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Property-based testing is a popular technique for automatically testing semantic properties of a program, specified as a pair of pre- and post-conditions. The efficacy of this approach depends on being able to quickly generate inputs that meet the precondition, in order to maximize the set of program behaviors that are probed. For semantically rich preconditions, purely random generation is unlikely to produce many valid inputs; when this occurs, users are forced to manually write their own specialized input generators. One common problem with handwritten generators is that they may be *incomplete*, i.e., they are unable to generate some values meeting the target precondition. This paper presents a novel program repair technique that patches an incomplete generator so that its range includes every valid input. Our approach uses a novel enumerative synthesis algorithm that leverages the recently developed notion of *coverage types* to characterize the set of missing test values as well as the coverage provided by candidate repairs. We have implemented a repair tool for OCaml input generators, called Cobb, and have used it to repair a suite of benchmarks drawn from the property-based testing literature.

ACM Reference Format:

1 Introduction

Property-based testing (PBT) is an increasingly popular methodology for automatically testing rich semantic properties of systems, with PBT frameworks targeting most mainstream programming languages, including Java [48, 56], JavaScript [14], Rust [58], Haskell [8], Python [24], Scala [59], and OCaml [55]. In recent years, PBT frameworks have been effectively applied to a number of real-world settings. Prominent examples include validating real-world commercial storage systems [6], ensuring the correctness of formal specifications against modern architecture and operating system artifacts [57], and generating executable specifications of automotive software components [44].

PBT frameworks require two key components from users: *executable properties* that capture the expected input-output behaviors of the system under test (i.e., pre- and post-conditions), and *test input generators* that generate random values of the input types. The values produced by an input generator are used to validate a system's behaviors, after filtering out any values that do not meet the stated precondition. A generator is simply a nondeterministic program that samples from a space of values, supplying a *family* of inputs against which programs are tested. As a simple example, the following generator for integer trees randomly chooses one of the two constructors of **int** tree using a nondeterministic choice operator, \oplus , and then recursively fills in any of its arguments:

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Many PBT frameworks have some support for automatically deriving a default generator for an arbitrary algebraic datatype using a similar strategy- genTree is effectively what is produced by a deriving Arbitrary clause in QuickCheck, for example. Conceptually, a default generator naïvely samples from the space of possible values at random: genTree n produces trees of random integers of height at most n, for example. Unfortunately, many programs under test impose sparse preconditions on their inputs, i.e., a property that an arbitrary input is unlikely to satisfy: e.g., valid postal addresses, well-structured XML documents, red-black trees, or well-typed expressions. If we use genTree to test a function that expects valid binary search trees containing at least three elements, for example, we will have to throw away roughly 95% of the values the generator produces. As the precondition grows more restrictive, the overhead of simply filtering the results of a default generator becomes too great for most users, especially when testing is part of continuous integration [19]. When this occurs, the standard recourse is to manually write an input generator that produces the desired set of inputs. This process is unsatisfactory for end-users, however: a recent study of industrial users of PBT frameworks identified the need for handwritten generators to "be a source of friction for many participants" [18], with practitioners stating that writing generators by hand was a "tedious" and "high-effort" process.

An important challenge when writing generators tailored to a particular precondition is identifying which values *not* to enumerate– a generator that only produces a restricted set of values will miss valid parts of the input space, while one that is too permissive will waste work enumerating terms that are discarded by the testing framework. While PBT frameworks like QuickCheck can report how many terms do not to meet a precondition, signaling when a generator is too permissive, they do not provide similar feedback about the inputs an overly restrictive generator will fail to produce. To address this problem, Zhou et al. [67] recently proposed *coverage types*, a type system for reasoning about the values a generator *must* yield. Intuitively, a function that fails to type check against a particular coverage type $\overline{\tau} \rightarrow [v:b \mid \phi]$ will fail to evaluate to at least one value that satisfies the predicate ϕ . Unfortunately, while coverage types can help developers identify when the range of a generator is missing certain values, it still falls to the developer to extend the generator so that its outputs cover those values. Simply using the default generator to augment the outputs of an incomplete generator suffers from the same problems as the naïve sample and filter approach: as our experiments in Section 6.2 show, this strategy fails to meaningfully extend the coverage of an incomplete generator in most scenarios. Thus, a more targeted approach is needed.

In this paper, we propose an approach that frees the developer from this obligation by *auto-matically repairing* an incomplete generator so that it is complete with respect to a user-specified property. Our approach uses a novel program synthesis algorithm which leverages coverage types to build patches that are guaranteed to fill in any gaps in a generator's coverage. In contrast to the traditional type-guided program synthesis setting, in which valid solutions are defined by the *absence* of unwanted/unsafe behaviors, the success of our repairs is defined by the sorts of behaviors they *add*. This qualitative difference manifests in meaningful ways in the design of our algorithm: in contrast to the safety specifications used by traditional deductive synthesis techniques, a top-level specification of the set of missing values provides limited guidance on how coverage duties should be decomposed and delegated among the subexpressions of an incomplete generator. On the other hand, it is straightforward to combine partial solutions that only contribute a piece of the missing coverage into a complete solution. Our algorithm leverages this capability to construct "minimal" solutions, i.e., ones that augment the existing generator with just enough new behaviors to fill in

<pre>1 let rec genEvens (n : int) : int list =</pre>							
2 if n == 0	<pre>2 if n == 0 then [int_gen()]</pre>	<pre>2 if n == 0 then [2 * int_gen()]</pre>					
3 then [0]	3 else	3 else					
4 else [2]	4 [int_gen()] ⊕	4 (* [2 * int_gen()] ⊕ *)					
	<pre>5 int_gen() :: genEvens(n - 1)</pre>	<pre>5 2*int_gen() :: genEvens(n - 1)</pre>					
$\{1: il \mid 1 = [0] \lor 1 = [2]\}$	<: $\{1: il \mid 0 < len(1) \le n+1\}$:>	$\{l: il \mid \texttt{all_evens}(1) \land 0 < \texttt{len}(1) = \texttt{n} + 1\}$					
$[1:il \mid 1 = [0] \lor 1 = [2]]$:> $[1:il 0 < len(1) \le n+1] <:$	$[1:il \mid all_evens(1) \land 0 < len(1) = n + 1]$					

Fig. 2. Three sized generators for non-empty lists of even numbers, followed by refinement and coverage types for their bodies. The direction of the subtyping relation on the types in each column is included. We use *il* as an alias for **int list**.

any coverage gaps– for almost all the incomplete generators in our experimental evaluation, 100% of the values produced by their repaired counterparts satisfy the target precondition. As we shall see, our approach can also be used to solve sketch-based synthesis problems [61, 63], wherein users provide an incomplete generator comprised of only the control flow structure the final generator should use, relying on our repair algorithm to generate the program fragments needed to satisfy the target coverage property.

Fig. 1 presents a high-level overview of our repair algorithm and its two main phases. The first phase of our algorithm sets up the repair problem, which the second phase then solves.

Our system takes two inputs: an incomplete generator and a coverage type specifying the inputs that the generator needs to cover. The first phase begins by characterizing the current and missing coverage of the input generator using coverage types. It then builds a program sketch containing typed holes; the typing context of each hole captures all the local variables that can be used to complete it. The algorithm's second phase then uses this information to complete the sketch, employing an enumerative synthesis procedure to find terms that can be used to patch each of its holes.

In summary, we make the following contributions.



Fig. 1. Overview of our proposed pipeline.

- Show how coverage types can be used to diagnose and formally specify missing coverage of test input generators.
- Present a novel synthesis algorithm that leverages coverage types to intelligently repair incomplete test generators.
- Implement this approach in a tool, Cobb, and demonstrate its efficacy by using Cobb to automatically repair a suite of incomplete generators targeting a rich class of datatypes and semantic properties drawn from the property-based testing literature.

2 Overview

Before presenting the technical details of our approach, we begin with a brief review of coverage types and then walk through an end-to-end example of our repair procedure. Fig. 2 presents three generators for lists of integers: all three are examples of *sized* generators [10], which use a parameter, in this case n, to bound the number of recursive calls and thus ensure termination. The generator

on the left of Fig. 2 nondeterministically produces a singleton list containing 0 or 2, the middle one yields all non-empty lists of integers whose length is less than n+1, and the range of the rightmost generator is lists of even numbers containing *exactly* n+1 elements.

Immediately under each generator is a refinement type [26], {I:intlist $| \phi \rangle$, and a *coverage type* [67], [I:intlist $| \phi \rangle$, whose qualifiers ϕ captures semantic properties of the values the generator outputs. Although the two types are syntactically similar, their semantic interpretation features an important difference: each refinement type describes a *superset* of the actual range of the corresponding generator, and each coverage type encodes a *subset* of its actual outputs. This relationship is captured by the subtyping relation for each kind of type. The refinement type in the middle column is a supertype of the types on either side of it, so it can be assigned to the generators on both the left and right. Coverage types, in contrast, invert this relationship: all three coverage types in the figure can be assigned to the generator in the middle, since each type describes a subset of the outputs it "covers".

Importantly, users will get a type error when checking a generator against a coverage type whose qualifier is satisfied by values that fall outside its range: while $\{l: int list | l = [i] \land 0 \le i \le 2\}$ is a perfectly reasonable refinement type for the leftmost generator, we cannot type it against a similar coverage type, $[l:int list | l = [i] \land 0 \le i \le 2]$, because [1] is not one of its outputs. As another example, suppose that a user wants a generator for non-empty lists that contain even integers, a property that is captured by the following type signature:

$$genEvens : \underbrace{n: \{n: int \mid n \ge 0\}}_{refinement type} \rightarrow \underbrace{[1: int list \mid \neg empty(1) \land len(1) \le n + 1 \land all_evens(1)]}_{coverage type} \qquad (\tau_{Ev})$$

The n parameter of this function type has a standard refinement type that stipulates that the function expects a non-negative argument. The return type is a coverage type stipulating that the range of the function includes every non-empty list of even numbers containing at most n+1 elements. Notably, τ_{Ev} is not a valid type for the generator on the right of Fig. 2— it cannot produce lists containing fewer than n+1 elements— but it is possible to extend this generator so that it can output such lists by uncommenting the expression on line 4. We will now describe our approach for automatically generating these sorts of patches for incomplete generators.

