Lecture 20

- BW vs. FW Stability, which is better? ✓
- Simple Ex. of BW stability ✓
- Numerical Stability of Sherman-Morrison
- FW/BW Stability + Condition #. ✓

- If A is not sym, how to compute f(A)? ✓
- Does BW stable LU mean small perturbation to A doesn't change results and we have the exact soln? ✓

BW

- You solve the right problem

⊛ Algs ⊕
How it's implemented Matters!

FW Error

- Small Error or a bound on Error vs. input perturbation.

⊕ Math ⊕

+ perturbation

w/ Mathematically Exact function

- fund. Sensitivi

(but not always)

$$f(x) = \sqrt{x}, \quad \boxed{0.5 \le |x| \le 1.5}$$

FW Error is Sensitv

$$\frac{\frac{f(x+\delta x)}{f(x)}}{\frac{\delta x}{x}} \Rightarrow \frac{f(x+\delta x)\, x}{\delta x \; f(x)}$$

$$f'(x) = \frac{1}{2} x^{-1/2}$$

$\delta x \to 0$

$\Rightarrow \quad \frac{f'(x)\cdot x}{f(x)}$

$\frac{1}{2}\frac{1}{\sqrt{0.5}} \quad \cdots \quad \frac{1}{2}\frac{1}{\sqrt{2}}$

$\uparrow$

$\simeq 0.6 \qquad\qquad 0.37$

Alg:

```
for i=1 to 60
    x ⇐ √x
for i=1 to 59
    x ⇐ x^2
```

If

Then

ALG

ALG(

ALG(x) is a
BW stable ~~Alg~~ Algorithm of f(x).

$(x) \cong f(x+\delta_x)$

$x) - f(x)$ ⇒ Is a condition problem.

$f(x+\delta_x) - f(x)$.

Practical Answer

- "LU is Exact on Computer" Ⓘ
(Ignore Caveats)

Theory / Math answer

- "LU on Systems w/o bad growth factors is as accurate as it can be".

$f(x,y) = x+y$

ALG: $x \oplus y = (x+y)(1+\varepsilon)$

$= x(1+\varepsilon) + y(1+\varepsilon)$

$= f(x(1+\varepsilon), y(1+\varepsilon))$

$= f(x+\varepsilon x, y+\varepsilon y)$

$\delta x, \delta y.$

If A is

but sh

$f(A)$

$f(x) \Rightarrow$

then Comp

non - sym,

l diag, then

$= X f(\Lambda) X^{-1}$

$-$ approx (x)
(polynom, ratl. fun)

$f(A)$

$\to$ approx $(x) = p(x)$

$$= \sum_{i=0}^{K} \alpha_i T_i(x)$$

$T_i(x) =$ ith chebychev poly.

"$T_i(x) = x^i \Rightarrow A^i$"

$$= \sum_{i=0}^{K} \alpha_i \boxed{T_i(A)}$$

$$\frac{p(x)}{(q(x))^{-1}} = p(x)(q(x))^{-1}$$

$$\boxed{p(A)}\,\boxed{q(A)^{-1}}$$

Exam

$$f(x) = \sqrt{x}$$

$$y^2 - x = 0 \iff$$

$$y = \sqrt{x}$$

View as non-lin sys of Eqs,

Then use Newton's!

$$f(y) = y^2 - x$$

$$J(y) = 2y$$

$$f(y) \approx f(y_0) + J(y_0)(y - y_0) + \dots$$

$$f(y_0) + J(y_0)(y - y_0) = 0$$

$$y = y_0 - J(y_0)^{-1} f(y_0)$$

$$y^{(k+1)} = \underbrace{y^{(k)} - \boxed{\tfrac{1}{2}(y^{(k)})^T(y^{(k)})}}$$

$$A = \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 \\ -1 & 1 & \ddots & \vdots & 1 \\ \vdots & & \ddots & 0 & 1 \\ & & \ddots & & 1 \\ -1 & \cdots & \cdots & -1 & 1 \end{bmatrix}$$

$$A = 2I - \overset{L}{T} + \begin{bmatrix} - \\ -1 \end{bmatrix}$$

$$\|A\| \cdot \|A^{-}$$

$$f = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \end{bmatrix} \leftarrow$$

$$\begin{bmatrix} \ddots & \\ & -1 \end{bmatrix}$$

$$\underline{e_n^T f}$$

ll is !

$$A^T A =$$