Lecture 9

How to find a permutation s.t. Jacobi will converge? ✓

Why -= not = In Jacobi?

Why does Jacobi diverge in the video? ✓

Jacobi typo? ✓

Why does it converge?

When to use GS vs. Jacobi? ✓

Is there a performance opt. for Jacobi / GS when coding them? ✓

Given $\rho(A)$ can we deduce $\rho(M_M^{-1} N_M)$? ✓

[How to pick α in Richardson? ✓

↳ will some α be faster?

→ See past In class videos.

$\rho(A) \le \|A\|$.

Convergence / Divergence of Jacobi / GS

$\rho(D_M^{-1} N_M) < 1$

$\Rightarrow$ converge

$\rho(D_M^{-1} N_M) > 1$

$\Rightarrow$ divergence ⊛

(for at least some right hand side.)

$$(D_M^{-1} N_M)^K \underline{e}_0 = \underline{e}_K$$

Then

if e_0 is in an invariant subspace w/ bounded spectral radius, it will converge.

Suppose $\underline{e}_0$ was an eigenvector of $D_M^{-1} N_M$ w/ eigenvalue λ

Theorem:

Assumes diagonalizable.

Let $\underline{I}$ be a subset of indices corresponding to eigenvectors of $D_M^{-1} N_M$ s.t.

$$\underline{e}_0 = \underline{x}_0 - \underline{x} = \sum_{i \in I} c_i \underline{v}_i$$

where $\underline{v}_i$ is an eigenvector of $D_M^{-1} N_M$.

Then if $\max_{i \in I} |\lambda_i| < 1$,

Jacobi (GS) will converge.

Proof,

look at $c_i^{(k)}$ and show it goes to 0.

$$\underline{e}_k = \underline{x}_k - \underline{x} = \sum_{i \in I} c_i \lambda_i^k v_i.$$

Given $\rho(A)$ can we deduce

$\rho(D_m^{-1} N_m)$ for Jacobi?

Not in general.

Maybe some bounds.

Yes if $D_m^{-1} = I$.

If $L = D - A$ is a Laplacian matrix. T

$A = A^T$, $A_{ij} \geq 0$,

A is adj. matrix of a graph. $A_{ii} = 0$.

D = diag. deg matrix

$D_{ii} = \sum_j A_{ij}$.

$\sum_j L_{ij} = 0$.

L

De

$\rho(L)$

$\rho(D^{-1}L$

$\underline{e} = O(\underline{e})$

$= A\underline{e}.$

$\le 2 \cdot d_{max}.$

$) \le 2.$

How to find P s.t.
Jacobi will Converge?

I don't know ☹.

You can always find
a permutation s.t.
you can use
Jacobi by matching.

Let A be non-sing.
Then there is a permutation P
s.t. $A_{ij} \neq 0$ for
the entries $P_{ij} \neq 0$.

(look @ $\det(A)$)!

GS and Jacobi

$A = M - N$ C

Jacobi: $M = D$

$N = D - A$

Gauss S: $M =$ Lower triangular + Diag

$N = M - A$ (upper-triangular)

Eigenvalues of $M^{-1}N$ control convergence.

$M^{-1}N$ for GS

is "complicated".

See HogWild methods

for analysis of

"parallel" GS.