Our algorithm expects two inputs: a *coverage type* capturing the type of generators that produce all valid inputs, e.g., τ_{Ev} and an *incomplete generator* to repair. To illustrate the details of our approach, our walkthrough will use the following generator, which is an even more incomplete version of the one on the right of Fig. 2:¹

```
let rec genEvensinc (n : int) : int list =
    if n == 0 then err (* base case *) else err (* recursive case *)
```

Phase 1: Characterizing Missing Coverage. Given these inputs, our repair algorithm begins by identifying any target values not covered by genEvens. It does so by inferring a pair of coverage types, [1: int list $|\phi_{cur}|$ and [1: int list $|\phi_{need}|$, that respectively capture a) the current coverage of the input generator, and b) the coverage that it is missing. Intuitively, b) provides a semantic characterization of the term(s) we need to synthesize. In the case of genEvens_{inc}, the current coverage is [1: int list $|\perp|$, as the function is not guaranteed to output *any* values, and the missing coverage type is simply the return type of τ_{Ev} , i.e.,

$$[l: int list | \neg empty(1) \land len(1) \le n + 1 \land all_evens(1)]$$
 (τ_{EvNeed})

If we had used the generator to the right of Fig. 2 instead, ϕ_{need} would be:

[l:intlist | $n \neq 0 \land len(1) = 1 \land all_evens(1)$]

which indicates the generator needs to be able to stop early in order to be able to generate lists with fewer than n+1 elements. From here, our algorithm builds a *sketch* [63] of the complete generator

¹The err expression always throws an exception, so genEvens_{inc} does not produce any outputs.

[,] Vol. 1, No. 1, Article . Publication date: April 2025.

by inserting typed holes into each control flow path where a patch can be inserted to add coverage. Our motivating example results in the following sketch with two holes:

if n == 0 then
$$\Box_1 = \tau_{EvNeed}$$
 (* base case *) else $\Box_2 = \tau_{EvNeed}$ (* recursive case *)

Our algorithm also attaches a typing context to each hole in the sketch; intuitively, the typing context of a hole summarizes the control flow at that program point. The typing contexts for the holes in our sketch are

 $n: \{n: int \mid n = 0\} \vdash \Box_1 : \tau_{EvNeed}$ $n: \{n: int \mid n \neq 0\} \vdash \Box_2 : \tau_{EvNeed}$

Phase 2: Synthesis. The next phase of our algorithm synthesizes well-typed terms, $\Gamma_i \vdash e_i : \tau_{EvNeed}$ for these holes; replacing each \Box_i in our sketch with e_i produces a generator whose type is:

 $genEvens : n : \{size: int \mid n \ge 0\} \rightarrow [I:int list \mid \neg empty(1) \land len(1) \le n + 1 \land all_evens(1) \lor \bot]$

which is a subtype of τ_{Ev} , i.e., the target coverage type.

Observe that using the analogous refinement type as the specification of the missing coverage

 $n: \{n: int \mid n \ge 0\} \rightarrow \{l: int list \mid \neg empty(l) \land len(l) \le n+1 \land all_evens(l) \lor \bot\}$

admits numerous solutions that are incongruous with our intended use of genEvens as a test generator, including the function on the left of Fig. 2. The first hole can be filled with any singleton list containing an even number, for example, including [0], [4], [n], [2*n], [4*int_gen()]. The first three of these expressions are consistent with the bias used by many program synthesizers, Occam's razor, which prioritizes the "smallest" program among candidate solutions [4, 21, 64].

A larger challenge is that when repairing an incomplete generator, a description of the behaviors a patch must add does not provide much guidance on how to decompose those behaviors into independently solvable subproblems. To see why, consider how we type check the else branch of the generator in the middle of Fig. 2. We cannot type check the use of \oplus in this branch against a coverage type [1:intlist | ϕ] by simply independently checking its subexpressions against [1:intlist | ϕ], as neither covers all the required values- indeed, if either subexpression did so, the generator wouldn't need the other expression at all. Thus, in order to type check $e_1 \oplus e_2$ against [1:intlist | ϕ], we need to come up with types [1:intlist | ϕ_1] and [1:intlist | ϕ_2] for e_1 and e_2 , check them against those types, and then check that the joint coverage of those types is sufficient, i.e. [1:intlist | $\phi_1 \lor \phi_2$] <: [1:intlist | ϕ_1] and [1:intlist | e_2 , we can use e_1 and e_2 to help infer [1:intlist | ϕ_1] and [1:intlist | ϕ_2], but a top-down, type-directed synthesis algorithm does not have e_1 and e_2 in hand; its job is to generate both terms from their types. Unfortunately, there are many possible ways to divide the coverage responsibilities of a \oplus expression between its subexpressions, each of which results in a different set of synthesis goals, and it is not clear how to choose between them.

As a consequence, our synthesis procedure instead adopts a bottom-up approach: iteratively generating a set of partial solutions that can be combined to construct a complete answer. Our algorithm maintains a pool of candidate terms that it uses to generate new terms; this pool grows as the algorithm proceeds. At each iteration of the loop, the algorithm uses a *syntactic* cost function to prioritize the generation of certain terms. Section 4.3 provides more detail on our cost function, but intuitively, smaller terms and terms with larger coverages like generators and recursive calls have lower cost. Fig. 3 provides some examples of

$\Sigma_0 \equiv \{ [0], [n], 1, 3, [], Leaf, \}$
[int_gen()],}
$\Sigma_1 \equiv \{2 \times int_gen(), genEvens(n-1), \ldots\}$
$\Sigma_2 \equiv \{0 :: genEvens(n-1),$
n :: genEvens(n-1),
<pre>int_gen() :: genEvens(n-1),}</pre>
$\Sigma_3 \equiv \{\text{genEvens}(n-1) + \text{genEvens}(n-1), \}$
2*int_gen() :: genEvens(n-1),}

Fig. 3. Example sets of enumerated terms, where the cost of the elements of Σ_i is less than cost of the elements of Σ_{i+1} .

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Fig. 4. A subset of the join semi-lattice built for \Box_2 in genEvens, where $\Gamma_2 \equiv n : \{n: int | n > 0\}$ and $\phi(1) \equiv \neg empty(tail(1)) \land len(tail(1)) \le (n-1) + 1 \land all_evens(tail(1)).$

terms at different cost levels for our running example.

After generating all the terms at the current cost threshold, our algorithm infers a coverage type for each expression, and uses this *semantic* information to prune out any terms that are unsafe, not useful, or redundant. In the case of terms containing a recursive call, for example, type checking ensures that the first argument to each recursive call is structurally decreasing, ensuring that a generator using such a term will terminate. Our algorithm also uses the inferred types to safely discard any terms that do not provide new coverage. If the pool of candidates already contains the term int_gen(), for example, there is no reason to add int_gen()+1 or int_gen()+int_gen() to it: all of these expressions generate the same terms, and thus have the exact same coverage type. We only add terms that satisfy these semantic conditions to the pool of enumerated terms.

The final step in our algorithm's enumeration loop is to check if a valid completion for any of the holes in the sketch has been found. To do so, our algorithm maintains a set of any enumerated terms which have the same base type as the missing values, under the typing context for that hole. This set is partially ordered by the subtyping relation on the types inferred for elements by our type inference algorithm, TyInfer:

$$e_1 \sqsubseteq e_2 \equiv \mathsf{TyInfer}(\Gamma_k, e_1) <: \mathsf{TyInfer}(\Gamma_k, e_2)$$

Fig. 4 shows an example of part of this poset for \Box_2 in genEvens_{inc}. As we have seen, if the terms e_1 and e_2 have the coverage types τ_1 and τ_2 where $\tau_1 <: \tau_2$, then e_2 is only guaranteed to generate a subset of the outputs of e_1 . Thus, this poset tracks the relative coverages of the candidate solutions (as determined by TyInfer) our algorithm has enumerated so far. The top element in this poset is the default generator for our target type, capable of enumerating every list of integers. Its two children only produce a subset of its outputs; they are sibling nodes because neither is a subtype of the other, i.e., neither has a set of outputs that subsumes the other's. Importantly, this poset forms a join-semilattice: given any two terms, we can build a term that covers both sets of inputs by joining them together via our nondeterministic choice operator: $e_1 \subseteq (e_1 \oplus e_2) \supseteq e_2$. Our implementation of this poset does not need to maintain these sorts of elements; it can always use \oplus to reconstruct them on demand: as a consequence, there is no need for Fig. 4 to explicitly include

$$n: \{n: int \mid n > 0\} \vdash [\emptyset] \oplus [2*n] : [I: int list \mid len(1) = 1 \land hd(1) = 0 \lor len(1) = 1 \land hd(1) = 2*n]$$

To check if it has found a solution for a hole, our algorithm first walks down this lattice looking for an element with the same type as the hole, returning that element as the solution if so. The poset corresponding to Fig. 4 for \Box_1 contains a such direct solution, for example:

$$n : \{n: int \mid n = 0\} \vdash [2 * int_gen()] : \tau_{EvNeed}$$

$$(p_1)$$

, Vol. 1, No. 1, Article . Publication date: April 2025.

