Central limit Theorem.

X_i: # of heads in one flip of a fair coin.

X_1 X_2 .... X_N : n independent flips of the coin.

Mean μ

Variance σ^2.

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

What do you think happens to μ as $N \to \infty$?

$\mu \to \frac{1}{2}$ as $N \to \infty$ with probability 1

Central limit theorem:

Given $X_1 \ldots X_N$ as i.i.d r.v.s

for sufficiently large N, values of r.v. A_N

(the average of N r.v.s) are normally distributed

around μ with variance σ^2/N

$$\mathrm{Var}(A_N) = \frac{1}{N^2}\sum_{i=1}^{N} \mathrm{Var}(x_i) = \frac{\sigma^2}{N}.$$

Consider the coin flip experiment.

Consider A_{1000} : average number of heads in 1000 fair coin flips.

Mean of A_{1000} is 0.5

$$Var(A_{1000}) = \frac{Var(X)}{1000}$$

$$Var(x_i) = E(x_i^2) - (E(x_i))^2$$
$$= 0.5 - (0.5)^2 = 0.25$$

$Var(A_{1000}) = 0.00025$

$Std.\ Dev(A_{1000}) = 0.0158$

Normal dist. is within one std. dev 68% of the times and within two standard dev. 99.7% of the time.

$\therefore$ A_{1000} is between 0.4684 and 0.5316 99.7% of the times

Monte Carlo Integration

1. Computing Pi

2.

$xyz \leq 1$

$\cancel{\theta}$. $-5 \leq x \leq 5$, $-5 \leq y \leq 5$, $-5 \leq z \leq 5$

$$\iiint_V \underbrace{\gamma(x,y,z)}_{\text{density}} \, dxdydz$$

say $\gamma(x,y,z) = e^{0.5z}$