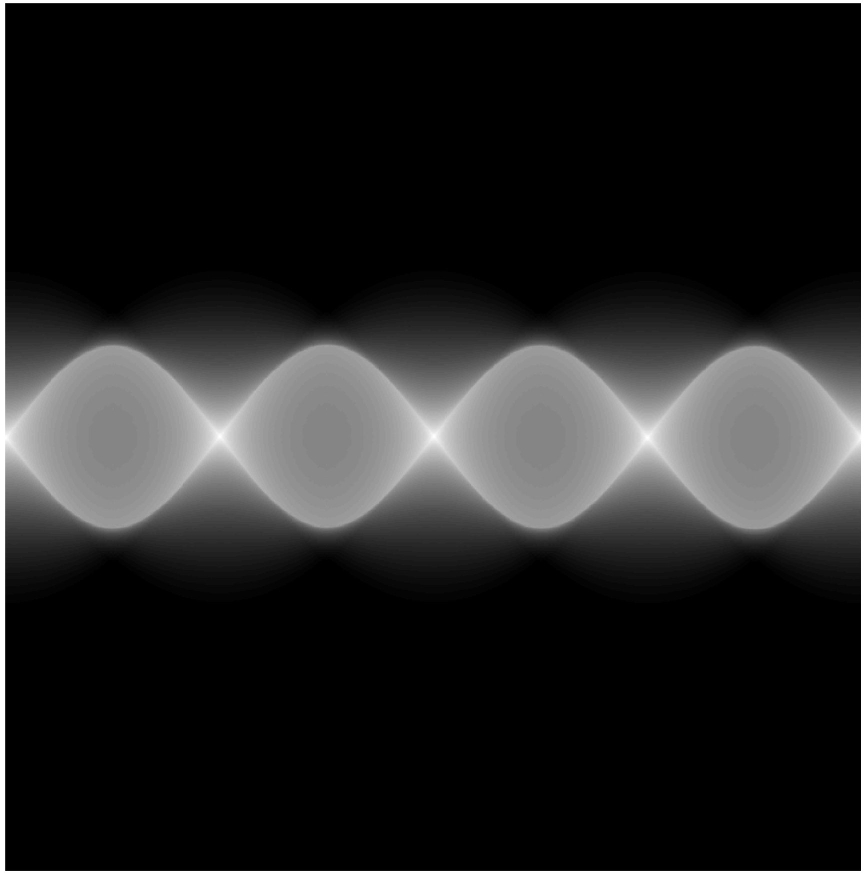




[Images: J. Kukal, 2009]

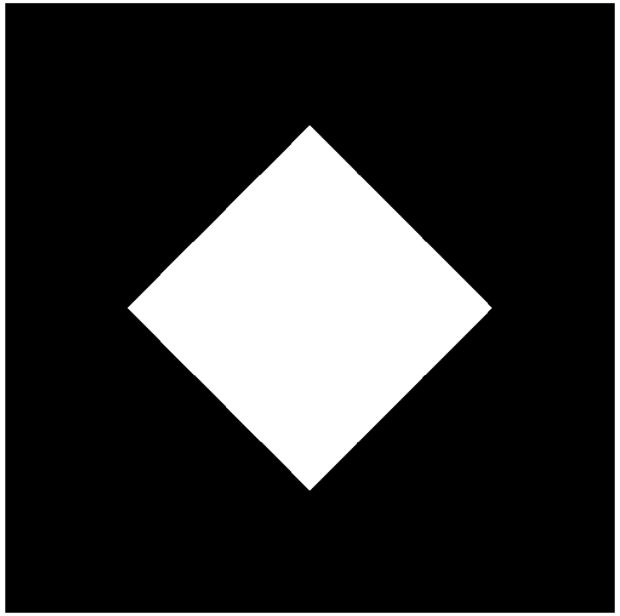
$|x|^{2/3}+|y|^{2/3}\leq 1$ in RT

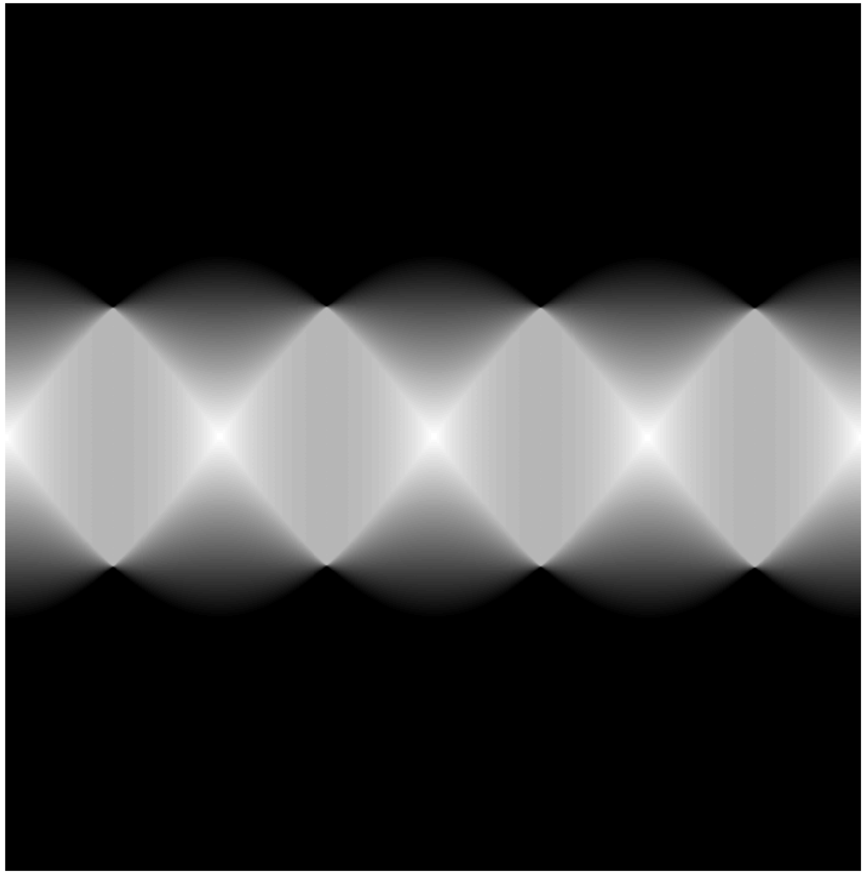




[Images: J. Kukal, 2009]

