Lecture 11

- HW3a due next week
- Midterm next week!

LU: $L_1 = I - \underline{v}\,\underline{e}_1$

$U_1 = I - \underline{e}_1\underline{u}^T$ ✓

What are $\underline{v}$, $\underline{u}$ here?

If $|\lambda_1| = |\lambda_2|$ what can we do w/ the power method ✓

$A + \gamma I$? How do you get back to $\underline{\lambda}, \underline{v}$ for A

- If $|\lambda_2| = 1$ it seems power very slow. What to do?

- for ~~Infor~~ Simple is there a way how many iters to hit ε to in Roundoff ?

d, I then

method is

✓

methods,

to estimate

are requir

lance?

✓

· Is not pivoting useful? ✓

· In power method

$$\frac{A^k x_0}{\|A^k x_0\|} = \begin{bmatrix} \text{scale} \cdot \overset{<1}{\boxed{\frac{\lambda^k}{\sigma^k}}} \gamma_{0,1} \\ 1 \end{bmatrix} \to e_1$$ ✓

But scale also depends on k.
Why doesn't this cause problems

· Is LU unique ?? → Yes if A is non-sing.
AND you specify
diag elements of L or U.

↳ What props require?

↳ Sparse matrices?

for simple methods

↳ Richardson, Jacobi, etc.

Can we get a bound on work?

— for steepest descent, we will work out a bound soon.

Jacobi/Richardson/GS

$$A = M - N$$

$$M\underline{x}^{(k+1)} = b + N\underline{x}^{(k)}$$

$$\underline{e}^{(k)} = (M^{-1}N)^k \underline{e}^{(0)}$$

If you can bound $\rho(M^{-1}N)$, then,

$O(\log_{\rho(M^{-1}N)} \varepsilon)$ iters

Suffice to drive

$$\|e^{(k)}\| \le \rho(M^{-1}N)^k \|e$$

(Exact, if A is normal).

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$^{(0)}\|$.

If $\rho(M^{-1}N) \le \theta < 1$, then θ^k is a proxy for the error after k steps

Scale$^{(k)}$ $\cancel{\frac{\lambda_1^k}{\sigma^k}[\gamma_0]}$

Scale$^{(k)} \Longrightarrow \frac{1}{\gamma_0}$

on overall $\|X^{(k)}\|_2$

$\|X\|_2$ is orthog. invariant, so

sym. $\|VX^{(k)}\|_2 = \|X^{(k)}\|_2 =$

So Scale$^{(k)} \to \frac{1}{\gamma_0} :=$

↙ Const.

to get

=1.

of A is

=1.

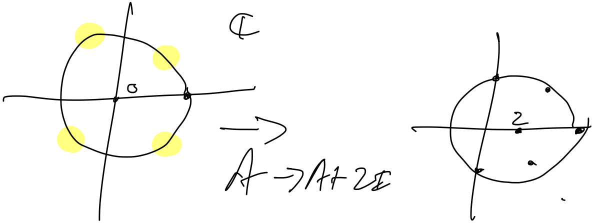
If $\lambda_2 \neq \lambda_1$ and $|\lambda_2| = |\lambda_1|$,

then $A + \gamma I$ will shift things

so $|\lambda_2| \neq |\lambda_1|$. (for some γ.)

lets say $\lambda_2 = -\lambda_1$

or



$A \to A + 2I$

↑ run power method

then subtract 2 from each λ-val.

If $\lambda_1 = \lambda_2$, then...

1. Use subspace iteration.

2. You can get an eigenvector.

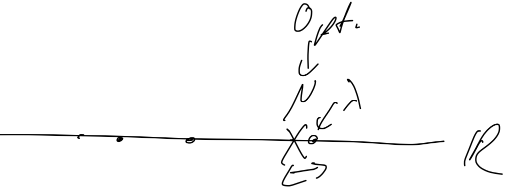
3. You can get both by looking at polynomial = relationships among 3 vectors.

- Write out $x_k \to x_{k+2}$ as a polynomial in λ-vals, then solve.

K

Alg:

for



$(A - \mu I)^{-1}$.

let $\mu, \underline{x}$ be an est. of $\lambda_{max}, \underline{x}$

hile

a few iters

$$\underline{x}^{(k+1)} = (A - \mu^{(k)} I)^{-1} x^{(k)}$$

$$\mu^{(k)} = \frac{x^{(k)T} A x^{(k)}}{x^{(k)T} x^{(k)}} ..$$

LU

almost

$$L_1 = \underline{I} - \underline{v} e_1 \ , \ U_1$$

wrm chi.

$$A = \begin{bmatrix} \alpha & \underline{u} \\ \underline{v} & R \end{bmatrix}$$

$$L_1 = I - \frac{1}{\alpha} \underline{v} e_1^T$$

$$L_i = I -$$

first col of A.

almost

$= \underline{I} - e_1 a^T$ first row of A.

$\underline{v}_i \underline{e}_i$

$PA = LU$ diag.

Then, $L \cdot D^{-1} D U$

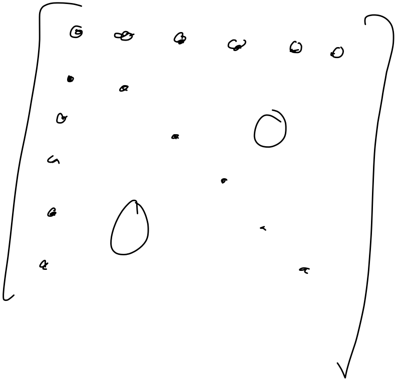
So LU is not unique.

UNLESS you specify the diagonal elements of L, U.

So if $\text{diag}(L) = \underline{I}$, then U is unique.

(and you use partial pivoting). w/ the same pivots.

A

=