Lecture 24

Last time

$$A\underline{x} = \lambda \underline{x}$$

Power Method

```
x = random
for i = 1 to ~ large iteration.
    $y = Ax$
    $x = y/\|y\|_2$
```

Idea:

$A^K \cdot$ random

→ Eigenvector

$x^{(0)} =$ rand

$y^{(1)} = A \cdot$ rand

$$x^{(1)} = \frac{A \cdot \text{rand}}{\rho}$$

$$y^{(2)} = A x^{(1)} = \frac{A^2 \cdot \text{rand}}{\rho}$$

$$x^{(2)} = \frac{A^2 \cdot \text{random}}{\rho^{(2)}}$$

$$x^{(k)} = \frac{A^k \cdot \text{rand}}{\rho^{(k)}}$$

Convergence

A = symmetric, real

$A = V \Lambda V^T$

$V = [\underline{v}_1 \; \cdots \; \underline{v}_n]$

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$A\underline{v}_i = \lambda \underline{v}_i$

also $\underset{\sim}{v}_i^T \underset{\sim}{v}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$$A^2 = (V \Lambda V^T)(V \Lambda V^T)$$

$$= V \Lambda \underbrace{(V^T V)}_{I} \Lambda V^T$$

$$= V \Lambda^2 V^T$$

$$A^K = V \Lambda^K V^T$$

$$\frac{A^k}{\lambda_{max}^k} \longrightarrow V \frac{\Lambda^k}{\lambda_{max}^k} V^T$$

λ_{max} = largest magnitude Eigenvalue

$\lambda_1 = 5$

$\lambda_2 = 3$ $\quad \lambda_{max} = 5$

$\lambda_3 = -2$

$\lambda_4 = -10$ $\quad \lambda_{max} = -10$

$$\begin{bmatrix} \lambda_1^k / \lambda_{max}^k & & \\ & \ddots & \\ & & \lambda_n^k / \lambda_{max}^k \end{bmatrix}$$

$$\Big\downarrow \text{ if } \lambda_1 = \lambda_{max}$$

$$\begin{bmatrix} 1 & & & \\ & \phi_2 & & \\ & & \ddots & \\ & & & \phi_n \end{bmatrix} \text{ as } k \to \infty \longrightarrow$$

$$|\phi_i| < 1$$

$$\begin{bmatrix} 1 & & 0 \\ & c & \\ 0 & & \ddots & 0 \end{bmatrix}$$

Claim

$$\rho^{(k)} \approx \lambda_{\max}^{(k)}$$

If $x^{(k)} \approx \underline{v}_1$

$$A\underline{v}_1 = \lambda_{\max} v_1$$

$$\|A v_1\| = |\lambda_{\max}|$$

Convergen

$\lambda'_{\max}$ = 2nd largest magnitude

$$\overline{\lambda_{\max}.}$$

How to get

vectors?

qr Q =

qr(vecto

& if $\underline{x}$

then

Multiple

Q factor in qr(A)

$) =$

is a vector

$$x = \underbrace{\begin{bmatrix} x/\|x\|_2 \end{bmatrix}}_{Q} \cdot \underbrace{\begin{bmatrix} \|x\|_2 \end{bmatrix}}_{R}$$

```
for i=1 to _
    x = qrQ(Ax)
```

---

let $X^{(0)} = I$

then

```
for k=1 to _
    $X^{(k)} = qrQ(AX^{(k-1)})$  →
```

$$Q^{(k+1)} R^{(k+1)} = qr(R^{(k)} . Q^{(k)})$$