Assignment 1: Order Analysis, Basic Proof Techniques, and Programming Overview

Due: Sept 13, 4:30 PM

Note: Absolutely no late submissions will be accepted. 50% of the points for the order analysis and basic proof techniques and 50% of the points for the programming project.

Order Analysis and Basic Proof Techniques

1. Show that if $f(n)$ is $O(g(n))$ and $d(n)$ is $O(h(n))$, then $f(n) + d(n)$ is $O(g(n) + h(n))$.
2. Show that $O(\max\{f(n), g(n)\}) = O(f(n) + g(n))$.
3. Show that if $p(n)$ is a polynomial in n, then $\log p(n)$ is $O(\log n)$.
4. Characterize the following summation (exactly) in terms of n:

$$\sum_{i=1}^{n}(3i + 4).$$

5. Show that $\sum_{i=1}^{n} i^2$ is $O(n^3)$.
6. Show that $\sum_{i=1}^{n} i/2^i < 2$. (Hint: try to bound this sum term-by-term with a geometric progression.)
7. An n-degree **polynomial** $p(x)$ is an equation of the form

$$p(x) = \sum_{i=0}^{n} a_i x^i,$$

   where x is a real number and each a_i is a constant.

   (a) Describe a simple $O(n^2)$ time method for computing $p(x)$ for a particular value of x.

   (b) Consider now a rewriting of $p(x)$ as

$$p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \cdots + x(a_{n-1} + xa_n)\cdots))),$$

   which is known as **Horner's method**. Characterize, using the big-Oh notation, the number of multiplications and additions this method of evaluation uses.

Programming Exercise

Programming Project 8, Page 375 of your text.