Variables		x, f, u,			
Data constructors	d ::=	() true false O S Cons Nil Leaf Node			
Constants	c ::=	$\mathbb{B} \mid \mathbb{N} \mid \mathbb{Z} \mid \ldots \mid d \ \overline{c}$			
Operators	op ::=	$d \mid + \mid == \mid < \mid mod \mid nat_gen \mid int_gen \mid$			
Values	v ::=	$c \mid op \mid x \mid \lambda x:t.e \mid fixf:t.\lambda x:t.e$			
Terms and	<i>e</i> , <i>s</i> ::=	$v \mid \text{err} \mid \text{let } x = e \text{ in } e \mid \text{let } x = op \ \overline{v} \text{ in } e \mid \text{let } x = v \ v \text{ in } e$			
		match v with $\overline{d\;\overline{y} ightarrow e}$			
Incomplete Terms		$ \Box : [v:b \phi]$			
Base Types	b ::=	unit bool nat int b list b tree			
Basic Types	t ::=	$b \mid t \rightarrow t$			
Method Predicates	mp ::=	emp hd mem			
Literals	l ::=	<i>c</i> <i>x</i>			
Propositions	$\phi ::=$	$l \mid \bot \mid \top_b \mid op(\bar{l}) \mid mp(\bar{x}) \mid \neg \phi \mid \phi \land \phi \mid \phi \lor \phi \mid \phi \Longrightarrow \phi \mid \forall u{:}b. \; \phi \mid \exists u{:}b. \; \phi$			
Refined Types	$\tau ::=$	$\llbracket v:b \mid \phi \rrbracket \mid \{v:b \mid \phi\} \mid x: \tau \to \tau$			
Type Contexts	$\Gamma ::=$	$\emptyset \mid \Gamma, x: \tau$			
		Fig. 5. λ^{TG} ++ syntax.			

If a direct solution is not available, our algorithm attempts to build a solution by joining together all the elements that would be immediate subchildren of an expression that had the target coverage type. While there is no immediate solution to \Box_2 in Fig. 4, it does contain two expressions that would be direct children of such a node: [2*int_gen()] and 2*int_gen () :: genEvens (n-1). The join of these expressions provides precisely the coverage required by \Box_2 :

$$n : \{n: int | n > 0\} \vdash [2 * int_gen()] \oplus 2 * int_gen() :: genEvens(n-1) : \tau_{EvNeed}$$
(p2)

Replacing the two holes in genEvens_{inc} with p_1 and p_2 results in the generator on the right of Fig. 2 with the fourth line uncommented, which provides exactly the desired coverage.

We pause here to highlight the distinguishing features of our algorithm: first is its use of the coverage type τ_{EvNeed} to precisely characterize the behaviors a repair must add to genEvens_{inc} to make it complete. While τ_{EvNeed} provides a semantic specification for the top-level synthesis problem, it does not provide much guidance on how to decompose that problem into independently solvable subgoals, e.g., when patching \Box_2 . Thankfully, the nondeterministic nature of input generators equips our bottom-up synthesis algorithm with a convenient mechanism to combine partial patches into a complete solution, a capability that it used to generate p_2 .

3 Language

To formalize our type-based approach to test generator synthesis and repair, we use λ^{TG} ++, a slightly modified version of λ^{TG} , a core calculus for input generators introduced by Zhou et al. [67]. This section reviews the key features of that original calculus, highlighting our extension along the way. The syntax of λ^{TG} ++ is shown in Fig. 5. The language is a call-by-value lambda-calculus with pattern-matching, inductive datatypes, and recursive functions. Programs are written in monadic normal-form (MNF) [23], a variant of A-Normal Form (ANF) [15] that allows nested let-bindings. λ^{TG} ++ is equipped with generators for numeric types– nat_gen and int_gen– which can evaluate to any number in their range with nonzero probability. These built-in generators suffice to express

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 $\Gamma \vdash s : \tau$

$$\frac{\Gamma \vdash \mathbf{WF} [v:b \mid \phi]}{\Gamma \vdash \Box : [v:b \mid \phi] : [v:b \mid \phi]} \text{ THOLE } \frac{\frac{\Gamma \vdash v:\tau_v}{\Gamma, \overline{y:\tau_y} \vdash e_i:\tau_i} \quad \overline{\nabla_i \tau_i = \tau} \quad \Gamma \vdash \mathbf{WF} \tau}{\Gamma \vdash (\text{match } v \text{ with } \overline{d_i \ \overline{y} \to e_i}):\tau} \text{ TMATCH}}$$

$$\frac{\frac{\Gamma \vdash \lambda x: b.\lambda f: (b \to \lfloor \tau \rfloor).e: (x:\{v:b \mid \phi\} \to f: (x:\{v:b \mid v \prec x \land \phi\} \to \tau) \to \tau) \quad \Gamma \vdash \mathbf{WF} x:\{v:b \mid \phi\} \to \tau}{\Gamma \vdash \text{fix} f: (b \to \lfloor \tau \rfloor).\lambda x: b. \ e: (x:\{v:b \mid \phi\} \to \tau)} \text{ TFIX}}$$

Fig. 6. Selected λ^{TG} ++ typing rules.

additional nondeterministic behaviors: the \oplus choice operator, for example, can be defined as:

 $e_1 \oplus e_2 \doteq \texttt{let} n = \texttt{nat}_\texttt{gen} () \mod 2 \texttt{ in match } n \texttt{ with } 0 \rightarrow e_1 \mid _ \rightarrow e_2$

Like its predecessor, λ^{TG} ++ does not include operators to bias how often values are produced, e.g., QuickCheck's frequency; including such an operator would not fundamentally impact the guarantees we provide for synthesized generators. λ^{TG} ++ is equipped with a completely standard small-step operational semantics, $e \hookrightarrow e'$, that mirrors that of λ^{TG} .

The only addition λ^{TG} ++ makes to λ^{TG} is an additional syntactic category of *incomplete terms*, *s* i.e., terms that contain one or more typed *holes*, $\Box : [v:b | \phi]$. Semantically, holes can evaluate to any $\Box : [v:b | \phi] \hookrightarrow v$ value satisfying ϕ (EHOLE), and thus act as a kind of semantic placeholder for a complete patch. Syntactically, our algorithm use holes to identify program points at which repairs can be inserted. Given an incomplete program *s* with *j* holes, we write *s*[\overline{e}] to denote the complete program where the *i*th hole has been replaced by e_i . The output of our repair algorithm is a *syntactically* complete λ^{TG} program, i.e. it does not contain any holes, that is also *semantically* complete, i.e. it can produce all inputs satisfying the target property.

3.1 Type System

 λ^{TG} ++ inherits the type system of λ^{TG} ; like that calculus, λ^{TG} ++ has three categories of types: *base* types, basic types, and refined types. Base types (b) include primitive types, e.g., unit and bool, and inductive datatypes, e.g., int list and bool tree. Basic types (t) extend base types with function types. As in other refinement type systems, refined types (τ) qualify base types with predicates in a decidable fragment of first-order logic (FOL). Importantly, however, type refinements have two distinct modalities: the qualifiers of coverage types ([$v : b | \phi$]) identify a subset of the values a nondeterministic expression must be able to evaluate to, while the qualifiers of refinement types $\{\{v: b \mid \phi\}\}$ characterize a superset of the values an expression may evaluate to. In order to express rich shape properties over inductive datatypes, we allow propositions to reference *method predicates*, boolean-valued functions on inductive datatypes like *emp*, *hd*, and *mem*. Using such predicates, it is straightforward to generate verification conditions that can be handled by an off-the-shelf theorem prover like Z3 [11]. In order to ensure that type checking is decidable, our type system restricts refinements to effectively propositional (EPR) sentences (i.e., prenex-quantified formulae of the form $\exists^* \forall^* \varphi$ where φ is quantifier-free). Following λ^{TG} , our type system allows function parameters to be qualified by refinements that specify when it is safe to apply a test generator, while a generator's return type is qualified using a coverage type that characterizes the values it is guaranteed to produce.

Typing

Algorithm 1: The high-level coverage repair algorithm (Repair)					
Inputs : s: incomplete program, Γ : typing context for s, $[v:b \mid \psi]$: target coverage type for s					
Output : Coverage complete repaired program e such that $\Gamma \vdash e : [v:b \mid \phi]$					
1 $[v:b \mid \psi_{cur}] \leftarrow TyInfer(\Gamma, i);$	Infer initial coverage of s				
2 $[\nu:b \mid \psi_{need}] \leftarrow Abduce(\Gamma, [\nu:b \mid \psi_{cur}], [\nu:b \mid \psi]);$	 Abduce missing coverage 				
$s (s', \overline{\Gamma_j \vdash \Box_j : [v : b \mid \psi_j]}) \leftarrow \text{Localize}(\Gamma, s, \psi_{\text{need}});$	 Identify repair locations 				
4 return Synthesize(Γ , s' , $[v:b \mid \psi_{need}]$, $\overline{\Gamma_j \vdash \Box_j : [v:b \mid \psi_j]}$);	 Synthesize patches for holes 				

Fig. 6 presents the key typing rules for λ^{TG} ++. The newly added typing rule for holes, THOLE, reflects the semantics of a hole as an oracle can produce any value satisfying the qualifier of its annotated type. The typing rule for match expressions, TMATCH, reflects that the coverage provided by match is the union (\vee) of the coverages of its branches, each of which may contribute a different set of values. This is in contrast to how branching control flow structures are treated in standard refinement type systems, where each branch can be independently checked against the type of the overall expression. The type system of λ^{TG} ++ enforces the same high-level properties as λ^{TG} 's: the typing rule for recursive functions, TFIX, for example, uses a well-founded relation on the first argument of a function to ensure that it is terminating. Similarly, its well-formedness relation ensures that refinements of argument and return types of functions have the expected modalities. The remaining typing rules are identical to those of λ^{TG} and are similar to other refinement type systems [26]²: the key differences are in the *semantics* of these judgements.

