Lecture 21

- If A is SPD do power subspace method do better than steepest descent? ✓
- Why do we need a poly expansion to prove Lemma 1

  $x^{(k)} = \sum_{j=0}^{k} c_j A^j \underline{b}$

  $(I-A)^k b = \text{poly}(A) \underline{b}$ ✓

- $A\underline{x} = \lambda Bx$ when B is not SPD? ✓
- How does B impact sensitivity? ✓
- Widespread uses for a adj matrix? ✓
- When does

  $\mathbb{K}_k(A, \underline{b})$

  $\mathbb{P}_k(A, b)$?

  form a complete space". Aren't those lin. dep.?

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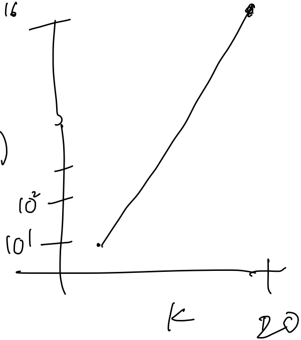
KIR

$B = [b, Ab, \dots, A^k b]$

Can we measure goodness w/ condition #?

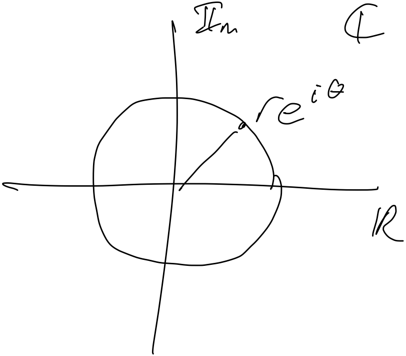
polar decomp! ✓

$A = [b, Ab, \dots, A^k b]$



20/60.

Polar Decomp.



Polar form:

- magnitude r
- angle θ

Polar Decomp

$$A = Q\,S$$

Orthog

" G "

Sym. pos. (semi) def.

(Suppose A is Square)

$$A = U\,\boxed{\Sigma V^T} \quad \text{SVD.}$$

$$= \underbrace{U V^T}_{Q}\;\underbrace{V \Sigma V^T}_{S}$$

$$x^T S x \geq 0$$

$Ax = \lambda Bx$ when B is not SPD.

QZ alg. to solve these.

→ pretty Esoteric.

Sensitivity to B?

→ Very sensitive! Big Impact!

→ Cond #'s of λ-vals.

① Pete Stewart is King of λ-val sensitivity!

Yes! they really take fewer iterations than steepest descent!

Why?

You can show

$$x_k = \text{Steepest descent}(A, \underline{b}, \epsilon)$$
$$\in \mathcal{K}_k(A, \underline{b})$$
$$= \text{span}(b, Ab, \ldots, A^{k-1}\underline{b})$$

$$x_k = \prod_{i=1}^{k} (I - \alpha_i A)\underline{b} = (I$$

$\alpha_i \rightarrow$ Step leng

In MINRES, we

$$\min \; \| b - Ax$$
$$\text{s.t.} \quad x^{(k)} \in$$

~~Need~~ to Need to inverse those

$-\alpha_1 A)\,(I-\alpha_2 A) \cdots (I-\alpha_k A)\,\underline{b}$

r.

compute

$(x)\|_2$

$\mathcal{K}_k(A,b)$.

~~$b, Ab, \ldots A^k \underline{b}$~~

~~is not linearly dep.~~

~~unless~~

~~$x = A^{-1} b$~~

if $b, Ab, \ldots A^k \underline{b}$

is linearly depend, then

you have an invariant

subspace of A.

$$\Rightarrow \; = A\overbrace{X}^{n\times k} = \overbrace{X}^{n\times k}\overbrace{Y}^{k\times k} \qquad k < n.$$

basis for subspace

coeffs in sub

let $S = \text{span}(b, Ab, \ldots, A^k b)$
then $Ax \in S$
for all $x \in S$.

~~Key fact for~~
Key fact for us:
for solving $Ax = b$

$\rightarrow$ if $b \in S$, then
$x \in S$.

$$\frac{A^{10000}\underline{b}}{\|A^{10000}\underline{b}\|_2} \text{ vs. } \frac{A^{10001}\underline{b}}{\|A^{10001}\underline{b}\|_2}$$

In Exact arithmetic, these are lin. indep.

But.....