Lecture 29

⊕ Reminder:

Final in < 1 week! ⊕

Why do subspace iteration / QR converge to λ-vals/vecs? ✓

Can we think of Reduction to tri/bi diag forms as a type of preconditioner? ✓

Can we use this to ~~aff~~ help $[K(A^TA)]$? ✓

- Does a bipartite matrix have pos. λ-vals? ✓

- Lanczos method to bring σ-vals. ✓

- When A has small condition #, is A^TA better than $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$? ✓

- Why not bidiag for λ-vals? ✓

Given A,
we can find
U, V s.t.
$U^T A V$ = Bidiag.
$U^T U = I$, $V^T V = I$.

Why not for λ-vals?

We need sym. transform
$X^{-1} A X$ or
$Q^T A Q$ for λ-vals.

if you have a matrix
$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$ then λ-vals
come in ± pairs!

$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \lambda \begin{bmatrix} u \\ v \end{bmatrix}$$

Then
$$\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} u \\ -v \end{bmatrix} = \begin{bmatrix} -A\underline{v} \\ A^T\underline{u} \end{bmatrix} = \begin{bmatrix} -\lambda u \\ \lambda v \end{bmatrix}$$

Why does Subspace / QR Converge?

$$\frac{A^k v}{\|A^k v\|} \Rightarrow q$$

Some λ-vec.

Same q.

$$qr(A^k I) \Rightarrow \underbrace{\begin{bmatrix} q & q_2 & \ldots \end{bmatrix}}$$

Not necessarily ordered this way!

Would this converge? When σ st

st. $\frac{A^k v - \sigma q}{\|A^k v - \sigma q\|}$

$$= -\lambda \begin{bmatrix} u \\ -v \end{bmatrix}$$

. $q^T(A^k \underline{v} - \sigma q) = 0$

$\longrightarrow \Longrightarrow q_2$

looking for zeros of $f(\lambda) = \det(A - \lambda I) \longleftarrow$ 1 Eq.

$$\boxed{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}}$$

n - Vars, n Eq.

$$\boxed{\begin{array}{c} Ax = \lambda x \\ x^T x = 1 \end{array}} \Rightarrow \boxed{\begin{array}{c} PAx = \lambda x \\ x^T x = 1 \end{array}}$$

$A^T A$

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less memory ✓

Much faster if $A^T A$ is small.

faster if 1m-by-1000?

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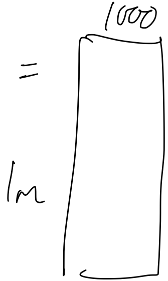
vs. $\begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$

$A =$



$QR = A$

SVD of R to get

$$\boxed{Q\, U_R}\, \Sigma_R V_R^T = A$$

Lanczos to find σ-vals.

Input A

form $B = \begin{bmatrix} 0 & A \\ A^T & 0 \end{bmatrix}$

Run Lanczos $\left(B, \begin{bmatrix} 0 \\ \underline{v} \end{bmatrix}\right)$

to get V_k, T_k,

Compute λ vals of $(\overline{T}_k)$

$\rightarrow \begin{bmatrix} 0 \\ \underline{v} \end{bmatrix}, \left\{\begin{matrix} A \\ 0 \end{matrix}\right.$

... lots

You get the

$A \rightarrow$

Whe

$\begin{bmatrix} \underline{v} \end{bmatrix}, \begin{bmatrix} 0 \\ A^TA\underline{v} \end{bmatrix}, \begin{bmatrix} AA^TA\underline{v} \\ 0 \end{bmatrix}$

d thimbu /algebra

bipartite Lanczos process..

U, V, B_k

B_k is bidiag.

$AV_k = U_{k+1} B_{k+1,k}$

$\begin{bmatrix} \alpha_1 & & \\ \beta_1 & \ddots & \\ & \ddots & \alpha_n \\ & & \beta_n \end{bmatrix}$ -