For the purposes of automatic test generator repair, the key property is that a well-typed λ^{TG} term e (i.e., a completed λ^{TG} ++ term) of $[v:b \mid \phi]$ can evaluate every value satisfying ϕ :

THEOREM 3.1 (TYPE SOUNDNESS [67]). A well-typed test generator of type $\vdash f : \overline{x_i : \{v : b_i \mid \phi_i\}} \rightarrow [v : b \mid \phi]$, when applied to well-typed arguments $\vdash v_i : \{v : b_i \mid \phi_i\}$, can evaluate every value satisfying $\phi[\overline{x_i \mapsto v_i}] : \forall v . \phi[\overline{x_i \mapsto v_i}, v \mapsto v] \Longrightarrow f \overline{v_i} \hookrightarrow^* v$

 λ^{TG} ++ is also equipped with a decidable bidirectional typing algorithm whose type synthesis (TyInfer) and type checking subroutines will play key roles in our repair algorithm.

4 Input Generator Repair

Our top-level repair algorithm, shown in Algorithm 1, closely follows the workflow illustrated in Fig. 1. Most of its functionality is delegated to three subroutines (Abduce, Localize, and Synthesize); this section will present the important details of each subroutine, focusing particularly on Synthesize, after discussing Repair. Repair takes the body of the target generator s (potentially with user-provided holes), a typing context for $s \Gamma$, and the target coverage type $[v:b | \psi]$. The algorithm is additionally parameterized over several items that it uses to construct repairs: a collection of typed components that Synthesize uses to enumerate terms, the set of method predicates used in the types of those components and by Abduce to characterize missing coverage, and axioms characterizing the semantics of those method predicates. To avoid cluttering our discussion, we leave these parameters implicit in the definition of Repair and its subroutines.

Repair begins by inferring two coverage types ψ_{cur} (line 1) and ψ_{need} (line 2). The former characterizes the current coverage of *s*, and the latter describes the coverage that *s* is missing. Repair then uses Localize (line 3) to construct a sketch *s'* that contains holes at each location in *s* where coverage should be added, as well as a set of contexts and types for each of its holes,

²The full set of typing rules and judgements are included in the supplementary material

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$$\begin{array}{ll} \text{if } n &= 0 \\ \text{then } [] \\ \text{else} \\ \text{let } h &= \text{int_gen() in} \\ \text{let } t &= \text{genIntList(n-1) in} \\ h &:: t \\ (a) \text{ genIntList} \end{array} \end{array} \xrightarrow{r \equiv [l: \text{int list} | \text{len(l)} \leq n]} \\ r_{\text{cur}} \equiv \begin{array}{ll} r \equiv [l: \text{int list} | n = 0 \implies \text{empty(l)} \\ \hline r_{\text{cur}} \equiv \begin{array}{ll} n \neq 0 \implies \exists h. \exists t. \text{len(t)} \leq n - 1 \land \text{hd(l)} = h \land \text{tl(l)} = t] \\ \hline \tau_{\text{need}} \equiv [l: \text{int list} | \text{len(l)} = 0 \land \text{len(l)} \leq n] \end{array}$$

Fig. 7. An incomplete generator for **int list** and the type inferred by Abduce for it.

 $\Gamma_j \vdash \Box_j : [v:b \mid \phi_j]$. Next, the algorithm builds the final generator by using Synthesize to patch each of the holes in s' (line 4).

4.1 Inferring Missing Coverage

The Abduce subroutine is used to infer a coverage type that captures a set of values missing from the range of a generator. Notably, Abduce may return a coverage type that is more general than necessary, i.e., it may capture a superset of the values needed to complete a generator. To understand why, consider the incomplete generator for length-bounded lists given in Fig. 7a, genIntList. This function always returns a list containing exactly n elements, so it fails to check against the target type τ shown in Fig. 7b. To perform this check, our type checker uses TyInfer to infer the type τ_{cur} for genIntList. In order to type as many programs as possible, TyInfer produces the most precise type it can, as even this simple example demonstrates. Importantly, this means that the complexity the type inferred by TyInfer is commensurate with the input program, e.g., the number of its control flow paths and the components it uses. Thus, directly using the missing coverage type results in an overly complex type, so Repair uses Abduce to find a simpler, but still precise characterization of missing coverage. In the case of genIntList, for example, Abduce will produce τ_{need} , which intuitively captures the coverage that needs to be added to genIntList.

Abduce is parameterized over a finite set of atomic formulas Φ that define the space of types it considers. Candidate solutions are types of the form $\bigvee (\bigwedge \overline{\phi} \land \bigwedge \overline{\neg \phi}) \land \psi$, where ϕ are drawn from Φ . Given a Φ containing len(1) = 1, empty(1), and n = 0, for example, the set of qualifiers considered by Abduce for genEvens includes:

- $len(1) = 1 \land all_evens(1)$: this covers all singleton lists of even elements,
- n ≠ 0 ∧ len(1) ≠ 1 ∧ all_evens(1): this covers all non-singleton even lists where the size parameter is non-zero,
- (len(1) = 1 ∨ empty(1)) ∧ all_evens(1): this covers even lists with zero or one elements.

From this solution space, Abduce adapts an existing learning-based specification inference algorithm [66] to find a coverage type that captures the missing outputs of the target generator:³

THEOREM 4.1 (Abduce IS SOUND). Given a set of atomic formulas Φ , a typing context Γ , the current coverage $[v:b \mid \psi_{cur}]$, and the target coverage $[v:b \mid \psi]$, Abduce $(\Phi, \Gamma, [v:b \mid \psi_{cur}], [v:b \mid \psi])$ produces a $\psi_{need} \in \{\psi' \mid \psi' = \lor (\land \overline{\phi} \land \land \neg \overline{\phi}) \land \psi\}$ such that $\Gamma \vdash [v:b \mid \psi_{cur} \lor \psi_{need}] <: [v:b \mid \psi]$. Moreover, ψ_{need} is a minimal solution in the solution space defined by $\Phi: \neg \exists \psi' \in \{\psi' \mid \psi' = \lor (\land \overline{\phi} \land \land \neg \overline{\phi}) \land \psi\}$. $\Gamma \vdash [v:b \mid \psi_{cur} \lor \psi'] <: [v:b \mid \psi] \land \psi_{need} \implies \psi'$.

4.2 Localization

The Localize subroutine inserts holes into a generator *s*, producing a sketch, *s'*, and a set of the locations in *s'*, $\overline{\Gamma_j \vdash \Box_j} : [v:b \mid \phi_j]$, that the subsequent Synthesize phase should repair.

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³The full definition of Abduce is included in the supplementary material.

```
\Gamma, \{n = 0\} \vdash \Box : \tau_{\text{need}},
                                                     1 if n == 0 then Leaf \oplus \square: \tau_{need}
1 if n == 0 then Leaf
2 else if lo + 1 < hi then
                                                     2 else if lo + 1 < hi then
                                                                                                           \Gamma, \{n \neq 0\}, \{lo + 1 < hi\},\
                                                                                                            x: [x:int | lo \le x \le hi] \vdash \Box : \tau_{need},
      let x = int_range lo hi in
                                                           let x = int_range lo hi in
3
                                                    3
        err
                                                            \Box: \tau_{need}
4
                                                    4
                                                                                                                  \Gamma, \{n \neq 0\}, \{hi \leq lo + 1\} \vdash \Box : \tau,
                                                        else \Box: \tau \oplus \Box: \tau_{need}
    else \Box : \tau
                                                     5
5
                                                                                                                 \Gamma, {n \neq 0}, {hi \leq lo + 1} \vdash \Box : \tau_{\text{need}}
```

Fig. 8. An incomplete generator for BSTs and the sketch and set of holes that Localize produces from it, where $\Gamma \equiv n : \{n : int \mid n \ge 0\}$, lo : int, hi : {hi : int $\mid lo \le hi$ }, genBST : n' : {n' : int $\mid n' < n$ } $\rightarrow \dots$ We use { ϕ } as shorthand for _ : {_:bool $\mid \phi$ }.

Algorithm 2: Synthesize repairs (Synthesize)

Inputs : *s*: A sketch containing *j* holes, $\Gamma_i \vdash \Box_j$: [$v:b \mid psi_j$]: a set of typing contexts for each hole, Γ: typing context, $[v:b | \psi_{need}]$: coverage missing from *s*, $[v:b | \psi]$: target coverage **Output** : A set of *j* expressions such that $\overline{\Gamma_i \vdash e_j}$: τ_{need} and a repaired generator $i[\overline{e_i}]$, where $\Gamma \vdash i[\overline{e_i}] : [v:b \mid \psi]$ 1 $\overline{Exp_i \leftarrow \emptyset}$; $\overline{Cand_i \leftarrow \emptyset}$; $\overline{e_i \leftarrow err}$; $\alpha \leftarrow 0$; while $\Gamma \nvDash s[\overline{e}] : [v:b | \psi]$ do 2 **foreach** $\Gamma_k \vdash \Box_k : [v : b \mid \psi_k] \in \Gamma_j \vdash \Box_j : [v : b \mid \psi_j]$ **do** 3 ▶ Skip if \Box_k has already been repaired; if $e_k \neq \text{err}$ then continue; 4 for $e \in genExp(Exp_k, \alpha)$ do 5 $\tau \leftarrow \mathsf{TyInfer}'(\Gamma_k, e);$ 6 if $\Gamma_k \vdash \tau \equiv [v:b \mid \bot] \lor \exists e' \in Exp_k$. $\Gamma_k \vdash e' : \tau' \land \Gamma_k \vdash \tau' \equiv \tau$ then 7 ▶ Discard *e* if unsafe or provides no useful coverage continue; 8 9 else $Exp_k \leftarrow Exp_k \cup \{e\};$ 10 if $\Gamma_k \vdash [v:b \mid \psi_k] \lt: \tau$ then $Cand_k \leftarrow Cand_k \cup \{e\}$; 11 if $\Gamma_k \vdash \bigoplus e : [v : b \mid \psi_k]$ then 12 $e \in Cand_k$ Cand \leftarrow argmin $\Gamma_k \vdash \bigoplus e : [v : b \mid \psi_k];$ 13 $Cand' \subseteq Cand \quad e \in Cand'$ $e_k \leftarrow \bigoplus e;$ 14 $e \in Cand$ $\alpha \leftarrow \alpha + 1;$ 15 16 return $s[\overline{e}]$

Intuitively, Localize builds s' by inserting holes at the end of each possible control flow path in s, recording the typing context and missing coverage at that point.⁴ When constructing s', Localize leaves any existing holes in s untouched, adding them to the set of repair locations; it also replaces any errs with holes, as such terms contribute no useful coverage. Fig. 8 shows the output of Localize on an incomplete generator for BSTs. Each of the four holes in the resulting sketch is accompanied by a typing context that extends the initial context with additional control flow information for that hole, e.g., $\{n = 0\}$, and local variables.

