Lecture 23

- Lanczos w/ Large Matrix vs. Arnoldi
  Yes! with Caveats.
- Krylov/power subspace good for Sparse Systems? Yes!
- V_k spans $\{b, \ldots, A^k b\}$
  But up to some limit? ✓
  Ex. $A = I$

In CG

$k = 8$ to $k = 10$

the error goes to zero! ✓

Why? How?

Reduce Round-off Error in Lanczos?

Why $\| \|b\| e_1 - T_{k+1} z \|$

using CG vs. Least Squares? ✓

To Compute

Lanczos (A, b, k)

We need ...

Size (A) → implicit in b.

$A \cdot \underline{v}$ → Matvec!

$$\underset{\substack{\downarrow \\ n \times k \\ \text{mem}}}{V_k}, \underset{\substack{\downarrow \\ k+1 \times k \\ \text{memory.}}}{T_k} = \text{Lanczos}(\sim, k)$$

$$A = \underset{\uparrow}{S} + \beta \underline{e}\,\underline{e}^T$$

$$\underset{\uparrow}{Sx} + \beta \underline{e} \underset{\uparrow}{(\underline{e}^T \underline{x})}$$

Why is T tridiagonal?

See ~~my~~ the lecture (same about polynomials + this!

It's the polynomials (:)

Max Dim of

$\operatorname{span}([b, Ab, \ldots, A^k b])$

is less than n.

~~Is it~~

It can be!

How does this relate to

V_k from Lanczos?

→ if

than

if A

You

still

$\text{Span}(\ldots) < n,$

$V_{k+1} = [V_k \;\; 0]$

$= I,$ and

run Lanczos on it,

stop after 1 iteration!

$$\left[\begin{array}{ccc|c} \alpha_1 & \beta_2 & & \\ \beta_2 & \alpha_2 & \beta_3 & \\ & \beta_3 & \alpha_3 & \\ \hline & & \beta_4 & \\ & & & \end{array}\right.$$

In any Matrix method

$\min \| b - A$

st $x \in K_k(A, b$

$\downarrow$

$\min \| \|b\| e_1 - T_k y$

st. $x = V_k y_k$

$\Downarrow$

Least squares

MINRES

power subspa

$x\|_2 \longrightarrow \underline{x}_k$

)

if A is sym. pos. def.

$$\min \frac{1}{2}\underline{x}^T A \underline{x} - \underline{x}^T \underline{b}$$

$$\Big\downarrow$$

$y_k\|_2$

$$\min \frac{1}{2} y_k^T T_k y_k - y_k^T \|b\| \underline{e}_1$$

$$\Downarrow$$

$$T_k y_k = \|b\| \underline{e}_1$$

CG

CG:

1 matvec

X norms

y axpys

2 dot prods

axpy $\Leftrightarrow$

$\underline{z} \leftarrow \alpha \underline{x} + \underline{y}$

norm $\Leftrightarrow$

$\gamma \leftarrow \|\underline{x}\|$

dot $\Leftrightarrow$

$\theta \leftarrow \underline{x}^T \underline{y}$

MINRES

1 matvec

x' norms

y' axpys

z' dot prods

$x' > x$

$y' > y$ (but

$z' > z$ not by much)