Lecture 27

- Multigrid w/ non-uniform grid/mesh spacing. (GS vs. Jacobi) ✓
- Does A need to be diag. dom. for Gershgorin ✓
- Why do Gershgorin disks intersect? ✓
- Matrix form of Multigrid like Incomplete Cholesky? ✓

- QR for non-sym? ✓
- Preconditioners for λ-vecs? $Ax = \lambda x$. ✓
- Use Ax operator only to get eigenvalue bounds from Lanczos. ✓
- For multigrid, do we get matrix values from the bigger matrix? or the original formulation ✓

- How does po
behave if
more than
vector in
(i.e. geom
>1.)

$A = \{$

wer method
there is
one dominan
the ~~s~~ subspace
for multilevels

B
1 3
1

$$\underbrace{XAX^{-1}}$$

Simi. transform preserves λ-vals. but changes λ-vecs.

- Since convergence depends on λ-vals, this won't really help.
- if angles between subspaces than it could, but that's more complicated
- SCALING! does help... ✓

Multigrid for non-uniform

- Use Algebraic Multigrid.

Also, think about adaption yourself!

Where do smaller probs come from?

AMG → matrix.

geometric
GMG → problem

care

Getting a matrix from Multigrid! ?

Yes! But it isn't a nice matrix.

You can compute i.e.

given $f(\underline{x}) = A\underline{y}$

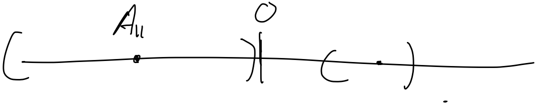
we can compute A by

$f(\underline{e}_1)$, $f(\underline{e}_2)$ ..., $f(\underline{e}_n)$

$= \underline{a}_1 \quad = \underline{a}_2 \quad \ldots \quad = \underline{a}_n$

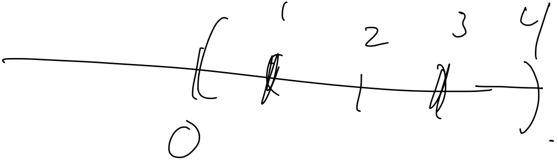
If A is strictly diag. dom.

Then $|A_{ii}| > \sum_{j \neq i} |A_{ij}|$ for all i.



If $|A_{ii}| \geq \sum_{j \neq i} |A_{ij}|$?

$$\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix}.$$



QR for non-sym

$A \Rightarrow Q_1 R_1$

$A^{(1)} \Leftarrow R_1 Q_1$

$A^{(1)} \rightarrow Q_2 R_2$

$A^{(2)} \leftarrow R_2 Q_2$

$Q_2^T Q_1^T A Q_1 Q_2$

QR is "wrong"
λ-vals.. Need

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)

?

Subspace methods

for non-sym.

other ideas.

If $A \neq A^T$, then there exist Q ortho s.t.

$QAQ^T = R$ where

R is upper tri.

(Schur Decomp.)

If $A = A^T$, then

$QAQ^T = R = \text{Diag}$.

Suppose you have $f(x)$ to compute Ax

But not A.

If A is sym. Run Lanczos!

If A is non-sym. Run Arnoldi.

idea1: Run Both, see what happens!

idea2:

x^TAy vs. y^TAx

$= y^TA^Tx$.

→ Est. λ-vecs for T

→ Est. λ-vecs for H_k.

Experiments

- geometric multiplicity
- QR on non-sym
- testing for symmetry.