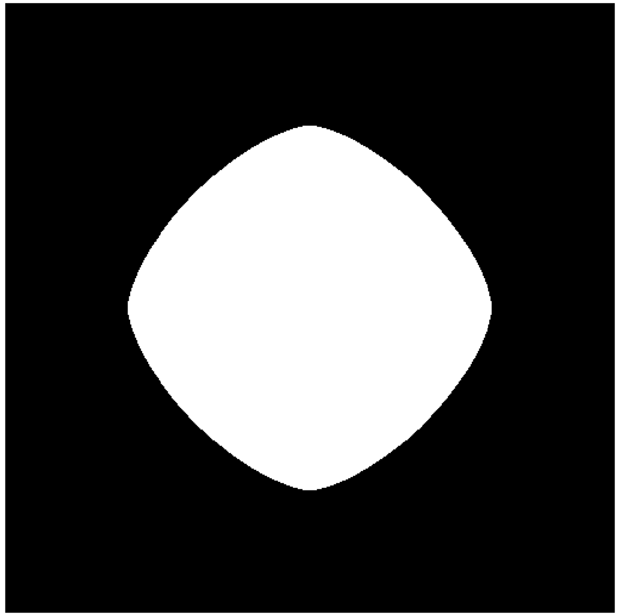
$|x|+|y|\leq 1$ in RT

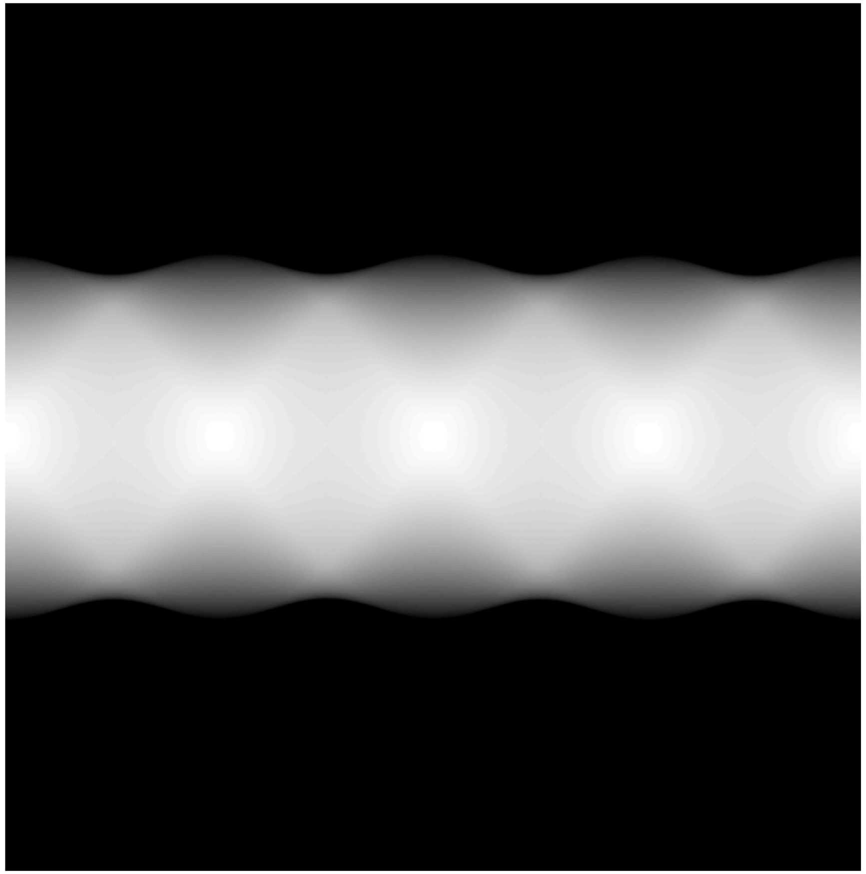




[Images: J. Kukal, 2009]

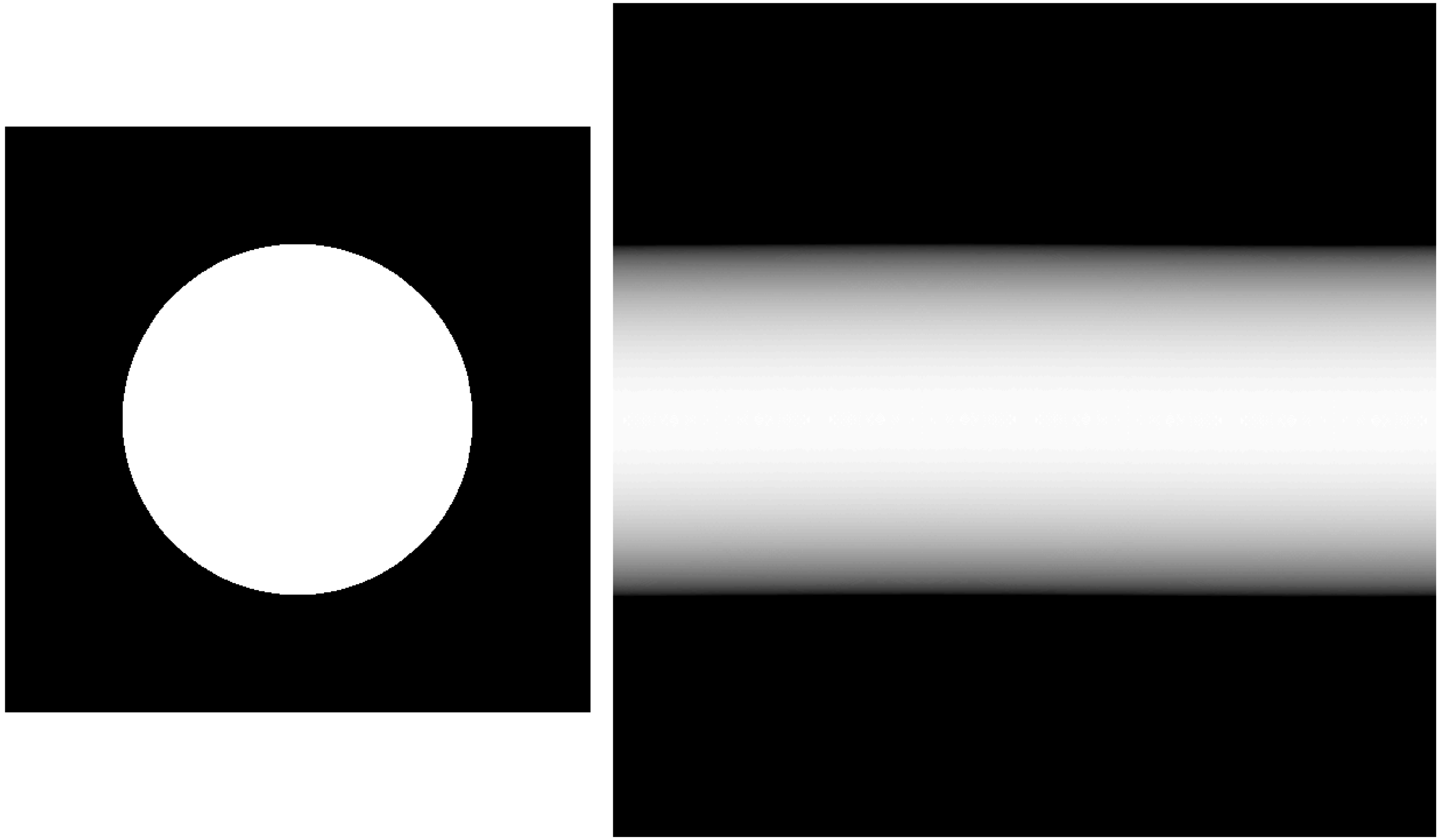
$|x|^{3/2} + |y|^{3/2} \leq 1$ in RT





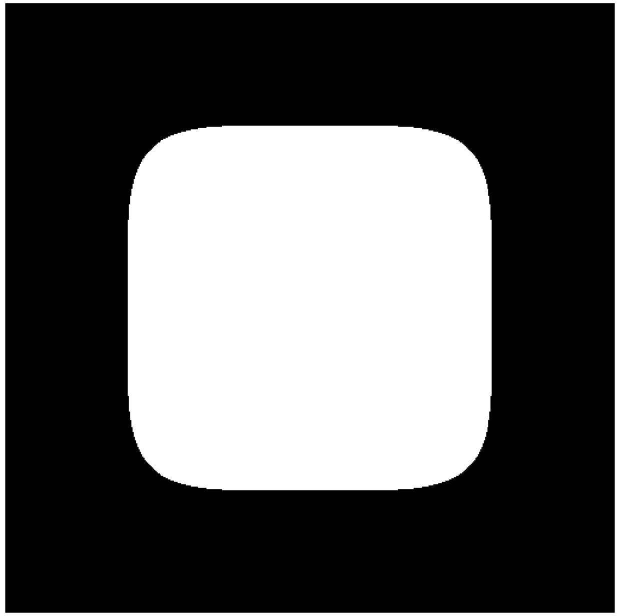
[Images: J. Kukal, 2009]

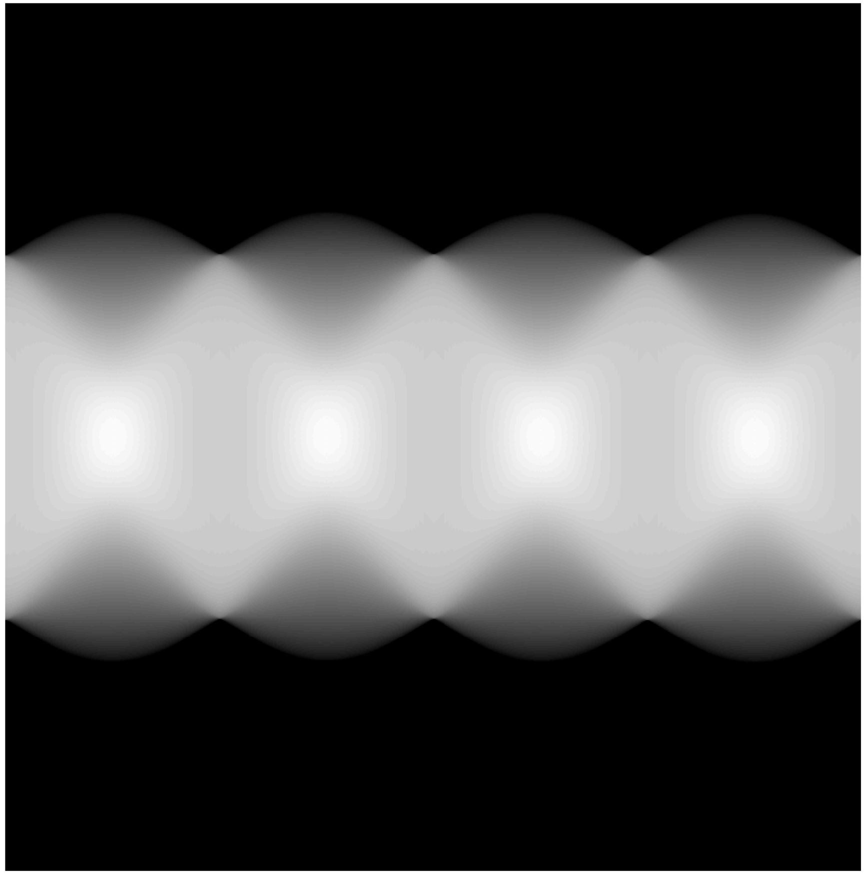
$|x|^2 + |y|^2 \leq 1$ in RT



[Images: J. Kukal, 2009]

