In this class:

- *A quick review of probability: random variables and expectation*
- *How to compute pi with a Monte Carlo method*
- *How to compute an integral with a Monte Carlo method*
- *The central limit theorem and what it says about Monte Carlo*

September 14, 2016

Monte Carlo Methods

Next class

HW Due, Catchup and Review!

G&C – Chapter 3

Next next class

Review for Midterm, Variance

Random Variables

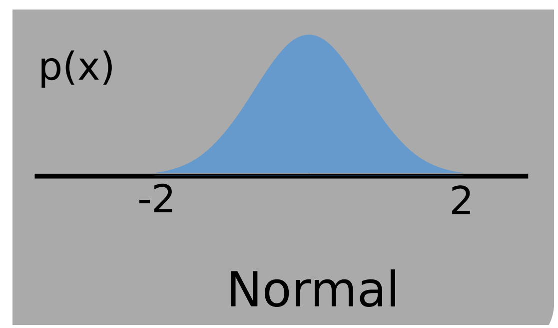
X is a random variable (R.V.)

- Discrete RV

  coin toss, dice roll, Monty Hall problem

- Continuous RV

  uniform, normal



Expectation

the "expectation" is akin to the "average"

$$E[X] = \sum_{v=\text{values of X}} v \cdot \text{Prob}[X = v]$$

E[dice] = 1·1/6 + 2·1/6 + 3·1/6 + …

$$E[X] = \int_{\mathcal{D}} xp(x)\, dx$$

Expectation

Linear $E[aX + bY] = E[aX] + E[bY]$
if a and b are non-random constants

Non-multiplicative $E[XY] \neq E[X]E[Y]$
(unless X and Y are independent)

Expectation Example

$$X \sim \text{Uniform}[0,1],\ Y \sim \text{Uniform}[0,1]$$

$$Z = XY$$

$$E[Z] = E[XY] = \int_0^1 \int_0^1 xy\, dx\, dy$$

$$E[Z] = 1/4$$

Preview of Central Limit Theorem

The CLT is why Monte Carlo works!

$X_1, X_2, X_3, ...$ are iid copies

independent, identically distributed

think independent dice rolls

$X_1 \sim X$ dice roll 1

$X_2 \sim X$ dice roll 2

Preview of Central Limit Theorem

The CLT is why Monte Carlo works!

$X_1, X_2, X_3, \ldots$ are iid copies

independent, identically distributed

The CLT gives a relationship between

$$\frac{1}{N}\sum_{i=1}^{N} X_i \quad \leftrightarrow \quad E[X]$$

Average of N samples of X.
What we compute with MC!

The formal expectation!