Lecture 8

- What does sub-mult mean for a matrix norm alone? vs. a matrix-norm / vector norm pair? ✓ ✓
- Example where Coord-descent is faster than Steepest descent? ✓
- How to compute a bound on α for Richardson. ✓
- Local min / Global min of convex funct. ✓

Sub-mult:

Idea:

$\|Ax\| \le$

f: vector norm

g: matrix norm

h: vector norm

$f(A\underline{x}) \le g(A)\, h$

$\|A\| \cdot \|x\|$

$\mathbb{R}^m \to \mathbb{R}$

$\mathbb{R}^{m \times n} \to \mathbb{R}$

$\mathbb{R}^n \to \mathbb{R}$

$C[\underline{x}]$

In fact,
Most matrix norms
are "well-defined"
on a $m \times 1$ matrix.

$$\|Ax\|_2 \le \underset{\text{2-norm.}}{\|A\|} \cdot \|x\|_2$$

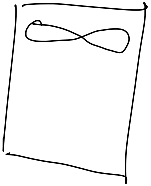
Matrix

What is

$\|X\|_\infty$?

⇒ largest elem

If ∞ is matrix-norm then



vector

∞-norm ($\underline{X}$) = largest elem too.

2-norm:

$\|\underline{X}\|_2$ = largest sing. val
as matrix-norm

$\|\underline{X}\|_2 = \sqrt{(\sum X_{ij}^2)}$ as vector norm.

Richardson: What α to use?

- Sym?
- Non-Sym?

$\rho(I-\alpha A) < 1$

and A is non-sym.

Use a matrix-norm;

$\rho(A) \le \|A\|$ for any induced matrix norm.

$\|I-\alpha A\|_1$ an

find α s.t. $\|I-\alpha A\|_\infty < 1$.

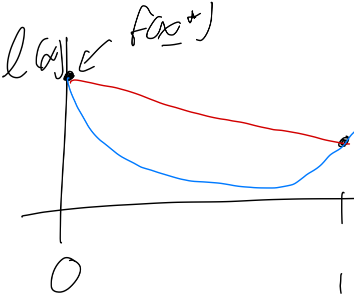
Local mins + Global mins of Convex function

Theorem: if $\underline{x}^*$ is a local min of a Convex function $f \; \mathbb{R}^n \to \mathbb{R}$, then $\underline{x}^*$ is also a global min!

Pf: By Contradiction

$\underline{y}^*$ is a global min that is not ~~a loca~~ x^* min.

look at $f((1-\alpha)\underline{x}^*$



~~(i.e. not x^*)~~

$+\alpha \underline{y}^*) = \ell(\alpha)$

$f(y^*)$

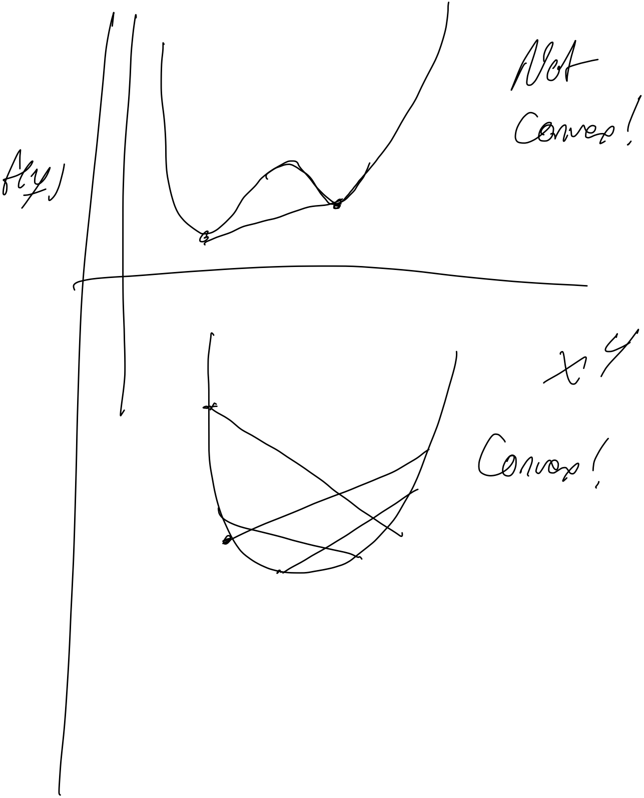
α

$f(\underline{x})$ is convex if

$f(\alpha \underline{x} + (1-\alpha)\underline{y}) \le \alpha f(\underline{x}) + (1-\alpha)$

for any 2 points $\underline{x}, \underline{y}$ in the domain of f.

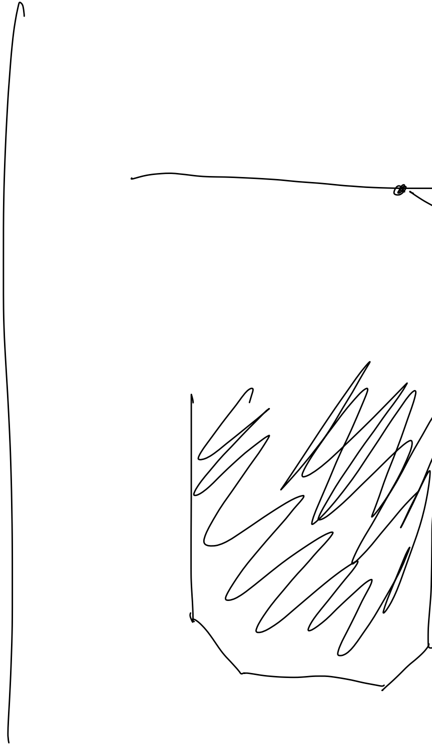
(and the domain of f must be a convex set.)

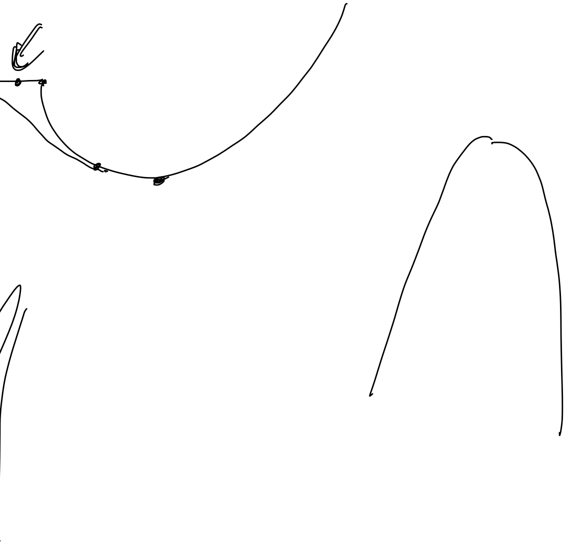


Not Convex!

x^4

Convex!





$H(x) \geq 0$. Convex.

$H(x) \leq 0$. Concave

$$\sum_{k=0}^{\infty} A^k = \cancel{A^{-1}} (I-A)^{-1}$$

if $\rho(A) < 1$

for sym/nonsym.

$A = X J X^{-1}$ is Jordan form.

$\sum A^k$

$J_i^k =$

$$\begin{bmatrix} 1 & 1 \\ \varepsilon & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A = X \begin{bmatrix} J_1 & & \\ & J_2 & \\ & & \ddots \; J_k \end{bmatrix}$$

$$J_i = \begin{bmatrix} \lambda_i & 1 & 0 \\ & \ddots & 1 \\ 0 & & \lambda_i \end{bmatrix}$$