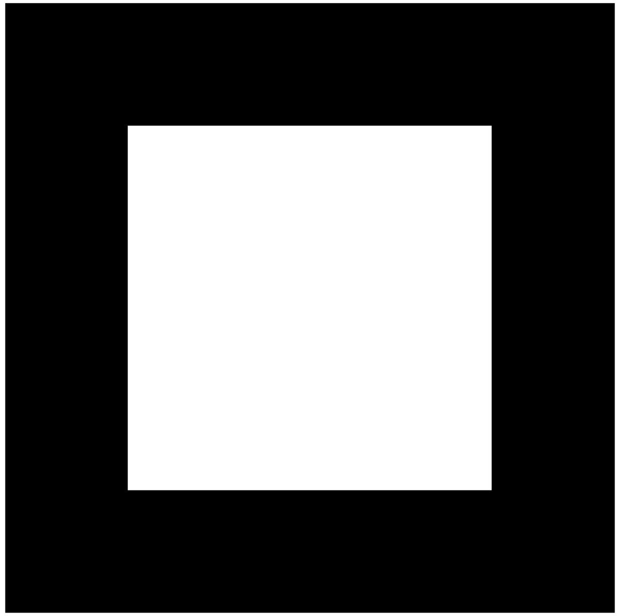
$|x|^6 + |y|^6 \leq 1$ in RT

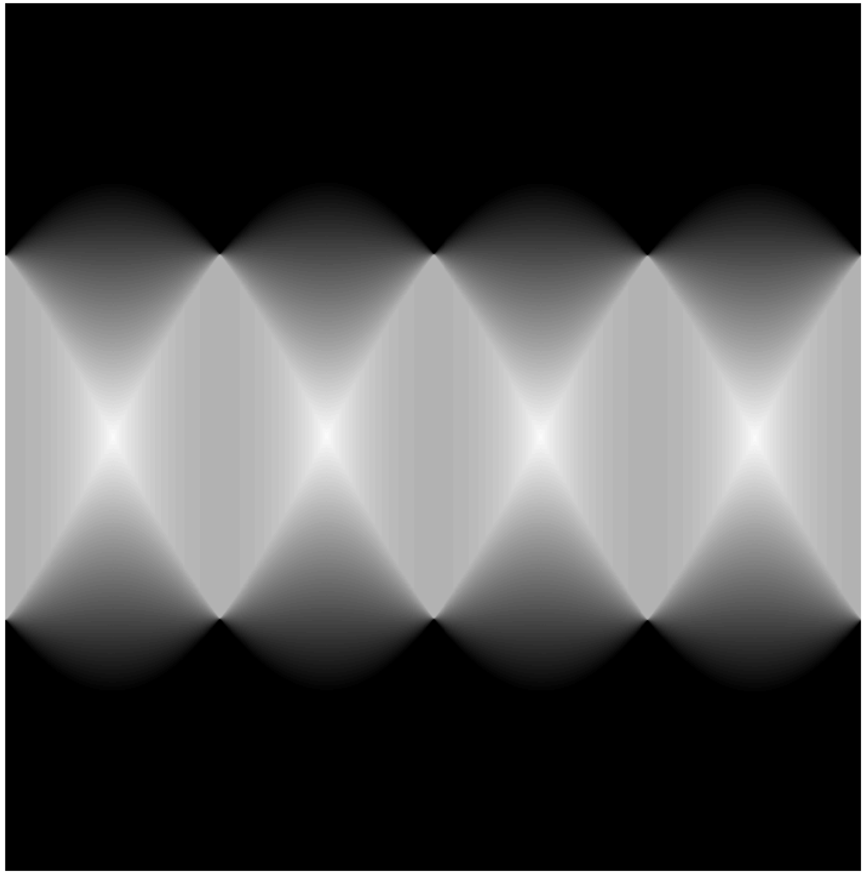




[Images: J. Kukal, 2009]

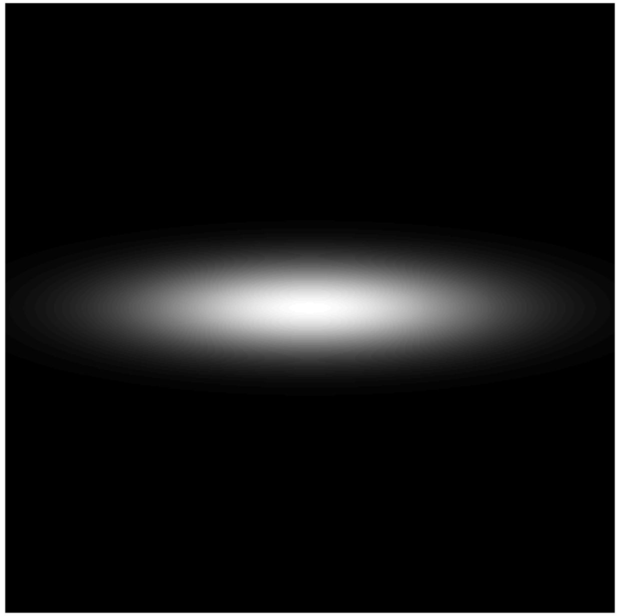
$|x|^n + |y|^n \le 1$ for $n \to \infty$ in RT

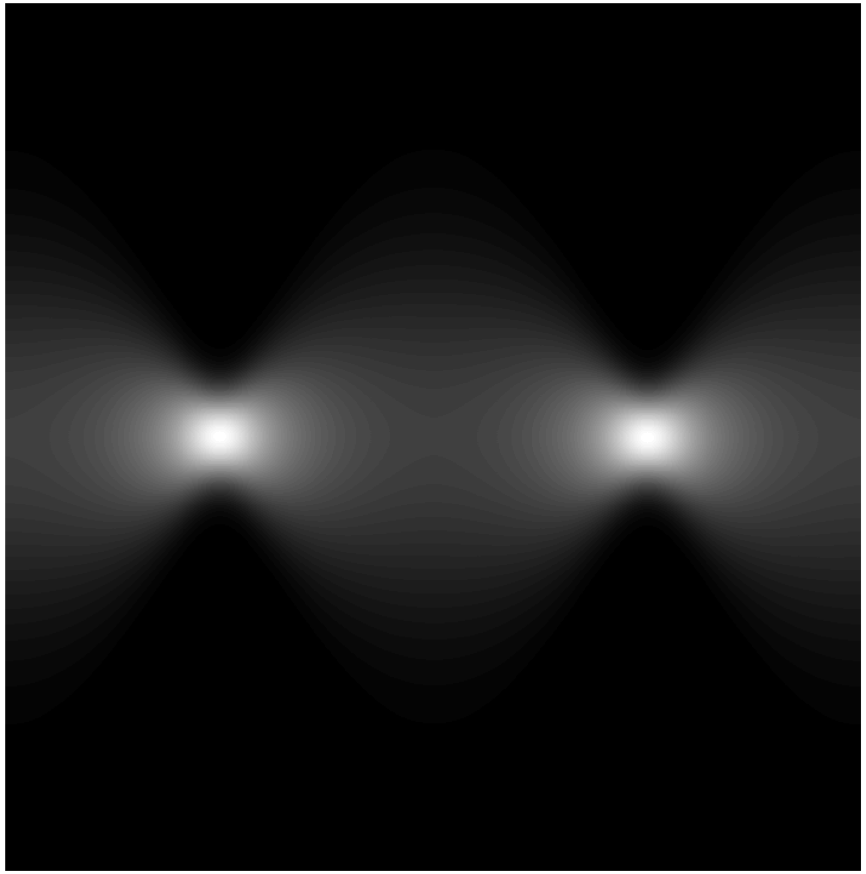




[Images: J. Kukal, 2009]

Smooth elliptic object in RT





[Images: J. Kukal, 2009]

