Lecture 26

- Use Multiple preconditioners? Jacobi + Cholesky? ✓
- How to determine the best preconditioner for a given type of matrix ✓
- Compare preconditioners to direct methods? Which is more efficient? ✓
- Can rank-K update be used as a preconditioner ✓
- How do you know if a problem needs a preconditioner?
  - ↳ Which one to use? ✓
- Common choices for K in incomplete Cholesky? ✓
- Can we use $\lambda(M^{-1}A)$ to construct M^{-1}? ✓
- Can we do Sparse Approx. Inverse w/ Block sparse matrices? ✓

→ SPAI ⟺

$$\min \|AX - I\|_F^2$$

s.t. X has known sparsity.

⇒ Solve for X column-by-column using least squares.

if Each column of X only has a few non-zeros.

⇒ These don't seem to work great !! $\boxed{\text{⊕ in Marc's study ⊕}}$

You should ALWAYS do Jacobi precond!

Consider other ideas too!

↳ Incomplete Cholesky.

⊕ Sometimes Multigrid ⊕

Incomplete Cholesky?

$M^{-1}A X - I$

↑

SPAI?

looks very diff than A!, you don't compute $M^{-1}A$!

If you known λ_i, v_i for extremal λ-vals

E.g.

$\lambda_1 \ldots \lambda_k, \lambda_{n-k} \ldots \lambda_n$

$v_1 \ldots v_k, v_{n-k} \ldots v_n$

Then you could try an preconditioner those away... it's like a deflation mat

$(I - 2VV^T)$ for some orthog. mat. V.

to these

deflate out.

Can Rank K-update be used as a preconditioner.?

Yes, if $A = \overset{\checkmark}{\text{sparse}} + \overset{\checkmark}{\text{rank}}\text{-}k$ update.

$A \to \tilde{A}$ by delete row/col

Then $\tilde{A}$ = Schur complement of

$$\begin{bmatrix} A & \underline{V}^T \\ \underline{U}^T & \underline{0} \end{bmatrix}$$

- Chris Maes PhD Thesis
- Alex Pothen's Studies on Computation Surgery.