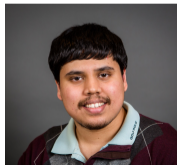


Estimation of Skill Distribution from a Tournament

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Laboratory for Information & Decision Systems
Massachusetts Institute of Technology

Conference on Neural Information Processing Systems (NeurIPS)
6-12 December 2020



- 1 Introduction
 - Motivation and Goal
 - Experiments
- 2 Contributions

Motivation and Goal

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Lord's the perfect venue for greatest one-day game ever played

England cricket team



Andy Bull at Lord's
Sun 14 Jul 2015 14:58 EDT
107 657

England's two Test matches will be marred after evening and New Zealand's Martin Guptill, Philadelphia Star-KUWRT/Getty Images

Where better for England's transgenerational trauma to end than the home of cricket, pretty as a picture in the evening sun?


The sun was out when England won the World Cup. It had emerged, at last, from behind the fleckles of white cloud and was shining low through the bright blue sky over the Grand Stand. The shadows stretched all the way to the wicket, the flags licked in the evening breeze, the pavilion glowed soft terracotta. The old place looked pretty as a picture, exactly how we see it in winter when thinking back on the games we saw, and the games we played, in the long, warm days of summer. For the 11 men in this England team, for the thousands in the ground, for the millions watching on TV, that is how they will always see it in their minds, years from now, when they think back on this match: the greatest World Cup final, back

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Often hailed the world's best, England's Premier League has earned that title this season

Doag McEneaney
Yahoo Sports May 3, 2015, 9:11 PM
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With Liverpool and Tottenham in the Champions League final, an English club will be crowned Europe's best for the first time since Chelsea (right) in 2012. (Lew Smith/Action)

Before Liverpool came back from 3-0 down earlier this week to shock Lionel Messi and Barcelona and advance to a second straight Champions League final, before Tottenham Hotspur pulled off a miracle of its own the next day against Ajax to join the Reds in the title match, even before Arsenal and Chelsea won their respective Europa League semis on Thursday to ensure that the continent's two major club tournament deciders would all-English affairs, this had been a banner season for the Premier League in European competition.

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Can we measure the level of skill in a game based on win-loss data from tournaments?

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Andy Bull at Lord's
Sun 14 Jul 2015 14:56 EDT
107 657

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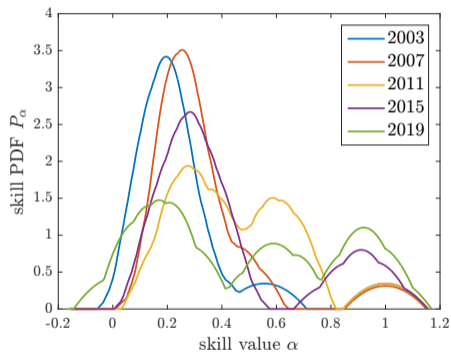
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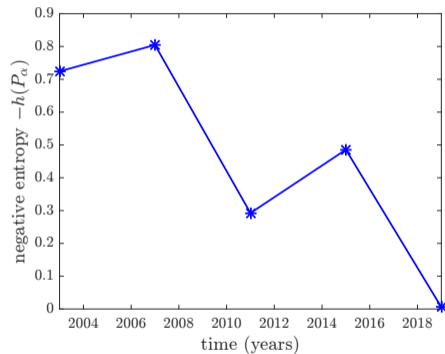
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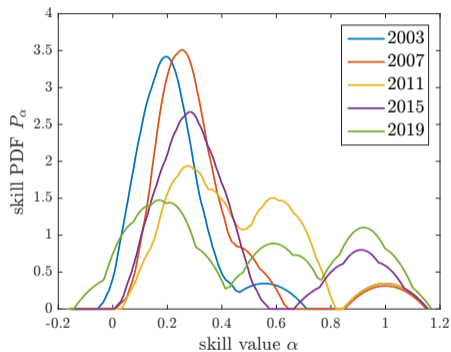
Estimated Skill Densities from Tournament Data