Radon Transform Properties

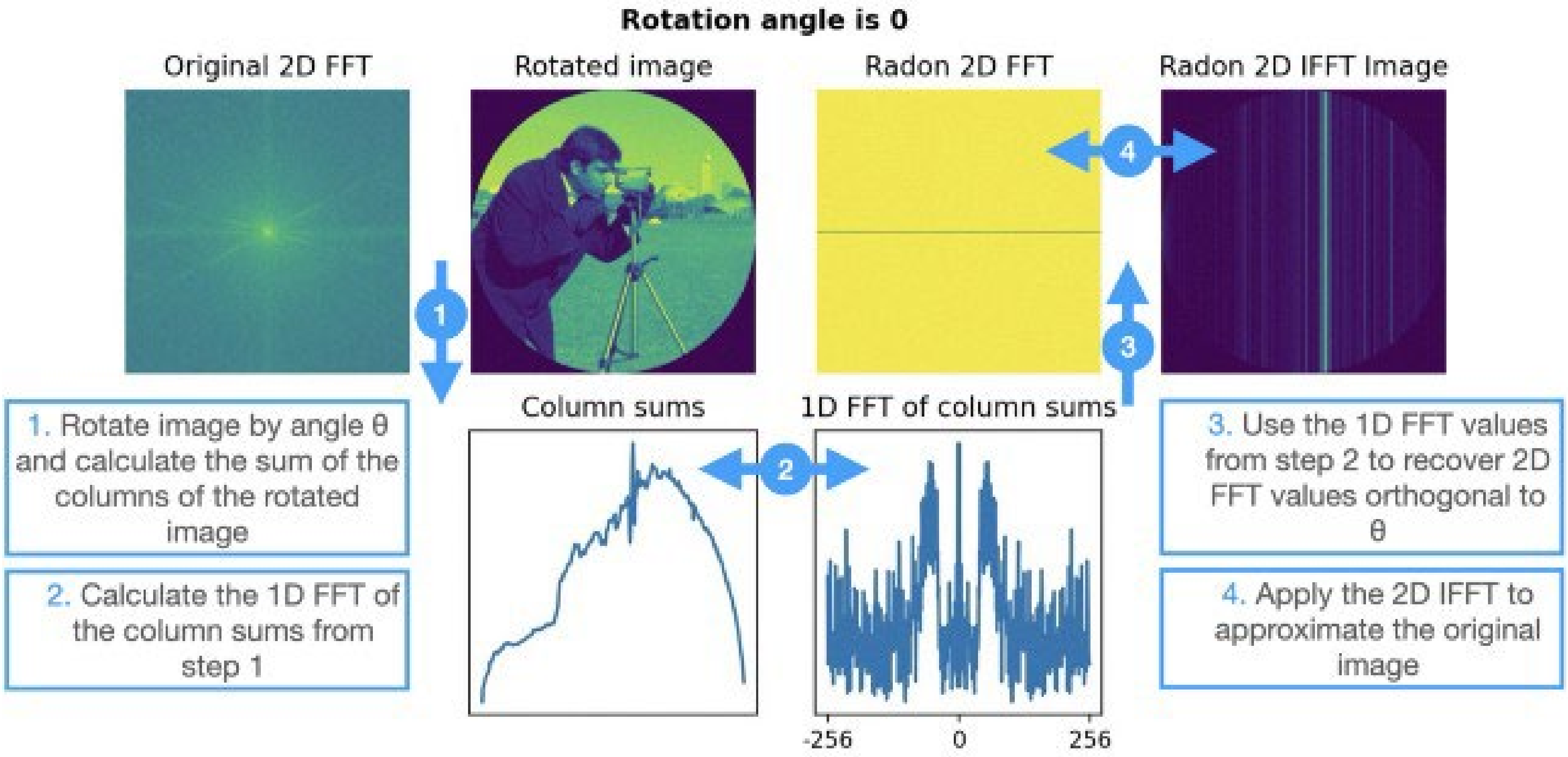
- Image of any f + g is F + G
- Image of cf is cF for any real c
- Rotation of f causes translation of F in θ
- Scaling of f in (x,y) causes scaling of F in s
- Image of a point (2D Dirac function) is sine wave line
- Image of n points is a set of n sine wave lines
- Image of a line is a point (2D Dirac function)
- Image of polygon contour is a point set

Radon Transform

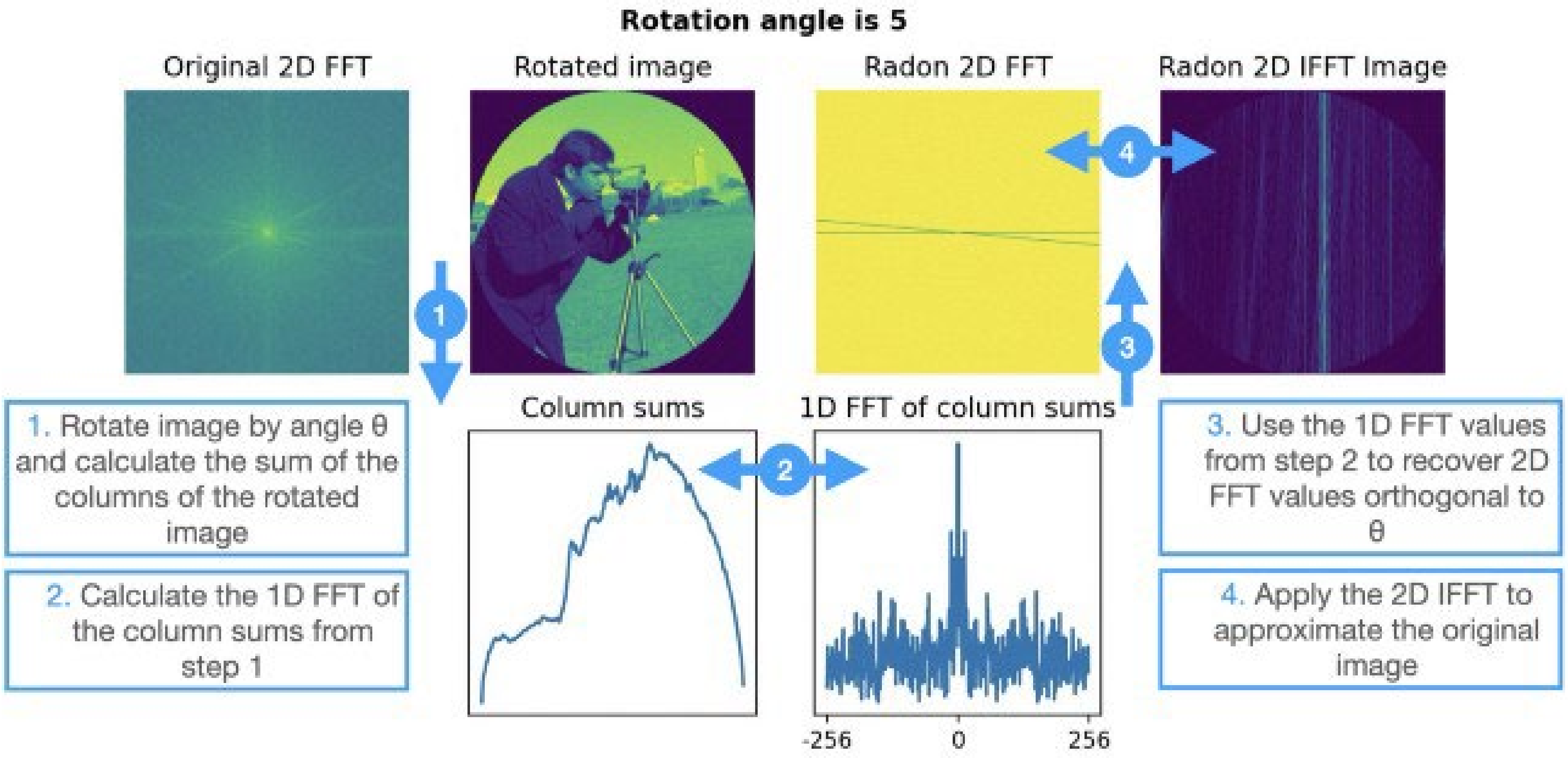
- Often used to reconstruct a 3D volume from 2D slices (in medical applications)
- Why? How?