4.3 Synthesizing Patches

The final subroutine of our algorithm is Synthesize, shown in Algorithm 2, which generates patches for the holes in the sketch built by Localize. As we saw in Section 2, while the type produced by Abduce provides a top-level goal for Synthesize; this type does not provide guidance on how coverage should be apportioned to subexpressions, e.g., in the presence of non-deterministic choice. For that reason, Synthesize uses a bottom-up approach, adopting an inductive-synthesis-style algorithm [1, 3, 40] to enumerate a pool of candidate repairs for each hole.

Synthesize maintains two sets of terms for each hole $\Gamma_k \vdash \Box_k : [v:b \mid \psi]$: a general pool of all type-safe terms enumerated so far, Exp_k , and a set of candidate patches $Cand_k$ that cover a portion of the hole's missing outputs. As discussed in Section 2, $Cand_k$ is equipped with a partial order that uses the coverage types of its elements, a property that Synthesize leverages when extracting candidate patches. Synthesize maintains hole-specific sets of terms because the validity of a repair depends on the context into which it is inserted: if it uses local variables or its safety depends on a particular set of path conditions, for example. The algorithm also keeps track of whether a meaningful repair has been synthesized for each hole, e_k ; these are initially set to err (line 1). Synthesize is implemented as a loop that iteratively *enumerates* terms below the current cost α (lines 5-11), *filtering* any enumerated terms that are not useful (lines 6-8), and then attempts to *extract* a complete patch for each hole from the (partial) solutions it has found. This loop terminates when the current set of patches are sufficient to ensure the current sketch has the desired coverage (line 2). Note that the loop can terminate without repairing every hole, e.g., if patching a user-provided hole ensures coverage completeness.

Enumerate. Synthesize uses the genExp subroutine to generate new terms for the current cost threshold (line 5). genExp is parameterized over a set of *seeds* and *components* that are used to construct candidate repairs. The seeds form the initial set of candidate repairs and typically consists of constants, e.g., 0, false and [], the default generators for the base types, e.g., int_gen and genTree, and any variables that are in scope for a particular hole. The components are used to construct new terms from previously generated expressions, and typically include built-in operators, datatype constructors, and functions. Seeds and components are both equipped with type signatures characterizing their coverage guarantees. The name of the generator being repaired is available via a hole's typing context, e.g., genBST in Fig. 8, enabling patches to make recursive calls. The signature of this component uses a refinement type to ensure that all recursive calls use strictly smaller arguments, the signature of genEvens, for example, is:

```
genEvens : \{n': int \mid n' \ge 0 \land n' < n\} \rightarrow [l: int list \mid \neg empty(l) \land len(l) \le size' + 1 \land all\_evens(l)]
```

genExp is parameterized over a cost function that it uses to prioritize certain elements in the search space, a common strategy in the program synthesis literature [1, 20, 21]. Cost functions are required to be monotonic— a term never has a smaller cost than any of its subterms— and stateless— a cost of a term is independent of the terms genExp has already seen. The cost function used in our implementation prefers terms with the following properties:

- **Recursive Calls** We assign a low cost to recursive calls, as they typically provide a large amount of coverage, tailored to the current repair.
- **Same Type as Target** A valid patch must produce values of the target coverage type, so we prioritize components that construct expressions with the same base type as the target over those that do not.
- **Seed Generators** Default type generators like int_gen() often provide useful coverage and are prioritized, alongside variables, over constant expressions.

⁴The full definition of Localize is included in the supplementary material.

Diverse Terms A naïve enumeration strategy will produce many terms that have repeated uses of the same components and seeds, e.g. Cons(x, Cons(y, [])). Recursive calls are likely to produce these sorts of terms in a more general way, so we prioritize expressions comprised of diverse subterms.

Filter. A key challenge in enumerative approaches to program synthesis is keeping the set of generated terms from becoming intractably large. Accordingly, Synthesize curates its pool of terms by discarding expressions that are redundant or unlikely to contribute to a solution (lines 6-8). While genExp employs a purely syntactic cost function to prioritize terms, Synthesize uses the semantics of a term when deciding whether it should be pruned. This semantic information is encoded in the coverage type inferred by TyInfer' (line 6). TyInfer' is a more restrictive version of TyInfer which aggressively discards function applications in order to filter potentially unsafe terms. TyInfer' strictly limits where type subsumption may be applied, so that the inferred coverage types of any function arguments do not include values that violate the refinement types of its parameters. TyInfer' implements the following modified version of the typing rule for function applications:

$$\frac{\Gamma \vdash v_1 \Rightarrow a : \{v : b \mid \phi_1\} \rightarrow \tau_1 \quad \Gamma \vdash v_2 \Rightarrow c : [v : b \mid \phi_2] \quad \text{Disj}(\neg(\phi_1), \phi_2) \equiv \emptyset}{\Gamma' \equiv a : [v : b \mid v = c \land \phi_1], x : \tau_1 \quad \Gamma, \Gamma' \vdash e \Rightarrow \tau \quad \tau' \equiv \text{Ex}(\Gamma', \tau) \quad \Gamma \vdash^{\text{WF}} \tau'}$$

$$\frac{\Gamma \vdash \text{let } x = v_1 v_2 \text{ in } e \Rightarrow \tau'}{\Gamma \vdash \text{let } x = v_1 v_2 \text{ in } e \Rightarrow \tau'}$$

This stronger rule ensures, e.g., that Synthesize's pools of terms only include recursive calls whose size argument is strictly decreasing.

If TyInfer' successfully infers a type for a term $\Gamma_k \vdash e : \tau$, Synthesize then decides whether to include e in Exp_k , using τ to judge whether the new expression supplies any meaningful coverage (line 7). It prunes any e that is not guaranteed to produce any values at all, i.e., its inferred type is equivalent to $[v:b \mid \bot]$. Synthesize also discards e if it has already enumerated a *coverage equivalent* term: intuitively, if two terms cover the same outputs, we only need to keep around the cheaper one, similarly to how other synthesizers use observational equivalence to prune enumerated terms [1, 33]. Any terms that are not filtered are then added to Exp_k (line 10); finally, Synthesize additionally checks whether e provides part of the target coverage, adding it to the poset of partial solutions if so (line 11).

Extraction. After enumerating all the useful terms for the current cost bound, Synthesize attempts to extract a patch from $Cand_k$, its set of partial solutions (lines 12-14). To do so, Synthesize first sees whether any solution can be built using $Cand_k$ by checking if nondeterministically joining together every element in $Cand_k$ provides the missing coverage (line 12). If so, Synthesize finds a minimal repair, by identifying a subset of $Cand_k$ that provides the necessary coverage (line 13).

In order to efficiently extract solutions, our implementation of Synthesize leverages the features of $Cand_k$, i.e., it is a join semi-lattice ordered using the types of the expressions it contains. The insight is that we can efficiently check if $Cand_k$ contains a complete repair by only examining the elements that are direct supertypes of the target type, since:

$$\Gamma_k \vdash \bigoplus_{e \in Cand'} e : [v : b \mid \psi_{\mathsf{need}}] \Rightarrow \Gamma_k \vdash \bigoplus_{e \in Cand_k} e : [v : b \mid \psi_{\mathsf{need}}]$$

where

$$Cand' = \left\{ \begin{array}{l} e \in Cand_k \\ \land \nexists e' \in Cand_k. \ [v:b \mid \psi_{need}] <: TyInfer'(\Gamma_k, e) \\ \land \nexists e' \in Cand_k. \ [v:b \mid \psi_{need}] <: TyInfer'(\Gamma_k, e') <: TyInfer'(\Gamma_k, e) \end{array} \right\}$$

To see how we leverage this to extract a patch from a poset of candidate solutions, we will show how to patch the hole $n : \{n: int | n \ge 0\} \vdash \square : [l: int list | len(1) \le n]$ using the lattice at the top of Fig. 9. We first check if there is a term in the lattice whose type is equivalent

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$$\Gamma \vdash e_{2} : [l:il \mid \neg empty(1)] \qquad \Gamma \vdash e_{5} : [l:il \mid empty(1)] \qquad \Gamma \vdash e_{5} : [l:il \mid empty(1)] \qquad \Gamma \vdash e_{5} : [l:il \mid empty(1)] \qquad \Gamma \vdash e_{4} : [l:il \mid \neg empty(1) \land len(1) < n] \qquad \Gamma \vdash e_{4} : [l:il \mid \neg empty(1) \land len(1) < n] \qquad \Gamma \vdash e_{2} : [l:il \mid \neg empty(1)] \qquad \Gamma \vdash target : [l:il \mid len(1) < n]$$

 $\Gamma \vdash \mathsf{e}_3 : [\mathsf{I}: il \mid \neg\mathsf{empty}(1) \land \mathsf{len}(1) = \mathsf{n}] \quad \Gamma \vdash \mathsf{e}_4 : [\mathsf{I}: il \mid \neg\mathsf{empty}(1) \land \mathsf{len}(1) < \mathsf{n}] \quad \Gamma \vdash \mathsf{e}_5 : [\mathsf{I}: il \mid \mathsf{empty}(1)]$

Fig. 9. A lattice of repairs before and after inserting target, where $\Gamma \equiv n : \{n : int | n \ge 0\}$ and $il \equiv int list$

to [1: int list | len(1) \leq n]. Since this fails, we instead insert a "dummy" node with this type into the lattice, producing the lattice at the bottom of Fig. 9. Observe that the inserted node is a direct parent of e_3 , e_4 , and e_5 , and that furthermore the join of these expressions has the coverage type n : {n:int | n \geq 0} \vdash [1:int list | empty(1) \lor len(1) = n \lor len(1) < n]. Since this type is equivalent to the type of the target hole, we have found a valid patch, which we return as the solution.⁵

5 Implementation

Cobb, our prototype implementation of the above approach, consists of about 3k lines of OCaml[35], and uses a modified version of Poirot [67] as its coverage type checker; this type checker uses Z3 [11] as its backing SMT solver. Cobb ingests and outputs sized generators in a DSL that closely mimics λ^{TG} ++. We have implemented this language as a shallowly embedded DSL in OCaml, and repaired generators can be directly executed using OCaml's QCheck framework [55]. Cobb is parameterized over the set of base types, components, and method predicates. It currently supports a number of standard OCaml primitive operations and datatypes, and includes built-in method predicates for expressing properties of these types, e.g., empty and sorted.