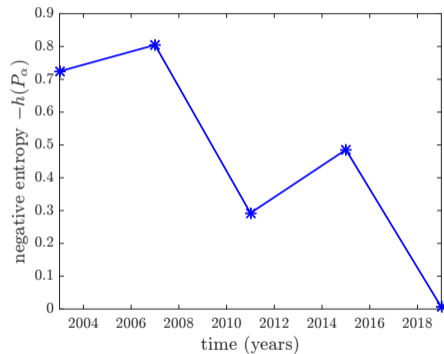
Negative Differential Entropies of Estimated Skill Densities



Estimated Skill Densities from Tournament Data

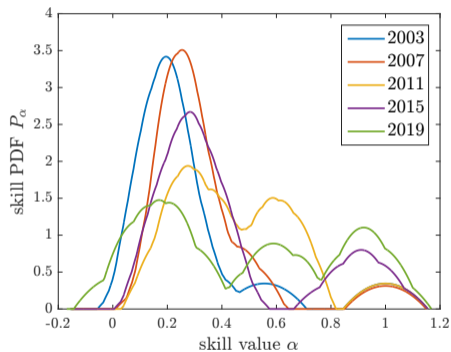


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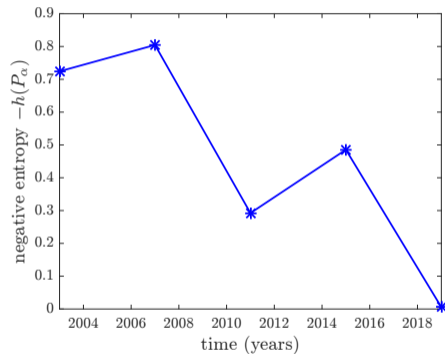


- **Entropy skill score:** Measures holistic **variation of skill levels** of teams
- High score = more “luck”, low score = more skill

Estimated Skill Densities from Tournament Data

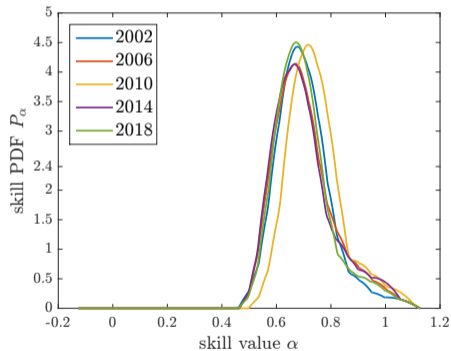


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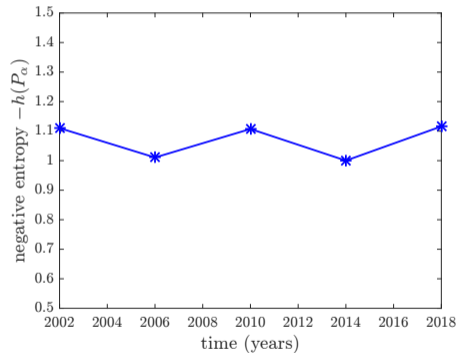


- **Entropy skill score:** Measures holistic variation of skill levels of teams
- High score = more “luck”, low score = more skill
- **Observation:** Skill scores of cricket world cup tournaments is decreasing

