Date + Time for Exam:

- Tomorrow (12/12) @ 8am
- Location: ~~Normal Classroom.~~
  Hampton 2123 (Normal Class room).

- SVD of Rank 1
- BW Stability of
- Calculate coeff of

  $A^{k+2}\underline{b}$

  when $A^{k+1}\underline{b} \in$
- If $A^{k}\underline{b} \in$ Span
  then all future powe
  ↳ How do we know
- LU BW stable i
  coffee shop.

Matrix ✓

the SVD ✓

} Lect 22 a/b

$(K_k(A,b))$ ✓

$(b, Ab, \ldots A^{k-1}b)$

n are too. ⊛

this is true?

n a

- fast way to get analytic SVDs ✓
- BW stability of sin(x)

$$A = \underline{u}\,\underline{v}^T$$

$$\begin{array}{cccc} & U & \Sigma & V^T \\ & \| & \| & \| \\ A = & \text{house}(\underline{u}) & \begin{bmatrix} \rho \\ 0 \\ \vdots \\ 0 \end{bmatrix} & \text{house}(\underline{v})^T \end{array}$$

$$\rho = \|\underline{u}\| \cdot \|\underline{v}\|.$$

house($\underline{x}$) = householder matrix to take $\underline{x} \to \|x\| \underline{e}_1$

R-SVD:

if A is very tall, (10^7) (10) $m \gg n$
then it looks "like" a vector:

$$QR(A) = \overset{m\times m}{Q}\ \overset{m\times n}{R}$$

R 10×10
upper-tri matrix.

$$SVD(A) = Q \cdot U_R \Sigma_R V_R^T$$

$$U_R, \Sigma_R V_R = svd(R)$$

$$\boxed{A^T A} = u v^T v u^T$$

$$= \underbrace{(v^T v)}_{K} \underline{u}\underline{u}^T$$

$$\|v\|^2 \cdot \boxed{u u^T}$$

$$\|v\|^2 \left(\|u\|^2 \cdot \frac{u}{\|u\|} \cdot \frac{u^T}{\|u\|} \right):$$

BW stability of the SVD.

↳ $\underline{U}^T\underline{U} \neq \underline{D}$ in FP, so U is not a FP orthogon matrix.

- $U^TU \neq I$ in FP, but $\hat{U}$ for ALG, U^* true soln, is small.

$(K_k(A$

If $A^k\underline{b} \in \text{span}(\underline{b}, A\underline{b}, \dots$

then $A^k\underline{b} = [\underline{b}, A\underline{b}, \dots A$

for some $\underline{c}$ ⟺

iff $A^k\underline{b} = \sum_{i=1}^{k} \boxed{c_i A^{i-1}\underline{b}}$

$$A^{k+1}\underline{b} = \sum_{i=1}^{k} c_i A \cdot A^{i-1}\underline{b} = \sum_{i=1}^{k} c$$

$$= A \cdot A^k\underline{b} =$$

$$= c_1 A\underline{b} + c_2 A^2\underline{b} + \dots$$

$$= c_1 c_k \underline{b} + (c_1 + c_2 c_k) A\underline{b} +$$

$, b)$

$A^{k-1} b]$

$^{k-1} b] \, \underline{c}$

then

$c_i A^i \underline{b}$

$c_k (c_1 \underline{b} + c_2 A \underline{b} + c_3 A^2 \underline{b} + \dots c_k A^{k-1} \underline{b}),$

"

$c_{k-1} A^{k-1} \underline{b} + c_k A^k \underline{b}$

$(c_2 + c_3 c_k) A^2 \underline{b} + \dots (c_{k-1} + c_k^2) A^{k-1} \underline{b}$

$$A^{k+2} \underline{b} = A \cdot A^{k+1} \underline{b}$$

$$= A \left(c_1 c_k \underline{b} + (c_1 + c_2 c_k) A \underline{b} + (c_2 + c_3 c_k) A^2 \underline{b} + \cdots (c_{k-1} + c_k^2) A^{k-1} \underline{b} \right)$$

$$= c_1 c_k A \underline{b} + (c_1 + c_2 c_k) A^2 \underline{b} + (c_2 + c_3 c_k) A^2 \underline{b} + \underbrace{(c_{k-1} + c_k^2)}_{= \rho} \boxed{A^k \underline{b}}$$

$$= \rho c_1 \underline{b} + (\rho c_2 + c_1 c_k) A \underline{b} +$$

$$+ (\rho c_3 + c_2 c_k) A^2 \underline{b} +$$

BW Stable Implement of

$\sin(x) \Rightarrow$

$-1 \le \sin(x) \le 1$ for all x.

Why?

BW Stable $\Rightarrow$

$$\hat{y} = \underbrace{\mathrm{mysin}(x)}_{\text{alg on computer}} = \boxed{\sin(x + \delta x)}$$

Where $\boxed{\dfrac{\delta x}{\boxed{|x|}} \le C u.}$ $\qquad \dfrac{\delta x}{\max(|x|, 1)} \le C u$

Is there a relationship between $\text{Cond}(A^k)$ and $\text{Cond}(A)$?

If $A \geq 0$, $A = A^T$, then

$\sigma_{max} = \lambda_{max}$

$\sigma_{min} = \lambda_{min}$

$$\kappa(A) = \frac{\sigma_{max}}{\boxed{\sigma_{min}}} = \frac{\lambda_{max}}{\lambda_{min}}.$$

$$\kappa(A^{(k)}) = \frac{\lambda_{max}^k}{\lambda_{min}^k} = (\kappa(A))^k.$$

If $A = A^T$

$\sigma_{max} = \max_i |\lambda_i|$

$\sigma_{min} = \min_i |\lambda_i|$

$$\kappa(A) = \frac{\sigma_{max}}{\sigma_{min}} =$$

$A^k \Rightarrow \sigma_{max} = \max_i |\lambda_i|^k =$

$\Rightarrow \kappa(A) = \kappa(A)^k$ too!

How do we know if

$\circledast$ $A^k \underline{b} \in \text{span}(b, Ab, \ldots, A^{k-1} b)$

We check β_{k+1} in Lanczos/Arnoldi!

if $\circledast$ holds, then

$\beta_{k+1} = 0$. (or is very very small)

if $k > n$, this must hold by subspace dimension.

If A

$A =$

when

Krylov +

$I +$

Will post

looks like

$I + B$

B is rand(r), then

tes r iteration.

(or $r+1$)

$\underline{ee^T} \Rightarrow 2$ iters.

- help sheet.