The guarantees provided by the Repair algorithm are *possibilistic*: the coverage guarantees of weighted and fair implementations of \oplus are the same. In practice, however, users often prefer generators that bias the choices of \oplus . If only one of its choices includes a recursive call, for example, a fair implementation of \oplus will bias a generator towards smaller values: genTree in the introduction generates Leaf nodes half of the time, for example. Thus, Cobb adopts the commonly used approach of using the bound of a sized generator to bias uses of \oplus operators [30]: after synthesis, Cobb applies a syntactic transformation to adjust the weights of \oplus in which only a single choice has a recursive call, weighting that choice according to the current bound. In practice, this means that Cobb produces generators that are initially biased towards recursive calls, but which are more likely to take the other choice as size decreases.

While it is not used in our evaluation, Cobb allows users to set an upper bound on the cost of enumerated terms. When this bound is reached, Cobb terminates and builds a best-effort repair using the current set of enumerated terms. To do so, it merges the current posets for each unsolved goal by pushing their typing contexts into the coverage type of each term using existential variables.

⁵Note that this extraction strategy crucially depends on using the precise type inferred by TyInfer' to order elements in the lattice. Using the subtyping relation on any valid type (via subsumption) would allow an element's children to provide more coverage than it does: under such a strategy, int_gen() could be a child of any element, for example!

```
let lt = rbtree_gen (inv - 2) false (h - 1) in
let rt = rbtree_gen (inv - 2) false (h - 1) in
Rbtnode (false, lt, int_gen(), rt)
```

Fig. 10. The relevant portion of the original generator and the repaired version found by Cobb for the sixth red-black tree variant.

It then searches the merged poset for an element that would be a direct parent of a term with the target type (if it existed in the poset)– in the worst case, this will be the default generator at the top of the poset. The resulting patch is used to complete the sketch.

6 Evaluation

Using Cobb, we have investigated four key questions about our approach to generator repair:

- **RQ1** Is our approach able to automatically find complete repairs for different kinds of generators, covering a diverse set of properties and datatypes, in a reasonable amount of time?
- **RQ2** How does our approach compare to alternative repair strategies that exclusively prioritize either safety or completeness?
- **RQ3** Is Cobb effective when used as a sketch-based synthesizer? Can it produce a generator from a skeleton that contains only the desired control flow structure of the target generator?
- **RQ4** How do our statically repaired generators compare to alternative complete input generation approaches that rely on run-time constraint solving?

All of our experiments were run on a 2020 M1 13-inch MacBook Pro with 8 GB of memory.

6.1 Synthesis of Coverage Complete Generators (RQ1, RQ3)

Our first set of experiments evaluate the ability of Cobb to automatically repair an incomplete input generator, and considers a diverse set of data types (e.g., lists, trees, and lambda terms) and target preconditions (e.g., sorted lists, balanced trees, and well-typed lambda terms) (**RQ1**). To build the incomplete generators used in our experiments, we took coverage-complete generators drawn from the existing PBT literature [28, 30, 66], and made them incomplete by replacing one or more of their branches with err. We construct multiple variants of each generator by removing different combinations of branches, including *sketches* of each generator which replace all its branches with err (**RQ3**). The coverage type specification used in each benchmark is a direct translation of the target precondition. Table 1 presents the results of using Cobb to repair each of these variants. The variants are (roughly) ordered by the amount of the functionality they lack, with the sketch acting as the final variant of each generator. The required repairs range from the relatively trivial— the first two sized list variants only require inserting an empty list ([]), for example— to the more substantial: repairing the red-black tree sketch requires synthesizing multiple recursive calls and applications of datatype constructors with very specific arguments.

For almost every variant, Cobb was able to find a repair that was equivalent to the term that had been replaced by err, modulo some syntactic differences (e.g., order of operations, normal form), including for every sketch (**RQ3**). A notable exception is the sixth variant of the red-black tree generator, shown in Fig. 10: while the original generator directly constructs a black node and its subtrees, Cobb finds a smaller, but semantically equivalent repair which makes a recursive call with the correct color/invariant arguments. Our cost function biases recursive terms earlier in the synthesis process because they produce similar coverage to that of our goal. In this case, the coverage supplied by the original branch can be fully realized by flipping the color in the recursive call and updating the size invariant.

Table 1. The results of using Cobb to repair incomplete generators. Benchmarks are annotated with their source: QuickChick [30] (*), Lampropoulos et al. [28] (*) and Zhou et al. [66] ($^{\circ}$). The middle set of columns characterize the complexity of the problem and the solution: the number of holes in the initial sketch (#Holes) and the size of the AST of the term that is synthesized (Repair Size). The last set of columns describe the effort required to find a repair: the number of terms enumerated (#Terms), the number of SMT queries (#Queries), the time it took to infer the missing coverage (Abduction), the time spent generating the final solution (Synthesis), and the total time needed to find a coverage complete generator (Total).

Benchmark	#Holes	Repair Size	#Queries	#Terms	Abduction (s)	Synthesis (s)	Total Time(s)
Sized List 1	1	1	31	3	0.76	0.51	1.34
2	1	1	30	3	1.95	0.52	2.54
3	1	14	77	10	2.95	2.88	5.89
4	2	2	38	6	2.62	0.61	3.31
5	1	20	94	10	2.67	3.62	6.37
6	2	15	84	13	3.14	2.99	6.19
7	2	21	100	13	3.95	3.61	7.64
8	1	20	93	10	2.86	3.73	6.67
sketch	2	21	99	13	4.41	3.72	8.21
Even List 1	1	11	100	17	6.07	3.05	9.25
2	1	11	169	23	6.26	8.54	14.93
3	1	17	172	26	6.06	10.94	17.08
4	2	22	247	40	10.17	11.07	21.37
5	1	33	231	26	10.39	16.15	26.7
6	2	28	250	43	10.46	13.38	23.91
sketch	2	44	308	43	14.7	17.74	32.59
Sorted List 1	1	1	27	3	0.13	0.37	0.54
2	2	16	151	74	2.59	6.02	8.65
sketch	3	17	154	77	1.99	5.91	7.94
Duplicate List 1	1	1	28	3	5 78	1 28	7 11
2	1	18	89	19	2.98	13 93	17
sketch	2	19	96	22	3.95	13.55	17.55
Unique List 1	1	1	26	2	0.13	0.42	0.58
2	2	7	70	12	0.83	6.46	7 32
sketch	3	, 8	71	14	0.05	6.10	7.82
	-						
Red-Black Tree 1	1	1	156	5	62.02	2.17	66.85
2	1	1	154	6	61.27	2.29	66.27
3	1	14	232	45	54.65	4.27	61.26
4	1	34	362	95	59.31	10.97	71.26
5	1	31	328	87	52.45	8.61	72.44
6	1	17	219	51	56.02	2.88	60.73
sketch	4	108	673	232	78.52	20.88	100.83
BST 1	1	1	66	4	22.81	4.34	47.17
2	1	1	71	5	21.18	4.54	42.37
3	1	1	65	4	24.26	6.05	51.44
4	1	33	877	589	18.26	129.98	160.79
sketch	3	41	941	597	51.19	145.84	215.25
Sized Tree 1	1	1	37	3	3 87	0.75	4,97
2	1	1	36	3	4 06	0.78	5.18
3	1	18	159	14	3.87	9 38	13 41
sketch	2	25	182	17	8.74	10.21	19.04
Complete Train 1	· -			- /	0.77		1.00
Complete Tree I		1		3	0.67	0.6	1.32
4 alzotab	1	10	00	14	1.18	4.92	0.10
SNELCH		19	- 7)	1/	10	4/1	0.00

For all these benchmarks, Cobb was able to find a complete generator within a reasonable amount of time (**RQ1**), with the time taken roughly correlated to the complexity of the target property and the functionality that was removed. In general, most of the repair time is spent on calls to Z3, with Abduce and Synthesize dominating the total runtime. In general, longer abduction times correspond to a more complex specification and more method predicates, while longer synthesis times correspond to a larger space of candidate patches. As expected, repairing the sketch of each

benchmark takes the largest amount of time, as they are missing the most coverage. The BST sketch, for example, requires Cobb to explore one of the largest search spaces of all our experiments, with the final repair synthesizing a pair of recursive calls with non-trivial arguments. On the other end of the spectrum, the target coverage type of the red-black tree benchmarks enforces several non-trivial invariants, resulting in some of the largest abduction times. Most of the remainder of the total time is spent type checking the completed generator; these times are consistent with those reported by Zhou et al. [67].