- Lets jump down to 2D from 1D slices…

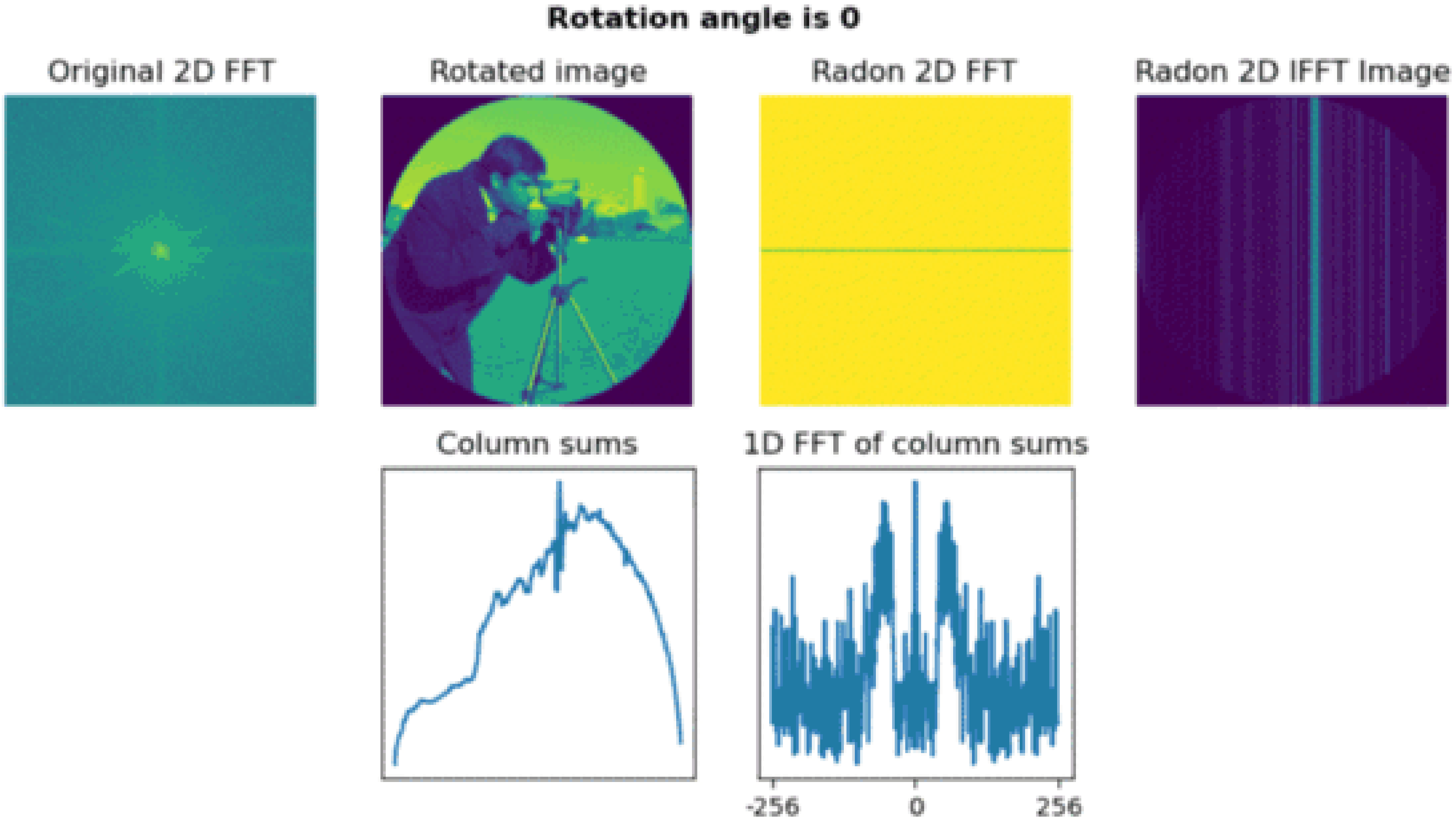
Radon Transform Based Reconstruction



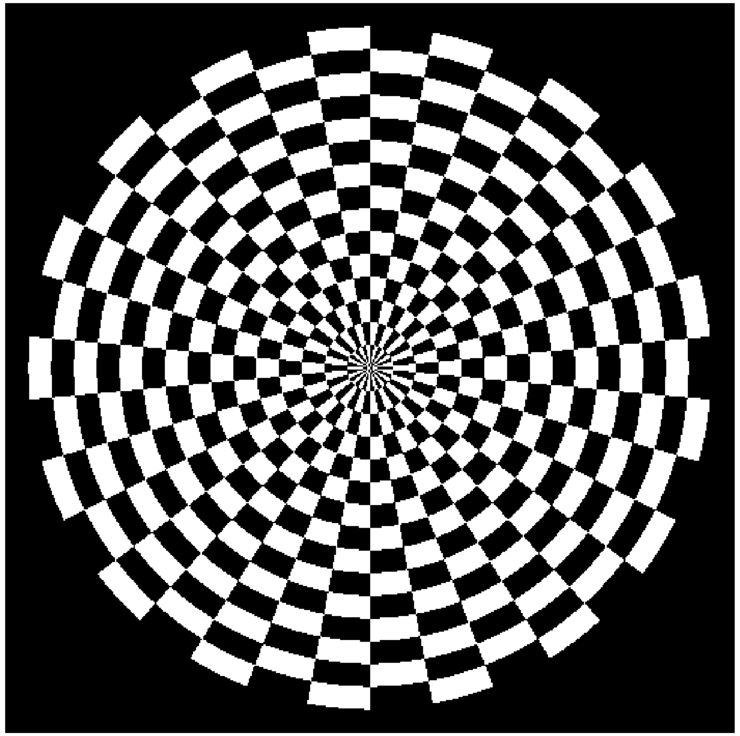
Radon Transform Based Reconstruction

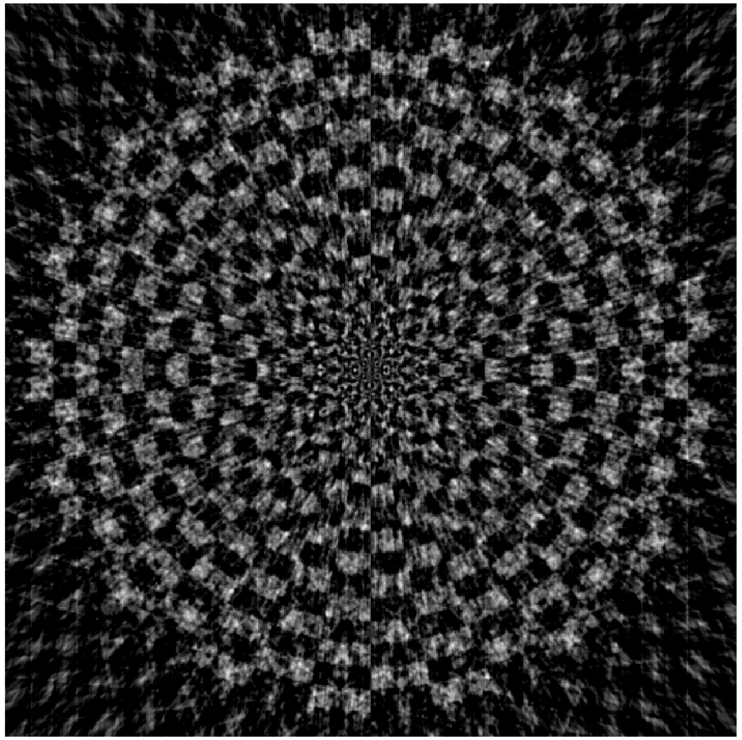


Radon Transform Based Reconstruction



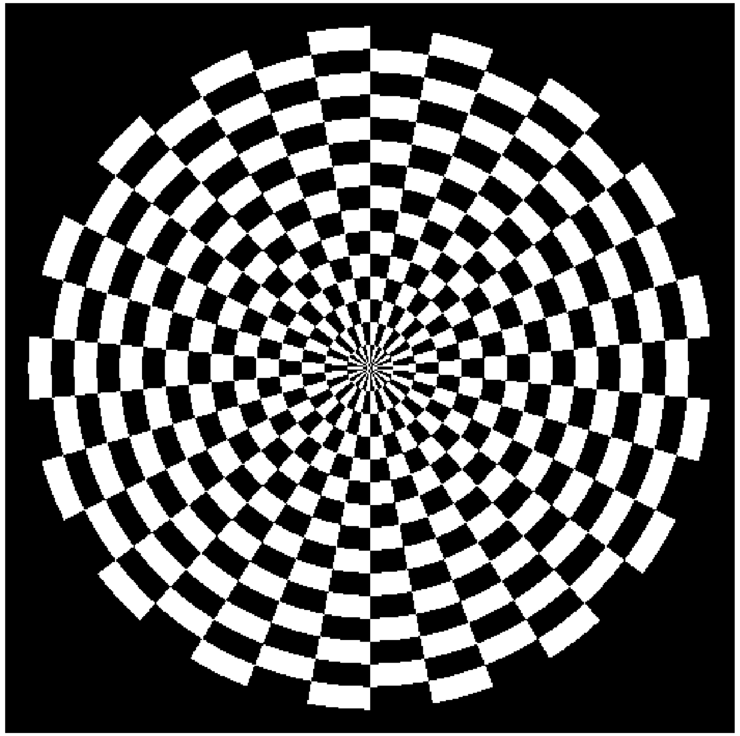
Reconstruction from 32 angles

