Lecture 5

⊕ Not around on Thursday!

- Pf of Neumann Series ✓

  Convergence (MM+P → ?)

  → typos? left/right mult?

  What is $\rho(I-A)$?

- Sparse matrix w/ most non-zero on the diag? ✓

- CSC makes it look like row order is the fast variable

  → is this right? ✓

Why not CSR vs. CSC? ✓

Why is random access common? (Eg. A[5,6]) ✓

What is a matrix is ambiguous! "makes sense" isn't precise. ✓

Pf of Neumann series ✓

Convergence

CSC vs. CSR in Julia. ✓

Why not store Sparse matrices like Matlab

Where do ~~Toeplitz~~ right shift matrices come up?

→ Take a class on signals...
(★ time lagged signal)

CSC vs. CSR.

★ Why not like Matlab?
I don't know!
Both Matlab/Julia use CSC!

CSC → CSR in Julia?

A → A' (fake transpose/adjoint) ★ this is lazy!

Copy(A') → Convert to CSC of A^T.

copy(A')' → CSR
Representation of A.

⊛ Use judiciously!

CSC stores start of each column, not each row, so we get fast access to random column. (not row.).

→

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Why random access to Sparse Matrices?

$T \, t_{140}$ ↙ a single one in Entry 140

$= T[:, 140]$

Sparse matrix · Sparse vector.

diagonal Entries.

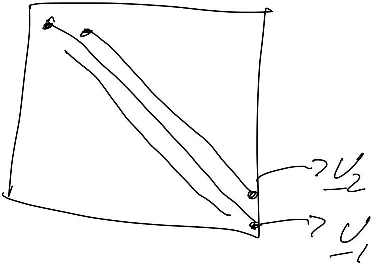
$$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \; e_i^T (T e_j).$$

Diagonal Matrices!

→ Just

→ Use Diag. / Tri Diag / K-Diag or

those plus an Indexed Matrix.



Diagonal. Does this work w/ sparse vector?

Pf of Neumann series.

Goal: show $A: n \times n$

$$\left(\sum_{k=0}^{\infty} A^{k}\right)(I-A) = I$$

or $$(I-A)\left(\sum_{k=0}^{\infty} A^{k}\right) = I$$

Why: for square matrices, left/right inverses are the same!

I

$(I$

I

$-A$

$S_\ell =$

S_ℓ

But

$S_\ell (I$

$$\overbrace{+A+A^2+A^3+\ )}^{\text{Converge.}}(I-A) =$$

$$+A+A^2+A^3+\ldots$$

$$-A^2-A^3-A^4-\ -\ldots \quad = \quad I$$

$$\sum_{k=0}^{\ell} A^k \quad \oplus \quad \text{Always true!}$$

$$(I-A) = I - A^{\ell+1}$$

if $A^{\ell} \xrightarrow{\ell\to\infty} 0$, than

$$(I-A) \longrightarrow I.$$

So conclusion

If $A^{\ell} \to 0$

than $\sum_{k=0}^{\infty} A^{\ell}$

$\lambda(A)$ is λ any Eigenvalue.

-/ theorem $\max_{\lambda} |\lambda(A)|$

$\iff \rho(A) < 1$

$= (I - A)^{-1}$