# Lecture 3

Sparse

- Optimization w/ Sparse Matrices
- How sparse data works in Julia
  CSC vs. CSR.
- Circulant vs. Hankel vs. Toeplitz (typo on Page 25?)

Types

- Eigenvalue propagation
  Hankel / Toeplitz cfs

Expected Values.

Why is Expected Value

$$X_{\{-3\}} = 1 + \frac{1}{2} X_{\{-2\}} + \frac{1}{2} X_{\{-4\}}$$

Why is 1 here?

- Scopes in Julia?
  local? global?
  for?

- When is it worthwhile to use sparse matrices? vs. dense matrix in Julia

- Example of Toeplitz + Hankel ✓

→ How to use Permutation Matrix ✓

- Defn of Circulant Matrix? ✓

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- Candyland

↳ $p_2 = T t_{140}$

Should this be

$p_2 = T(i,k) t_{140}(k)$

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Circulant Matrices

$$n\left[\overbrace{\begin{array}{ccccc} a_0 & a_1 & \cdots & & a_{n-1} \\ a_{n-1} & a_0 & a_1 & \cdots & a_{n-2} \\ a_{n-2} & a_{n-1} & a_0 & a_1 & \cdots \\ \vdots & & & & \\ a_1 & a_2 & \cdots & a_{n-1} & a_0 \end{array}}^{n}\right]$$

Constant diagonal

Wrap around Entries

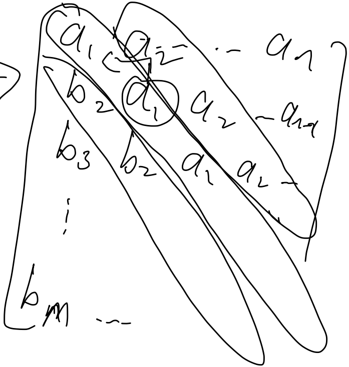
Add mod term on index ←

Toeplitz

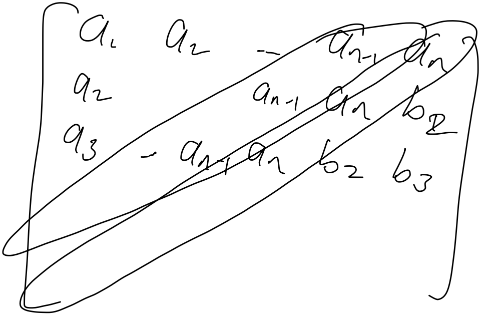
Shift right.

Hankel.

shift left



$$\begin{bmatrix} a_1 & a_2 & \cdots & \cdots & a_n \\ b_2 & a_1 & a_2 & \cdots & a_{n-1} \\ b_3 & b_2 & a_1 & a_2 & \cdots \\ \vdots & & & & \\ b_m & \cdots & & & \end{bmatrix}$$



$$\begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \\ a_2 & & a_{n-1} & a_n & b_2 \\ a_3 & \cdots & a_{n-1} & a_n & b_2 & b_3 \end{bmatrix}$$

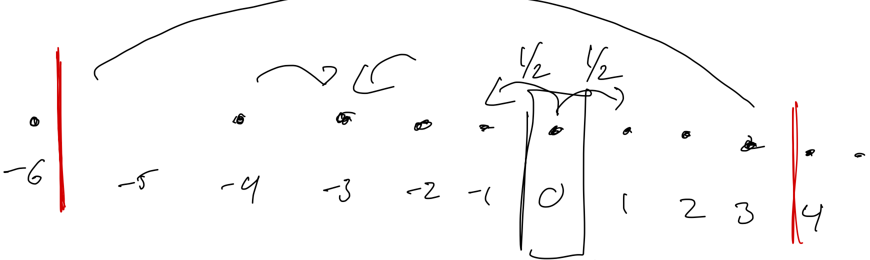
Const diagon

Constant reverse diag.

$A_{ij} = A_{i-1,j-1}$ or

⊛ Starting cases.

$A_{ij} = A_{i-1,j+1}$



Idea: Let $X_{[i]}$ = Expected ~~Length~~ Steps I'll last starting at state $i$.

$$X_{[-6]} = 0$$

$$X_{[4]} = 0$$

$$X_{[-5]} = 1 + \frac{1}{2} X_{[-6]} + \frac{1}{2} X_{[-4]}$$

$$\vdots$$

$$X_{[i]} = 1 + \frac{1}{2} X_{[i-1]} + \frac{1}{2} X_{[i+1]}$$

Permutation Matrices

→ have a single 1 in each row/col.

$$\underset{y}{\begin{bmatrix} x_2 \\ x_3 \\ x_1 \end{bmatrix}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$p = [2 \quad 3 \quad 1]$

$y = x[p]$ in Julia.

Optimization w/ Sparse Matrices + when to use sparse dense?

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