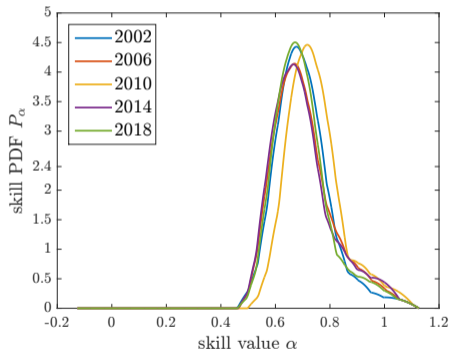
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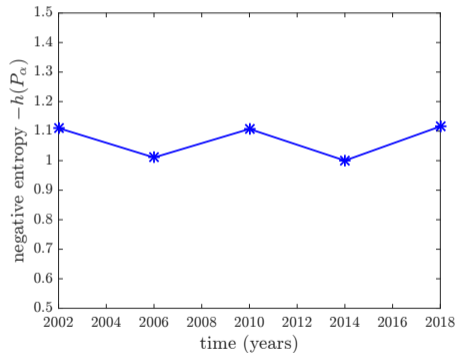
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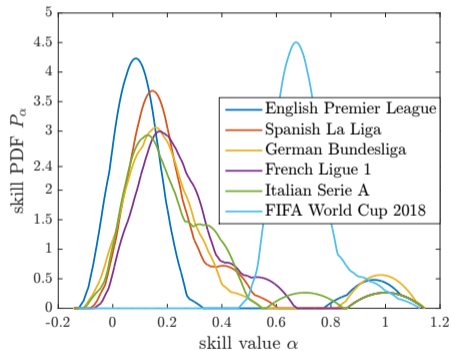


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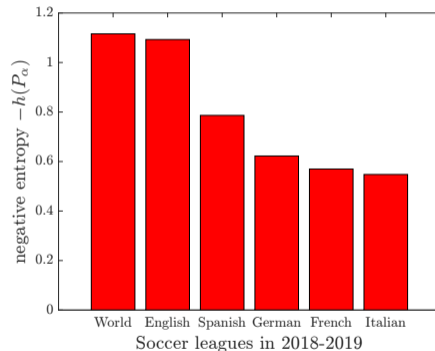


- **Observation:** Soccer world cups have remained unpredictable over the years

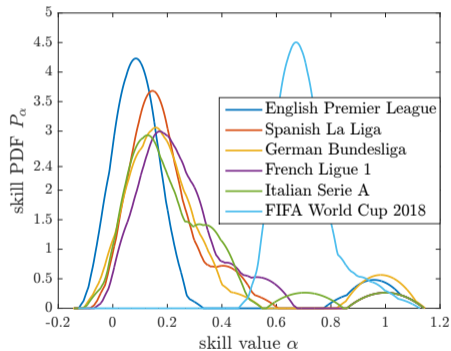
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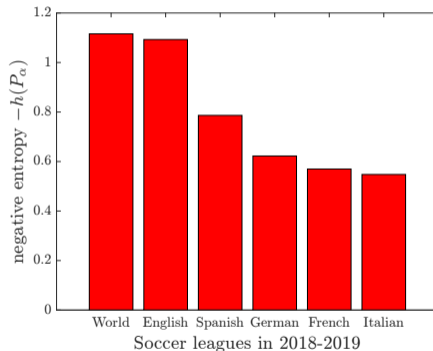
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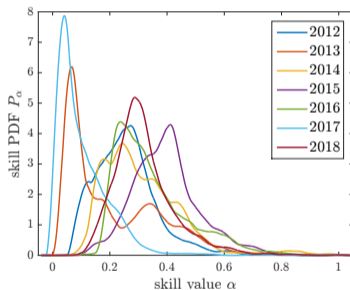
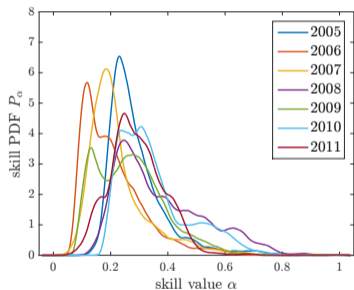


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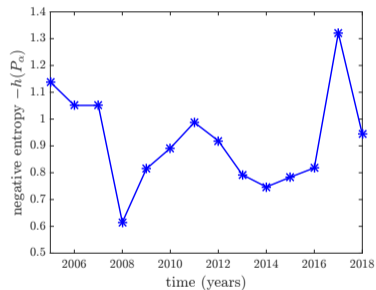


- **Observation:** Recover ranking of soccer leagues that is consistent with fan experience

