Lecture 12

- Schur + LU ✓
- Why safe to compute LU w/o pivoting on spd. ✓
- Why does Julia just check A[i,1] == 0 ? for singularity ✓

- $\underline{x} = S\underline{y} + T\underline{z}$ Splitting ✓
- F = lu(A), F\b vs. A\b
- Solving for λ vals of sparse matrices? ✓
- Partial vs. Complete pivoting in LU

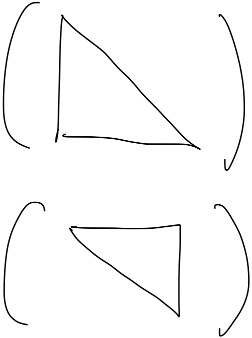
- Does coord. descent converge if A is not spd? (By analogy w/ Gauss-Seidel) ✓ ✓
- How does Julia compute P in LU?
  - ↳ Can you turn this off? Don't it Expensive??

Does Pivoting help spd?

- Back, Forward Solve? ✓

Back solve

Forward



$PA = LDU,$

$A\underline{x} = \underline{b}$

$PA\underline{x} = Pb$

Solve $\quad L D U \underline{x} = P\underline{b} = \tilde{b} \quad \nearrow \quad \underline{x} = \underbrace{U^{-1} \underbrace{D^{-1}(L^{-1} P \underline{b})}_{y}}_{z}$

$= L \qquad$ ~~$DU\underline{x} = y$~~

$\qquad\qquad$ ~~$Ly = \tilde{b}$~~ $\qquad U\underline{x} = \underline{z}$

$= U \qquad$ ~~$U\underline{x} = \underline{z}$~~

then $\qquad$ ~~$D\underline{z} = y$~~

λ-vals of Sparse Matrices.

→ power method works great!

Schur + LU

Why does this go fast?

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

y given $\underline{x}$.

Checking for Sing.

in Code.

Why does $A[i,i] == 0$ suffice?

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$\downarrow$ pivot

$$\begin{bmatrix} 2 & 4 \\ \underset{\to 0}{1} & 2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \boxed{0} \end{bmatrix} \leftarrow A_2$$

this will → fail in the first col.

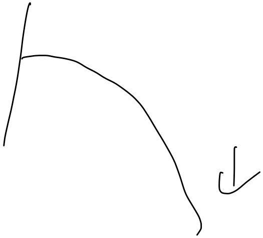
$$f(\underline{x}) = \frac{1}{2} x^T A \underline{x} - x^T \underline{b}$$

and A is non-sym.

then

$$g(x) = \frac{1}{2} A^T x + \frac{1}{2} A x -$$

If $A^T + A$ has a neg. λ-val.



⊛ Add Example to book of Coord Des.

Not Convergence

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Pivoting & SPD.

↳ Cholesky Decomp.

If A is spd. then after 1 step of Elimin

$A_1 \rightarrow A_2$

then A_2 is also spd.

$\underline{x}^T A \underline{x} > 0$

$\Rightarrow$

$e_i^T A e_i > 0$

$\Rightarrow$ all diag are positive!

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \swarrow$$

$\downarrow$ Pivot.

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

A \ b

- Is A
- Is A

if so,

try a

dec

dec

If flo

do LU

pivoting.

(in Julia

Okay? Is A upper triangular.
lower

sym.

then,

Sym. LU

amp / Cholesky

mp.

fails, then

w/ partial

F=lu(A), F \ b

This will always be faster than

A \ b

when lu is needed

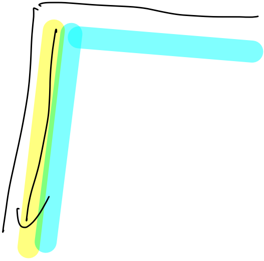
Complete pivoting

↳ find the largest mag. Entry in A and use it to pivot/ elimination ..

⊕ Example of LU not getting rank?

But complete pivoting does?

Rook pivot

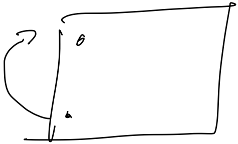


🟨 - partial

🟦 - Rook

whole - Complete

Rook + Complete Pivotin.

$$PAQ = LU$$