Nick Trefethen
has 2 talks
- 3:30 Today
- 3:45 Tomorrow

Course Notes / Book
has a very updated
section on PageRank.

- When to stop the power method? ✓
- Methods to determine permutation matrix ✓
- Other λ-vals besides spectral radius? (2x) ✓
- What does inv(A) do? ✓
- Speed up the power method?? ✓ ←

· Can we flip permutation in

$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

w/ Jacobi? ✓

(Eq Eq. 1 w/ row 1

· Why gradient ascent → power method? $\gamma \to \infty$

Why not just do gradient ascent? ✓

· for max $|x^T A x|$

s.t. $\|x\|^2 = 1$

Why do we ensure A is pos-def? ✓

pos. semi-def ok? ✓

· Why does power method converge? ✓

· Doesn't PCA require λ-vals of non-sym matrices? ✓

Other Examples?

(

2

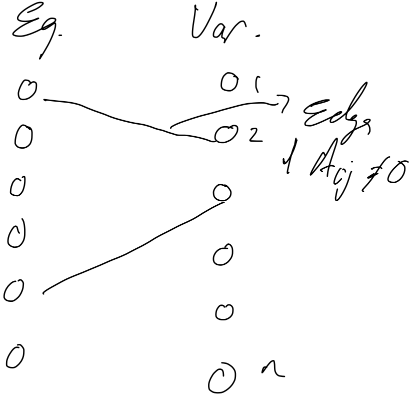
n

A

in

ass

Va



perfect matching
this graph is an
ignment of Eq's,
j s.t.
$A_{ij} \neq 0$.

In

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

if you do
parallel update w/
Eq 1, Var 1
Eq 2, Var 2,

I think it won't
Converge.

→ Todo:

Try it! or
work out

$$\rho(D_M^{-1} N_M)$$

$$\rho\left(\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\right)$$

inv(A) in Julia/Matlab

- Uses a finite term. alg / Elimination alg.

to solve

$$AX = I.$$

>1.c

$\Rightarrow$ n Systems of linear eq.s

Don't use inv(A) on Exams! (or HW).

Unless Explicitly asked for!

Why Sym. A for λ-vals / λ-vecs

- It's Easier.
- Only app. of non-sym λ-vals is

Are there any λ-vals w/ a positive Real Component.

- PCA uses λ-vals of a Sym. Matrix!

?

→ It's a Covariance matrix:

If X is a matrix of Data, PCA looks at λ-vals of

$$\tilde{X}^T \tilde{X} \text{ or }$$

$$\tilde{X}\tilde{X}^T$$

(dep. on orientation).

Power Method works great on non-sym matrices!

When to Stop power method?

$$|\lambda^{(k)} - \lambda^{(k+1)}| \,/\, \max($$

is an Estimate of

(*)

$$\boxed{\begin{array}{c} \|A\underline{x}^{(k)} - \lambda^{(k)}\underline{x}^{(k)}\|_2 \le \\ \lambda^{(k)} = x^{(k)T} A x^{(k)} \end{array}}$$

(assum $\|x^{(k)}\|_2 = 1$)

$(|\lambda^{(k)}|, |\lambda^{(k+1)}|) \le \varepsilon\sqrt{n}$

dim factor

$\downarrow$

$\sqrt{n} \cdot \varepsilon \cdot |\lambda^{(k)}|$

$\uparrow$ machine prec. 10^{-16}

2^{-52}

$\uparrow$ Matrix Scale.

→ Truncation

Convergence of $|\lambda^{(k)}|$.

$\underline{x}$ is an Eigenvector of

$A\underline{x} = \lambda \underline{x}$

$\Rightarrow$

If you are lucky and have an Error/Convergence formula for your matrix. Then run until that is Small Enough!.

- Deflation to find $\longrightarrow$ $A \rightarrow$ $\longrightarrow$
- Inverse iteration

→ apply power method to A^{-1} instead of A!

power method on $(A - \sigma I)^{-1}$ will find λ's close to σ.

polys in A.

$$(A - \lambda v v^T) = B$$

when $\lambda, \underline{v}$ is an Eigenvector.

$$B\underline{v} = (A - \lambda v v^T)\underline{v}$$

$$= A\underline{v} - \lambda \underline{v} \underbrace{(\underline{v}^T \underline{v})}_{=1}$$

$$= \lambda \underline{v} - \lambda \underline{v} = 0.$$

$$A - \lambda_1 v_1 v_1^T - \lambda_2 v_2 v_2^T - \lambda_3 v_3 v_3^T \; \vdots$$

Why does power method Converge?

$$\frac{A^k}{\rho(A)^k} \to \underline{v}\underline{v}^T$$

Where $\underline{v}$ is an Eigenvector.

for $A = A^T$,

$$A = V \Lambda V$$

$$\|\Lambda\| = \rho($$

$$\frac{\|A^k\|}{\rho(A)^k}$$

$$\frac{\|\Lambda^k\|}{\rho(A)^k}.$$

T

for matrix 2-norm

$\|A^k\| = \|\Lambda^k\|$

because V is orthogonal.

A)

for 1, 2, ∞ norms.

$\le 1.$

$\to \Lambda$ is diag.

$$\frac{\Lambda^k = \begin{bmatrix} \lambda_1^k & & \\ & \ddots & \\ & & \lambda_n^k \end{bmatrix}}{\lambda_{max}^k}$$

$$\to \frac{\|\Lambda^k\|}{\rho(A)} = 1.$$

$$\frac{\lambda_2^k}{\lambda_1^k}$$

λ_2 ← 2nd largest

λ_1 ← largest

$\lambda_2 < \lambda_1$

if $\underline{v}$ is an Eigenvector

and $\underline{x}$ is arbitrary,

then

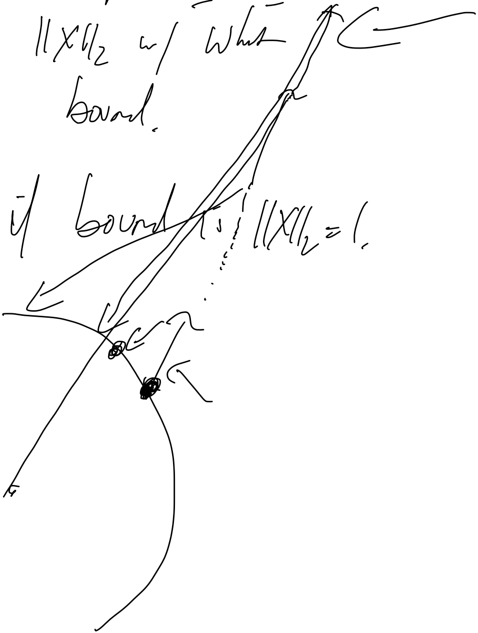
$$\frac{vv^T\underline{x}}{\|vv^Tx\|} = \frac{\sigma\underline{v}}{\|vv^Tx\|} = \underline{v}$$

Unless... $\underline{v}^T\underline{x} = 0$.

Gradient Ascent vs. Power Method

max $\underline{x}^T A \underline{x}$

$\|x\|_2$ w/ what bound.



$\|x\|_2 = 1$

$(I + \gamma A)$

~~$\|A\|_F$~~

$$x^{(k)} \to \frac{x^{(k)} + \gamma A x^{(k)}}{\|x^{(k)} + \gamma A x^{(k)}\|}.$$