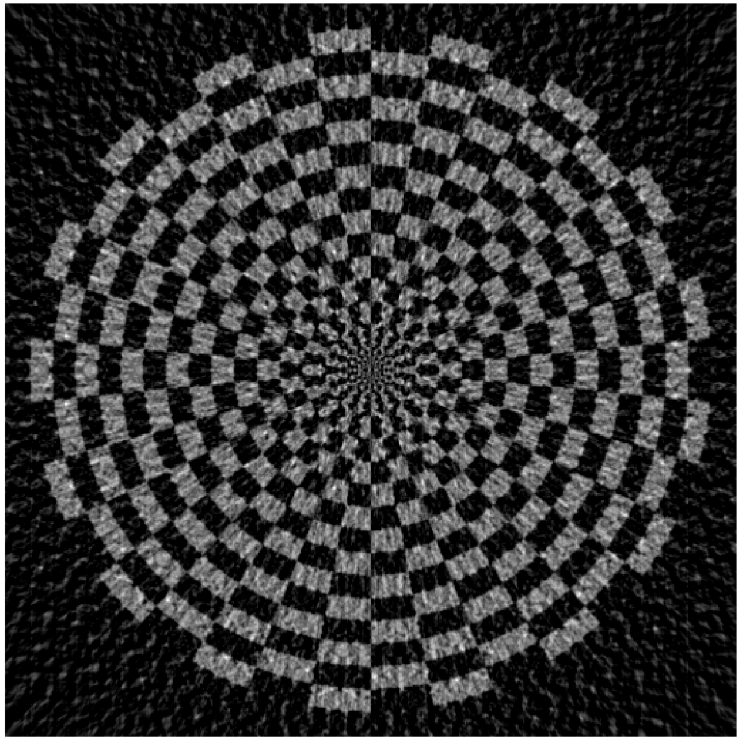




[Images: J. Kukal, 2009]

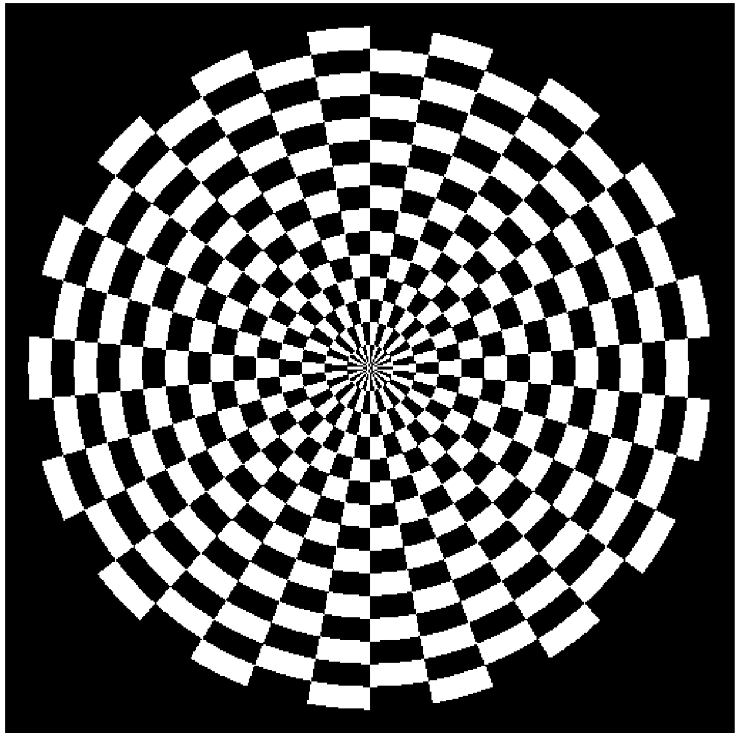
Reconstruction from 64 angles

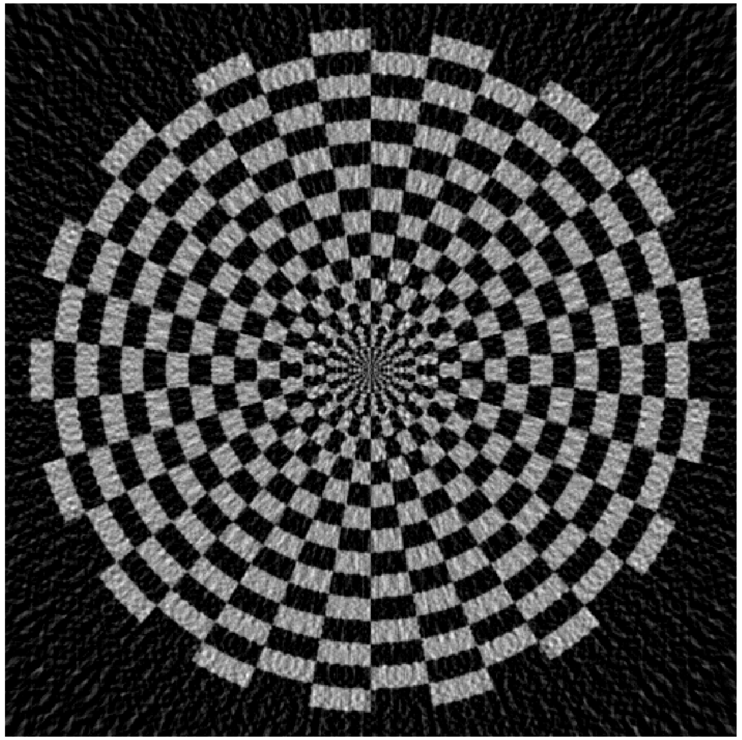




[Images: J. Kukal, 2009]

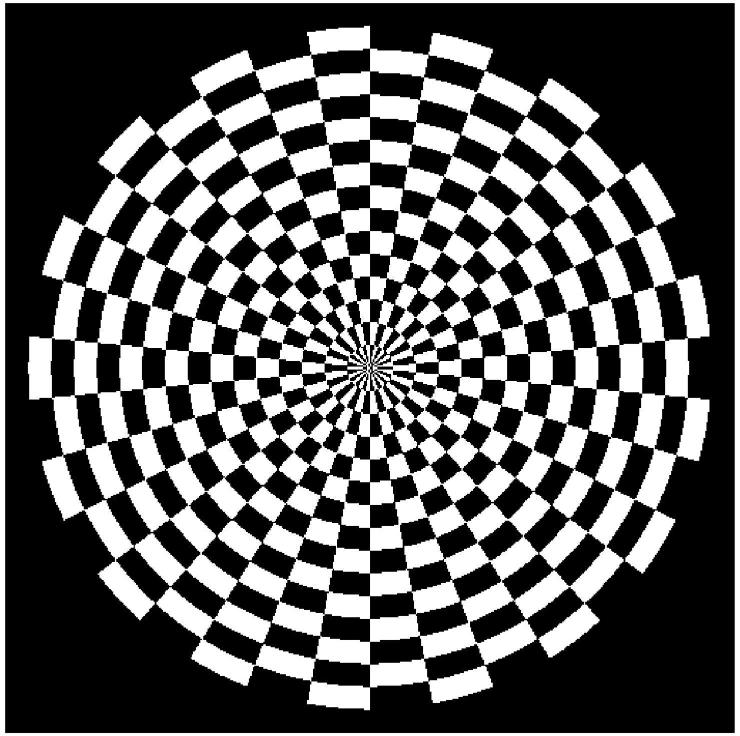
Reconstruction from 96 angles

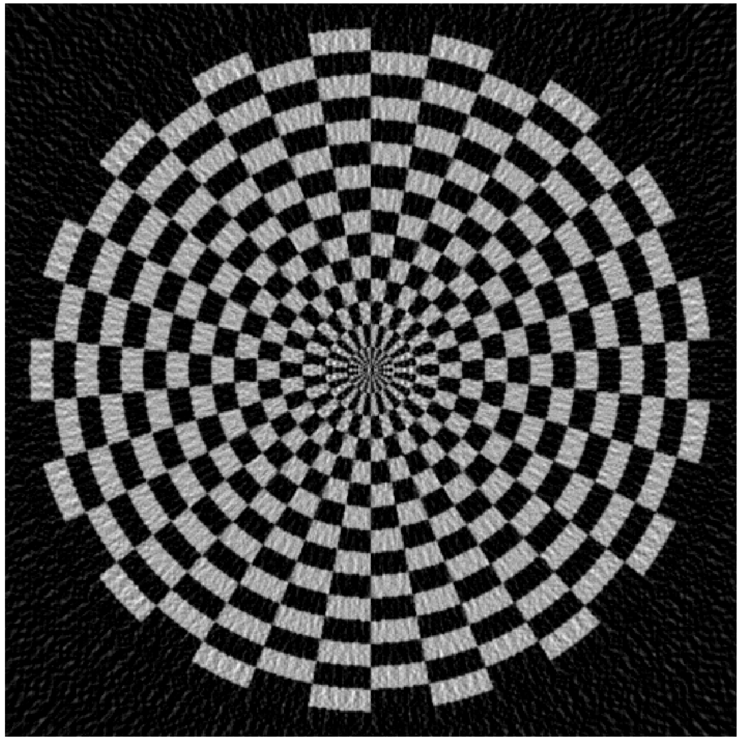




[Images: J. Kukal, 2009]