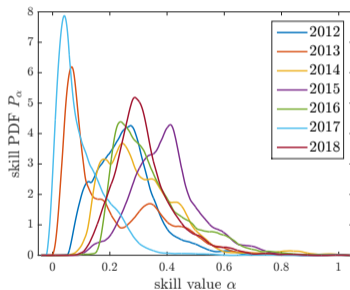
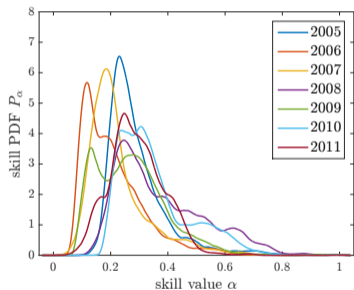
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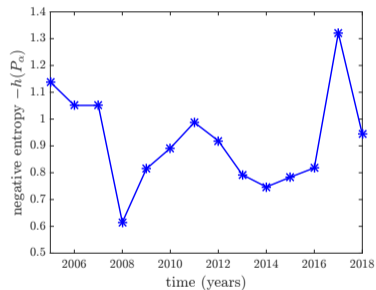
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Negative Entropies of Estimated Skill Densities



- **Observation:** Skill score is minimum during the **Great Recession in 2008**

1 Introduction

2 Contributions

- Formal Setup
- Estimation Algorithm
- Theoretical Results

Formal Setup

- Unknown probability density of skill levels P_α on \mathbb{R}_+

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- **Goal:** Learn P_α from observation matrix $Z \in [0, 1]^{n \times n}$ with

$$Z(i, j) = \begin{cases} \mathbb{1}\{\text{games observed between } i, j\} \frac{1}{k} \sum_{m=1}^k Z_m(i, j), & i \neq j \\ 0, & i = j \end{cases}$$

Estimation Algorithm

Assume P_α is bounded, in an η -Hölder class, and has support in $[\delta, 1]$.

Algorithm Estimating P_α from Z

Input: Observation matrix Z

Output: Estimator \hat{P}^* of unknown P_α

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Step 1: Skill parameter estimation using rank centrality algorithm [NOS12, NOS17]

- 1: Construct stochastic matrix $S \in \mathbb{R}^{n \times n}$ with $S(i, j) = \frac{Z(i, j)}{2np}$ for $i \neq j$, whose rows sum to 1
- 2: Compute leading left eigenvector $\hat{\pi}_*$ of S such that $\hat{\pi}_* = \hat{\pi}_* S$
- 3: Compute skill level estimates $\hat{\alpha}_i = \frac{\hat{\pi}_*(i)}{\|\hat{\pi}_*\|_\infty}$ for $i = 1, \dots, n$

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Step 2: Kernel density estimation using Parzen-Rosenblatt method [Ros56, Par62]

- 4: Compute bandwidth $h = \Theta(\log(n)^{\frac{1}{2\eta+2}} n^{-\frac{1}{2\eta+2}})$
 - 5: Construct $\hat{\mathcal{P}}^*$ using appropriate kernel $K : [-1, 1] \rightarrow \mathbb{R}$: $\hat{\mathcal{P}}^*(x) \triangleq \frac{1}{nh} \sum_{i=1}^n K\left(\frac{\hat{\alpha}_i - x}{h}\right)$
 - 6: **return** $\hat{\mathcal{P}}^*$
-

Summary of Minimax Estimation Results: (in red)

Estimation problem	Loss function	Upper bound	Lower bound
Smooth skill density	mean squared error	$\tilde{O}(n^{-1+\varepsilon})$	$\Omega(n^{-1})$ [IK82, Tsy09]
BTL skill parameters	relative ℓ^∞ -norm	$\tilde{O}(n^{-1/2})$ [CFMW19]	$\tilde{\Omega}(n^{-1/2})$
BTL skill parameters	ℓ^1 -norm	$O(n^{-1/2})$ [CFMW19]	$\tilde{\Omega}(n^{-1/2})$

Note: \tilde{O} and $\tilde{\Omega}$ hide $\text{polylog}(n)$ terms, and $\varepsilon > 0$ is any arbitrarily small constant.

Thank You!