Lecture 22

- Arnoldi vs. Gram-Schmidt? ✓
- for LDA, what are we trying to min/max ✓
- find minimum k s.t. $A^k b \in \mathcal{K}_k(A,b)$ w/o checking k, ✓ for a sing. matrix?
- use alt. basis for $\mathcal{K}_k(A,b)$ (Chebyshev/etc.) ✓
- If Arnoldi motivation is to get orthog. base, what about QR etc. ✓

- Why is CG slow when A is ill-conditioned? ✓
- Are Krylov/power subspace methods accurate if k is small? ✓

LDA max sep.

min. w/in class deviation.

$$\max \frac{x^TAx}{x^TBx} \begin{matrix} \leftarrow \text{sep.} \\ \leftarrow \text{w/in class} \end{matrix}$$

$$\min \frac{x^TBx}{x^TAx}$$

$$\mathcal{K}_k(A,b) = \text{span}(\underline{b}, A\underline{b}, \ldots, A^k \underline{b})$$

Arnoldi$(A,b,k) = Q_k, H_k$

where Q_k spans $\mathcal{K}_k(A,b)$ and $Q_k^T Q_{k} = I$.

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if A has few distinct λ-vals, then matrix power subspace method converge fast.

⊛

$$\hat{A} \quad \underline{x}$$

$$\begin{bmatrix} I & A & 0 \\ & I & B \\ & & I \end{bmatrix}$$

$$A^{B} b =$$

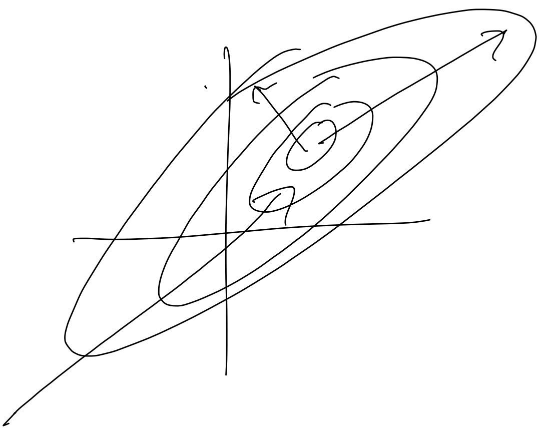
$$\begin{bmatrix} A & B \\ B^T & 0 \end{bmatrix}$$

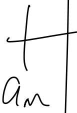
$= b$

$x = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

lin. comb of
$A^2 b, A^1 b, A^0 b \ldots$

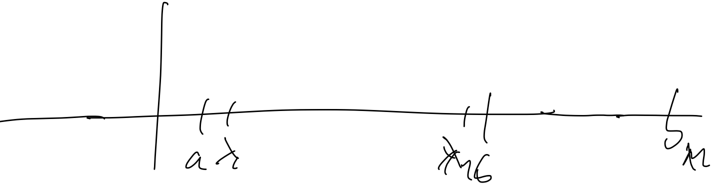
$\longrightarrow$





Why not Chebyshev polys vs. Monomials ?

To construct Chebyshev polys, you need bounds on the spectrum of A.



st $a \le \lambda_1 \le \cdots \le \lambda_n \le \boxed{b}.$

a, b -> Chebyshev polys =>

Q_k

an updat

q_r ([Q

$i \; R_k = qr([b, Ab, -A^k b])$

to Q_{k+1} by adding

$A^{k+1} b$

$k \; A^{k+1} \underline{b}))$