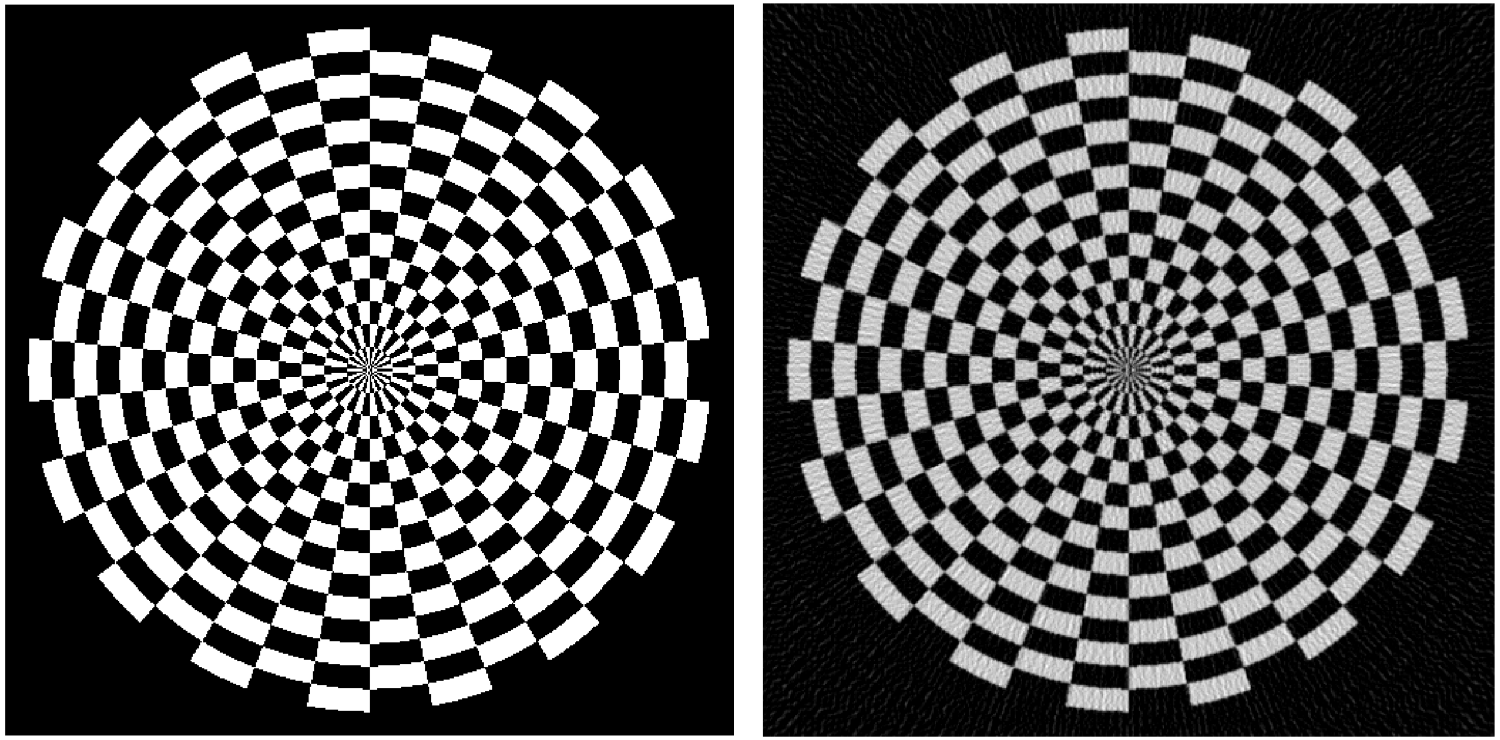
Reconstruction from 128 angles





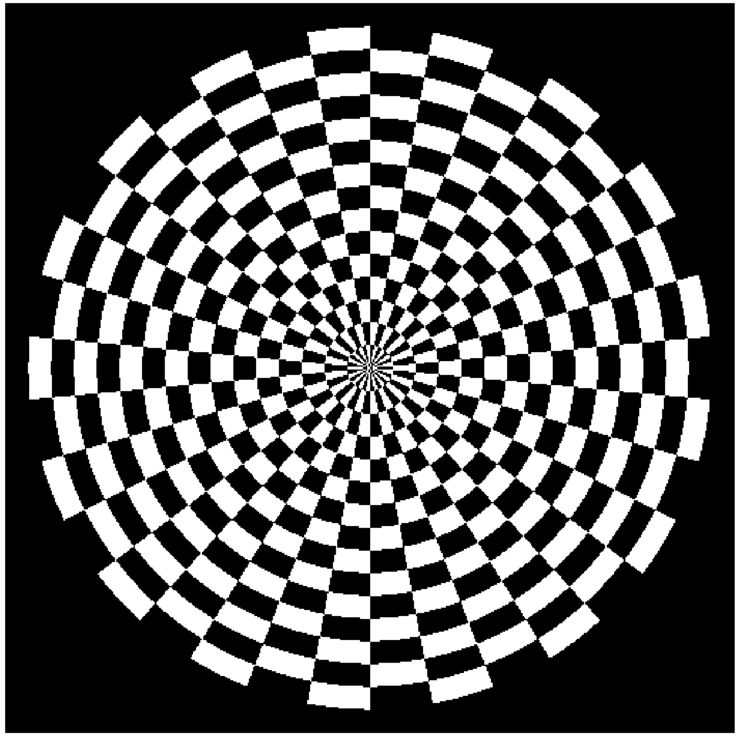
[Images: J. Kukal, 2009]

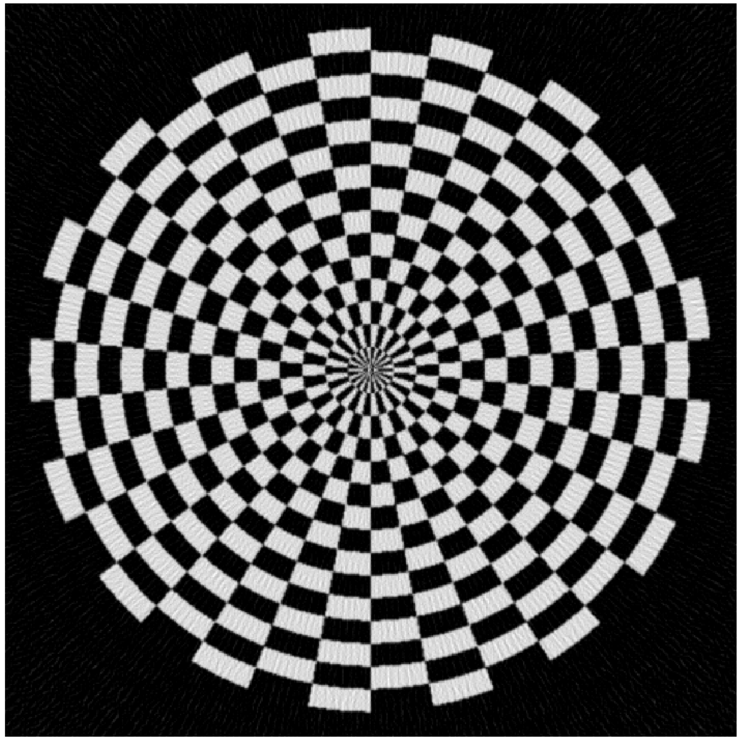
Reconstruction from 180 angles



[Images: J. Kukal, 2009]

