Lecture 13

⊛ Exam!

⊛ Rook Pivoting + failure of partial pivoting (Last Class) ✓

⊛ Office Hrs

↳ Tonight @ 9pm (ish) (≈ 45 mins)

1. How to build Q_1^T / R_1 in QR. ✓
2. LU vs. QR (111) ✓
3. Other norms in QR? ✓
4. Why is GS "unstable" / poor way to find QR ✓
5. gradient ascent + power method ✓
6. If A is not sym. how do we prove $\rho(A) < 1 \iff A^k \to 0$ → see Schur Jorda Canon For
7. When we do Variable Elimination for QR, why do we split $A = [\underline{a} \; C]$ Only not into rows? ✓

$$A = \begin{bmatrix} 0 & \\ \vdots & B \\ 0 & \end{bmatrix}$$

Partial pivoting will fail!

$$A = \begin{bmatrix} 0 & \cdots\ 0 \\ \vdots & B \\ 0 & \end{bmatrix}.$$

LU

- + Faster
- + More amenable to sparsity.

vs. QR

+ More stable/ (fewer Edge cases)

+ Works w/ Least Sq. directly.

+ Gives you an orthogonal basis.

QR (via Householder/ Reflector Matrices.)

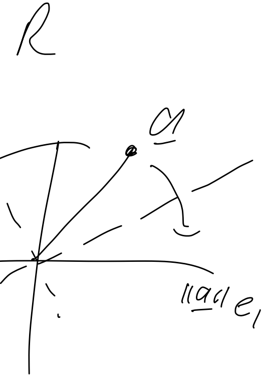
$$A \xrightarrow{Q_1^T A} \begin{bmatrix} \|a\| e_1 & B \\ 0 & \end{bmatrix} - \|a\| e_1 -$$

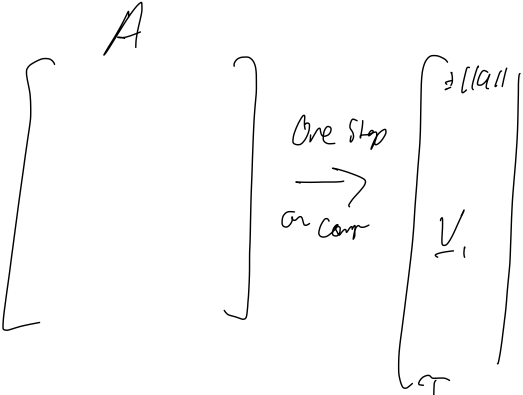
(Repeat on B[2:end, :])

$Q_1 = (I - 2\underline{v}\underline{v}^T)$

$\underline{v}$ = Something (check notes) sgn(a x||

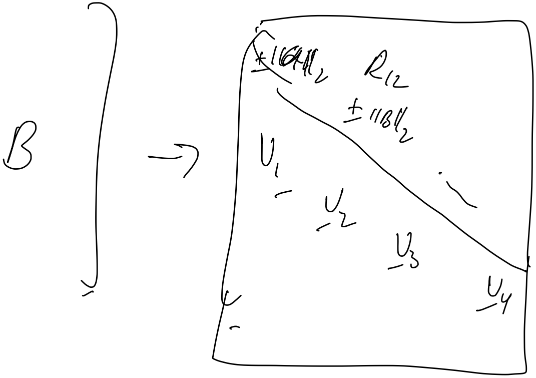
$$\underline{v} = \frac{\underline{a} - \alpha e_1}{\|\underline{a} - \alpha e_1\|_2} \qquad \alpha = \pm \|$$





There's a clever approach to get this to work.

Q



$(I - 2yy^T)\ldots$

Given you need to build Q as a prod

$$Q_1 \, Q_2 \, Q_3 - Q_{n-1}$$

$$(I - 2v_1v_1^T)(I - 2v_2v_2^T) \sim (I - 2v_{n-1}v_{n-1}^T)$$

Gradient Ascent

Max $f(x) \rightarrow f(x) = \frac{1}{2} x^T A \underline{x} \Rightarrow$

$g(\underline{x}) = A\underline{x}$

then try

gradient of f @ $\underline{x}_k$

$$\underline{x}^{(k+1)} = x^{(k)} + \alpha g^{(k)}$$

$$x^{(k+1)} = \underline{x}^{(k)} + \alpha A \underline{x}^{(k)}$$

Max $f(\underline{x})$ st $\|\underline{x}\|_2 = 1$

$$x^{(k+1)} = \frac{x^{(k)} + \alpha A x^{(k)}}{\| x^{(k)} + \alpha A x^{(k)} \|} \xrightarrow{\alpha \to \infty}$$

$$(I + \alpha A)$$

$$\Downarrow$$

$$x^{(k+1)} = \underline{x}^{(k)} + \alpha A\underline{x}^{(k)}$$

$$\frac{Ax^{(k)}}{\|Ax^{(k)}\|}$$