Case Study: Well-Typed Lambda Calculus Terms. As a final experiment, we also investigated Cobb's performance on a more challenging problem: repairing a generator for well-typed simply typed lambda calculus (STLC) terms [27, 49]. On its own, the reference generator is already quite complex, featuring multiple inductive datatypes and auxiliary functions. The specifications of the generator and these auxiliary functions are similarly intricate, requiring 15 method predicates. Simply checking the completeness of the reference generator is non-trivial, and takes roughly two minutes [67]. We developed three variants of the reference generator using the same methodology as our previous set of experiments. We additionally bound the space of candidate repairs in each experiment, by manually limiting the set of components used by Cobb to those occurring in the expression that was deleted from the reference implementation.

Despite the challenges inherent in this benchmark, Cobb was able to produce complete repairs for all three STLC variants, with the two simpler variants each taking less than two and a half minutes to repair.⁶ The final variant required a more substantial repair that involved multiple recursive calls and sophisticated reasoning, e.g., the patch must randomly divide the maximum number of applications allowed in recursively generated subterms. While searching for this patch, Cobb enumerates more than 1500 terms and issues almost 4500 SMT queries. Although Cobb is able to successfully find this patch, it takes almost 45 minutes to do so, with the bulk of the time being spent querying Z3. We suspect that optimizing these queries further should drive down the total runtime for all of our benchmarks; doing so is an important direction for future work.

Discussion. Taken together, these two sets of experiments provide evidence that Cobb's runtime scales reasonably well with the complexity of both the repair and synthesis tasks, suggesting the potential of our approach in future applications that depend on generating data that meets some desired property. Importantly, the cost of performing a repair is paid once: a repaired generator can be run normally, without any need to invoke an SMT solver.

6.2 Comparison with Alternative Repair Strategies (RQ2)

At a high-level, Cobb balances two competing concerns when searching for a patch, trying to find a repair that limits the number of 'useless' inputs that fail to meet the target precondition, while simultaneously ensuring it does not omit any 'interesting' values that do. This set of experiments compares Cobb to alternative strategies that exclusively prioritize one of these concerns (**RQ2**).

Completeness-Focused Repair. Our first set of experiments compares Cobb against an approach that only prioritizes completeness when searching for repairs. This admits an easy implementation: we simply fill in each hole inserted by Localize with the default generator for the base type of the hole, e.g., genTree or int_gen. This results in generators that are at least as complete as those found by Cobb, at the cost of potentially producing more 'useless' inputs. Thus, to compare the two strategies, we track how many values a repaired generator produces that violate the target precondition. We use each generator to produce 20k values, recording how many of these outputs satisfy the target precondition. Following prior work, we constrain the size parameter used in

⁶The supplementary material provides detailed numbers for each experiment.

Anonymous



Fig. 11. Comparison of Cobb with completeness-focused repair for list generators.



Fig. 12. Comparison of Cobb with completeness-focused repair for tree generators.

each experiment, adopting similar bounds to those works [27, 30, 60]; Section 6.4 provides more details on the bounds used. These experiments also address the feasibility of directly using a default generator to compensate for a generator's missing coverage.

Fig. 11 and Fig. 12 present the results of this experiment for each of the list and tree benchmarks from Table 1, respectively. Both tables also report the number of valid outputs produced by a default generator; this serves as a rough proxy for the restrictiveness of the target precondition. From left to right, each group of columns in the figures report the number of valid inputs produced by the default generator (purple), the generator repaired by Cobb (green),⁷ and the repaired version of each variant in Table 1 produced by a completeness-focused repair strategy (cyan), ending with the repaired sketch (blue). Unsurprisingly, the last variant performs comparably to the default generator: applying the completeness-focused repair strategy to genEven_{inc}, for example, results in a function that is effectively equivalent to the default **int list** generator.

While the generators produced by Cobb are consistently more likely to produce valid values than their completeness-focused counterparts, Fig. 12 shows that the latter strategy can be effective in certain cases, especially when pitted against the default generator. As expected, one such scenario is when the target property is relatively permissive, as is the case for our sized list and sized tree benchmarks. These generators only need to produce a value within the expected size; as the default generators show, roughly half of all randomly generated values satisfy this property.

Conversely, when the target specification is more restrictive, the alternative repair strategy is less effective. The complete tree benchmark falls into this category, as the subtrees of a randomly generated tree are unlikely to have a uniform depth. Similarly, the target preconditions used by our unique and duplicate list benchmarks are considerably tighter than that of the sized list benchmark: both require a list containing *exactly* size elements. In both cases, the coverage provided by the

⁷Fig. 11 and Fig. 12 uses the generator produced from the sketch for the generator produced by Cobb. Since the Cobb-repaired generators are semantically equivalent, so are their results on these benchmarks.



Fig. 13. Comparison to safety-focused repair.

repaired generators is mostly limited to when the size parameter is small, although uniqueness of list elements being a slightly more forgiving property.

A similar phenomenon occurs in the BST and red-black tree benchmarks, albeit in a more nuanced way. Both of these benchmarks feature semantically rich specifications, while still being somewhat more permissive than the previous three examples. Notably, the completness-focused repair strategy is effective for some of these benchmarks, in particular the third BST variant and the first red-black tree variant. For these two examples, the required repairs fall into execution paths which are exercised very rarely, so the default generator used in the repair is not given many opportunities to inject an invalid value into the output of the repaired generator. In the case of the third BST variant, for example, the repair is inserted into a branch in which bounds force the BST to be empty, i.e., lo + 1 = hi, a scenario that depends on a very particular sequence of nondeterministic choices. In the case of the red-black tree, the repair is only triggered when the generator is called with very specific values, namely when the input black height is precisely zero and the color argument is black. These sorts of corner cases are sometimes explicitly listed in handwritten generators in order to improve the likelihood they will occur; identifying and prioritizing these sorts of corner cases in repairs is an interesting direction for future work.

On most of these benchmarks, the generators repaired by Cobb almost always produce valid inputs, with the sorted list generator being the notable exception. This generator is unique among our benchmarks as it is the only benchmark in which the reference generator includes an err expression that is hit with some frequency. While errors are fine from the perspective of our coverage type system— the right sequence of nondeterministic choices always allows the generator to avoid them— because the original sorted list generator does not implement any kind of backtracking [27], these errors impact the number of sorted lists the repaired generator produces. As a result, the repaired generators in this experiment only have a reasonable probability of yielding a valid output for smaller lists.

Safety-Focused Repair. As the previous experiment showed, a repair strategy that only prioritizes completeness is likely to produce generators that output useless, i.e., invalid values. This set of experiments investigates whether a repair strategy that only considers safety will yield generators that are likely to omit interesting, i.e., valid values. Before doing so, observe that it is not obvious how to measure the incompleteness of a generator: even an incomplete generator can produce an infinite number of valid inputs. Our strategy is to instead compare the relative completeness of two generators, in this case by quantifying the number of outputs produced by a Cobb-repaired generator that could never be produced by a one repaired using a safety-focused strategy. In detail, we first ascribe a standard refinement type to the return type of each safe generator: intuitively, the qualifier of this type overapproximates the range of the generator. By negating this type qualifier, we can characterize the set of outputs that fall outside the range of a safety-repaired generator; any

valid outputs of a Cobb-repaired generator that satisfy this negated property cannot be produced by its safe counterpart.

As an example, one way to repair genIntList from Fig. 7a so that it is safe with respect to our target precondition is:

let rec genIntList_{safe} (n : int) : int list = if n == 0 then [] else []

We can ascribe the following refinement type to $genIntList_{safe}$, capturing the fact that it always returns the empty list:

$$\{n: int \mid n \geq 0\} \rightarrow \{1: int \ list \mid len(1) = 0\}$$

Thus, any value of the type $\{l: int \ list \mid \neg (len(l) \neq 0)\}$ must fall outside the range of genIntList_{safe}.

To carry out our experiment, we have developed a safety-focused repair strategy that replaces Synthesize with the repair that an off-the-shelf, safety-guided synthesizer [52] would generate for each hole. The hole corresponding to the base case of genSizedList yields the following synthesis goal, for example:

$$n : \{n : int \mid n = 0\} \vdash \Box_1 : \{1 : int \ list \mid len(1) \le n\}$$

To approximate the relative completeness of Cobb versus a safety-focused repair approach, we generate 20k values from the Cobb-repaired generator and record how many of these outputs fall outside the range of the safety-focused generator. Fig. 13 present the results of this experiment for each of the list and tree variant from Table 1. The taller the bar, the more (relatively) complete the Cobb-repaired generator. The safe versions of the unique and duplicate list benchmarks are coverage complete, so we omit them from Fig. 13.

On these benchmarks, the completeness of the safety-focused repair strategy tends to be an all-or-nothing proposition: in the case of the sized list benchmark, for example, the safety-focused strategy's prioritization of minimal terms yields generators that always return a [] term for six of the nine variants; this is precisely the repair needed by three of these, however. In contrast, the safety-focused strategy tends to be more effective when the set of valid repairs are strongly constrained by the arguments of a generator: the length of the output lists in the unique and duplicate benchmarks is completely determined by its size parameter, for example. The safe strategy is also effective when the required repair is a constant: the required repair in the first two variants of depth tree benchmark is a single Leaf constructor, for example.

In contrast, the safe repair strategy tends to perform poorly when the target precondition admits a number of safe repairs that are relatively cheap: a cost function that employs Occam's razor, for example, prioritizes small repairs that use variables or constants. Even when the target property is quite restrictive, a safety-focused strategy is biased towards repairs that produce values of the right "shape", but whose contents do not vary much, values that are unlikely to explore code paths that depend on those contents. Crucially, prioritizing completeness is not simply a matter of equipping a safety-focused generator with a different *syntactic* cost function: our complete repair for the red-black tree sketch, for example, requires synthesizing multiple terms, each of which are individually safe, and joining them together. Finding the right combinations of terms to join together requires *semantic* characterizations of both the missing coverage *and* the coverage provided by a candidate patch.