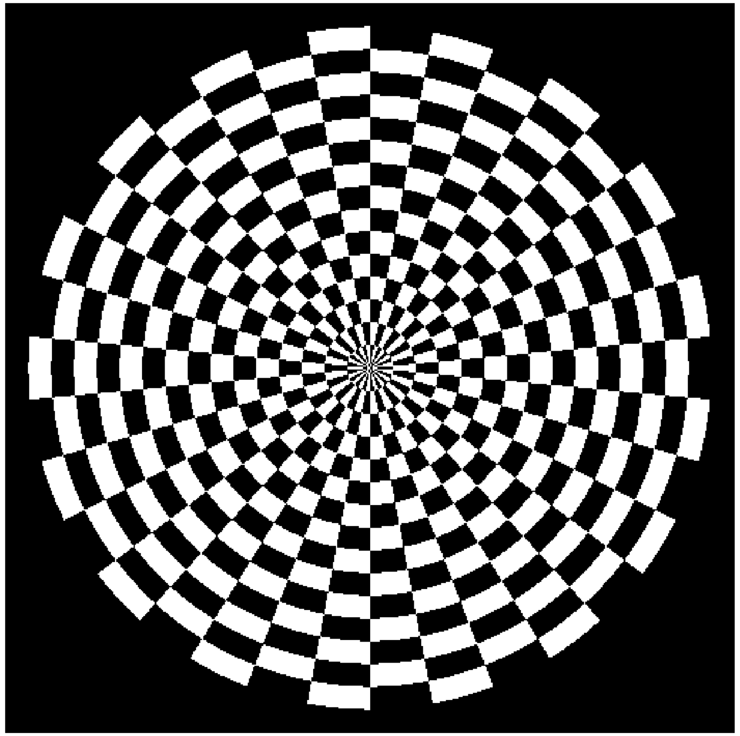
Reconstruction from 256 angles

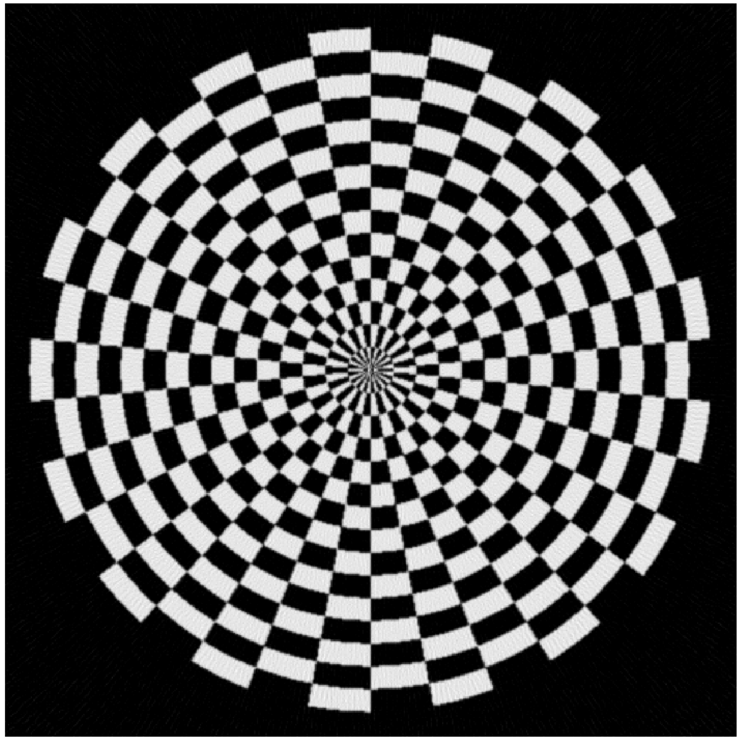




[Images: J. Kukal, 2009]

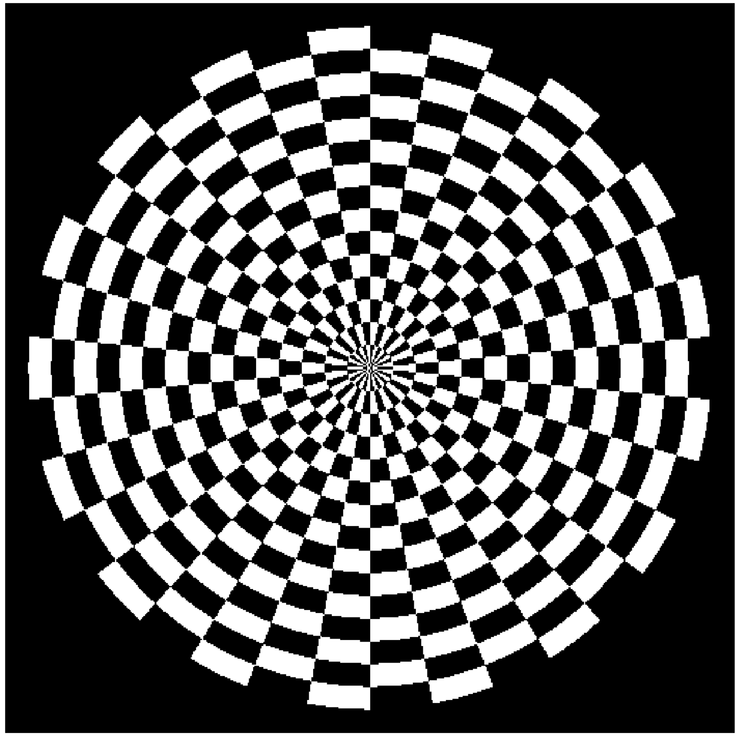
Reconstruction from 360 angles

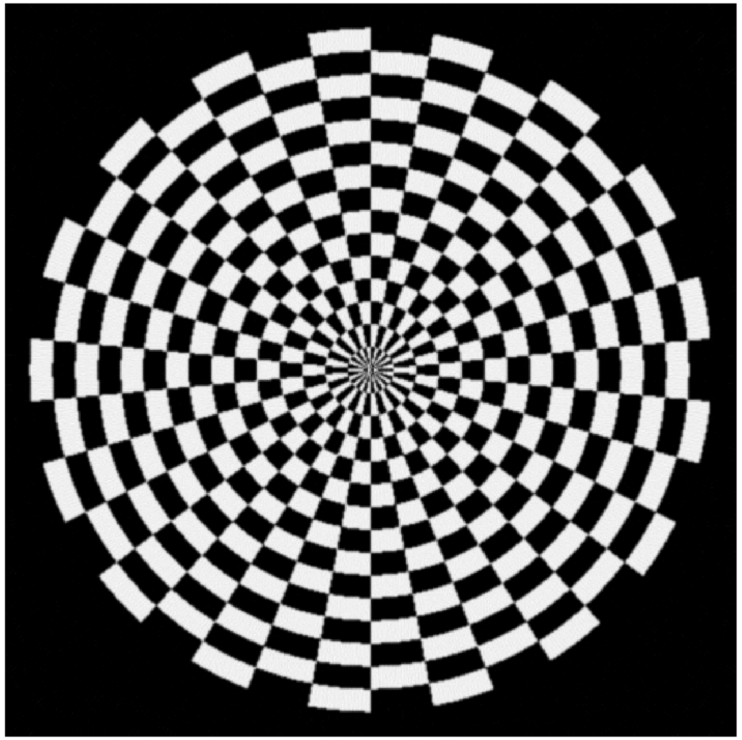




[Images: J. Kukal, 2009]

Reconstruction from 512 angles

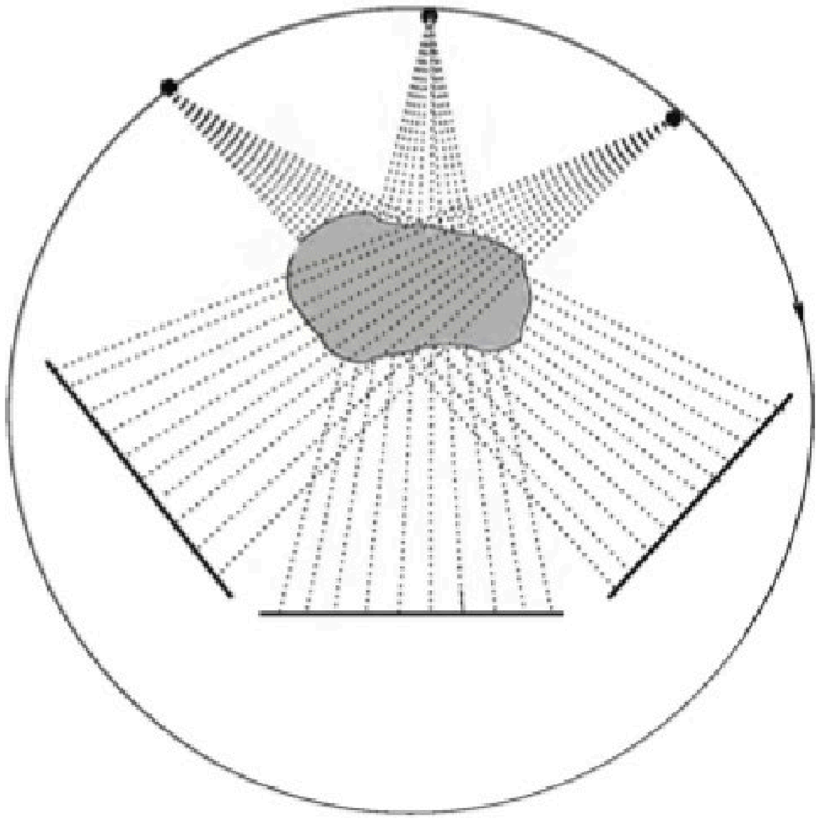




[Images: J. Kukal, 2009]

Medical Tomography

- Upgrade to 3D reconstruction from 2D images…



DL and Radon Transform

- https://ieeexplore.ieee.org/document/8950464

- https://ieeexplore.ieee.org/document/9507793

- chrome-extension://efaidnbmnnnibpcajpcglclefindmkaj/https://iopscience.iop.org/article/10.1088/1361-6420/aba415/pdf