Discussion. An important takeaway from both of these experiments is that while completenessand safety-focused repair strategies can yield useful repairs in certain situations, their efficacy is highly dependent on the particular problem, and there are many scenarios in which neither approach performs well. When the target property is weak, a completeness-focused repair can improve on the default generator, while a safety-focused strategy tends to work well when the specification is very strong. Neither approach tends to do well when the property falls somewhere



Fig. 14. Time needed to generate 1k+10k valid Red-Black Trees.

between these two extremes, e.g., our BST and red-black tree examples. Prioritizing the minimal coverage-complete repair enables Cobb to produce repaired functions that generate useful inputs without omitting any interesting values.

6.3 Comparison with Dynamic Test Input Generation (RQ4)

The ultimate goal of Cobb is to use symbolic reasoning to statically ensure that the set of values enumerated by a generator aligns with the complete set of values that meet the precondition of a function under test. One alternative approach is to instead use a theorem prover to dynamically generate these values during testing [27, 60]. To evaluate these alternative styles of test generation, this experiment compares Cobb with Luck [27], a tool which queries a constraint solver to produce values that satisfy a user-defined predicate written in a DSL. Fig. 14 reports the time needed for Luck to generate 1k and 10k red-black trees, respectively, against the time needed for Cobb to generate the same number of trees. As a baseline, the figure also reports the time needed by to produce 1k and 10k valid red-black trees by running the default generator in 'generate and filter' loop. We use an upper time limit of 5 minutes for all the experiments; only the baseline approach exceeds this bound.

While not a perfect apples-to-apples comparison— among other things,⁸ generators are implemented in different languages and frameworks— these experiments confirm the conclusions of Lampropoulos et al. [27] that run-time constraint checking imposes (at least) an order of magnitude amount of overhead over a generator that does not solve constraints at runtime. One takeaway from this experiment is that the overhead of dynamic constraint solving quickly matches the overhead required by our static repair approach— it takes Cobb roughly a minute to repair the red-black tree sketch, which is roughly the amount of time needed to generate 10k red-black trees with a black height of at least 5. Given that a repaired generator can be run without any additional constraint solving, the overhead required by the static repair approach seems reasonable for settings in which generators are repeatedly used, e.g., when using PBT in a CI setting [18].

6.4 Discussion and Limitations

While our sized generators are complete for an arbitrary size bound, the experiments in Sections 6.2 and 6.3 use a more limited range of bounds when generating values, as is common in the literature. All of our list benchmarks use QCheck's built-in generator for natural numbers, nat_gen(). This generator produces integers between 0 and 10000, and its distribution of outputs is skewed towards

⁸Unlike Luck, Cobb-repaired generators are not guaranteed to produce unique values, although the latter's use of int_gen() to produce (signed 63-bit) integers means they are statistically unlikely to produce duplicate trees.

smaller values. The bound in our tree benchmarks limits the height of the tree, which bounds the number of nodes in a tree at $O(2^{n+1} - 1)$. Simply using nat_gen() for these benchmarks can generate very large trees, so these benchmarks instead use a height between 0 and 12, chosen at random; this range is also used in prior works [27].

As mentioned in Section 5, Cobb only guarantees that a value can be generated with non-zero probability, and says nothing more about the likelihood that it will be generated. As a consequence, Cobb does not ensure that a repaired generator is fair, i.e., that every value can be produced with uniform probability. In our experiments, the distribution of a repaired generator's outputs largely depends on the structure of the original generator. Reasoning about the fairness of repaired generators and repairing unfair generators is an interesting direction for future work.

As with other bottom-up synthesis techniques, the number of components available to Cobb, and thus the size of its search space, impacts its performance, hence our use of a restricted set of components in the STLC benchmarks. Other bottom-up enumeration techniques have proposed several solutions to this problem, including [5, 7, 33, 34, 46]; while Cobb does not currently implement these strategies, they should also be effective in our setting.

7 Related Work

Generating Data Meeting Sparse Preconditions. A number of works have considered how to effectively generate data satisfying a sparse precondition. The proposed solutions can be roughly categorized as either dynamic and static approaches. Dynamic approaches attempt to directly ensure the validity of inputs as they are being generated [9, 17, 27, 39, 60], typically by relying on runtime constraint solving. Like Cobb, Target [60] uses refinement types to define the space of valid inputs. To generate values, however, Target queries an SMT solver for a model satisfying the type qualifier, and then converts the model into a value in the target language. To generate additional values, the SMT query is updated to explicitly exclude any models that have already been found. Another particularly popular strategy is to directly leverage the definition of the target precondition, lazily concretizing the value being generated in a way that ensures the constraint is satisfied and backtracking when constraints become unsatisfiable [9, 17, 27, 39], similar to the idea of narrowing in logic programming languages. The completeness of dynamic approaches is typically tied to the completeness of the underlying constraint solver: as long as the solver can return any satisfying value, so can the input generator. The need to solve constraints at run time imposes considerable overhead however; as discussed in Section 6.3, dynamic approaches can be orders-of-magnitude slower than their static counterparts, particularly when the target property is complex.

Cobb instead adopts a *static approach*. Static generation techniques avoid run-time constraint solving, and instead seek to construct input generators that are sound and complete *by construction*. Closely related to Cobb is a line of work that automatically builds generators for the QuickChick PBT framework [30] by compiling inductively defined relations into efficient generators [29, 50]. This pipeline uses a translation validation approach [51] to ensure the correctness of the resulting generators, producing formal proofs of their soundness and completeness in the Coq/Rocq proof assistant. Unlike Cobb, which is agnostic to how the target property is defined, these works impose a strict requirement that the target precondition be defined as an inductive proposition in Coq/Rocq, although recent work has considered how this restriction can be somewhat relaxed by, e.g., composing different inductive relations into a single unified definition [54]. This restriction is used to produce generators that closely follow the definition of the proposition itself; Cobb, in contrast, is able to synthesize and repair arbitrary programs that supply the desired coverage.

Generating a Good Distribution of Data. An orthogonal problem to coverage is the question of the *distribution* of outputs produced by a generator: a generator for trees that produces **Leaf** nodes 90%

23

of the time is less useful than one whose outputs are uniformly distributed, for example. A couple of tools have been proposed for statically reasoning about the distribution of a generator's outputs: Feat [13], for example, is a library for writing enumerators of datatypes that are guaranteed to produce a uniform distribution of the values of an algebraic datatype. The DRAGEN tool [42, 43], in contrast, uses a mathematical model to statically estimate the distribution of constructors produced a QuickCheck generator built from its frequency combinator, and uses those estimates to adjust the arguments of frequency to achieve a more desirable distribution. Extending Cobb to account for the distribution of a generator is an interesting direction for future work.

Program Synthesis. Automatically deriving programs from logical specifications of their behavior has been a goal of the programming synthesis community for almost half a century [37]. The overwhelming majority of program synthesis techniques use specifications that overapproximate the set of desired behaviors [2, 12, 41, 53], including those that also use refinement types to specify program behaviors [22, 25, 52]. The specifications used by Cobb, in contrast, stipulate an underapproximation of the desired behaviors. This impacts Cobb's repair algorithm, which composes partial solutions which do not individually satisfy the target specification, to build a complete solution. The sets of input-output examples used by inductive synthesis, or programming-by-example (PBE), techniques [1, 3, 16, 33, 34, 40, 47, 61, 65] also underapproximate the target program's behavior, although they do so much less comprehensively than coverage types. While similar to the bottom-up term enumeration strategy employed in PBE systems, Cobb's Synthesize procedure is able to take advantage of the more complete approximation provided by coverage types to, e.g., recursively call the generator being repaired before its full definition is known [40, 65].

Automated Program Repair. The goal of automated program repair (APR) is to automatically patch a buggy program with minimal user effort [32]. Most APR approaches use test suites to identify buggy behaviors: a valid patch is one that causes a program that was failing some tests to instead pass its suite. A notable exception is the work of Logozzo and Ball [36], which defines a good repair as one that decreases the number of statically detected assertion failures in a program without introducing any new ones. A major challenge in APR is finding repairs that generalize beyond a particular failing test [62], a problem that Cobb avoids thanks to the strong correctness specifications provided by coverage types. Like Cobb, several APR techniques rely on program synthesis to generate candidate repairs. Nguyen et al. [45], for example, employ symbolic execution to identify path constraints that cause tests to succeed or fail. These constraints are used as a safety specification for the target repair, which is then generated using component-based synthesis. An alternative strategy is to use angelic execution [31, 38] to identify concrete values that can be used to help a program pass a failing test; finding a patch that generates these values is an instance of a PBE problem. As previously discussed, the program synthesis techniques employed by both strategies use specifications that are fundamentally different from Cobb's.

8 Conclusion

When using a property-based testing framework to automatically test a program that has a restrictive or sparse precondition, users are typically forced to manually write a function that effectively generates values of interest. Alongside the additional burden this imposes on users of PBT frameworks, this process is also error-prone, as generators can be both unsound, producing values that do not meet the target precondition, and incomplete, incapable of producing all the values that meet the precondition. This paper presents a technique for detecting and repairing incomplete test input generators, leveraging coverage types to characterize the set of missing test values and the coverage provided by candidate repairs. Our repair technique uses a novel coverage-type guided enumerative synthesis algorithm to generate candidate repairs, employing

a lattice structure to store partial solutions so that they can be efficiently queried and combined to build a complete repair. We have implemented a repair tool for OCaml input generators, called Cobb, and have used it to repair a diverse suite of benchmarks drawn from the PBT literature. Our experiments demonstrate that Cobb can also be effective as a sketch-based synthesis tool for test input generators, suggesting its potential for further reducing a possible point of friction for users of PBT frameworks.

Data-Availability Statement

Our supplementary material includes an anonymized artifact. This artifact contains the OCaml source code for Cobb and of the modified dependencies, our suite of benchmark programs with results and scripts used to produce our experimental results. We intend to submit this artifact for evaluation by the artifact evaluation committee should this paper be accepted.

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