CS 515
Lecture 2

Objectives

↳ What is a matrix?

↳ Common notation

↳ Common matrix problems

↳ Intro to Julia!

My handwriting is bad. I'm so sorry ☹

Your Quiz
What is a m

- A 2d
- A linear
- A coordin a linear vector sp
- An elem
- I know
- It's a table of algebraic manip

atrix?

array / table of numbers

transformation

te representation of

transform between

spaces

e of $\mathbb{R}^{m\times n}$, $\mathbb{C}^{m\times n}$, $\mathbb{F}_2^{m\times n}$

it when I see it.

numbers where linear

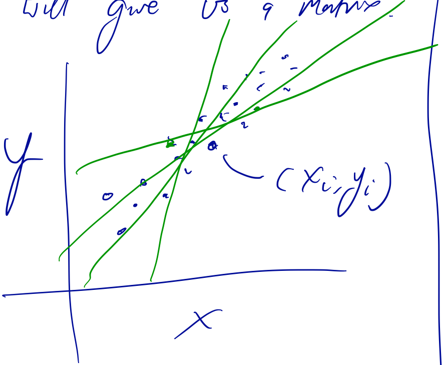
ulations make sense.

Is this a matrix?

$$\begin{bmatrix} 4 & 9 & 6 & 0 & 4 & 5 & 3 \\ 4 & 9 & 4 & 1 & 9 & 9 & 7 \\ 4 & 9 & 4 & 6 & 0 & 1 & 0 \\ 4 & 9 & 6 & 1 & 7 & 6 & 1 \\ 4 & 9 & 4 & 8 & 7 & 9 & 8 \\ 4 & 9 & 6 & 9 & 4 & 3 & 2 \end{bmatrix}$$

These are phone #s.

An Example problem that will give us a matrix!



Goal: find a linear relationship between

$$x \longrightarrow y.$$

The data are pairs

$$(x_1, y_1)$$
$$\vdots$$
$$(x_N, y_N)$$

There are a few matrices

$$X = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \\ x_N & y_N \end{bmatrix}$$

$$X = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 & x_2 \\ y_1 & y_2 \end{bmatrix}$$

?

$$\begin{matrix} x_1 \\ x_2 \\ \\ x_N \end{matrix} \Bigg]$$

$$\begin{matrix} x_N \\ -y_N \end{matrix} \Big]$$

Goal: find $c_1, \; c_2$

so that $y = c_2 x + c_1$

is a good fit to the point (x_i, y_i).

Our ^new goal

find c_2, c_1 to

make

$$(y_i - (c_2 x_i + c_1))^2$$

as small as possible for all points.

$$f(c_1, c_2) = \sum_{i=1}^{N}$$

We want to pick

make f as sm

Let $\underline{x}, \; \underline{y}, \; \underline{c}$

$$= \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

$(y_i - (c_2 x_i + c_1))^2$

c_1, c_2 to

all as possible.

$= \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$$f(\underline{c}) = \|y - \underline{x} c_2 - \underline{e} c_1\|_2^2$$

$$\|\underline{z}\|_2 = \sum_{i=1}^{m} (|z_i|)^2$$

$\underline{e}$ is the vector of all 1's

$$\underline{e} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} \underline{e} & \underline{x} \end{bmatrix}$

Then

$$f(\underline{c}) = \|y - A\underline{c}\|_2^2$$

Suppose we parameterize the line as

$$y = \underline{d}_2 (x - 1) + \underline{d}_1$$

so $c_2 = d_2$ and

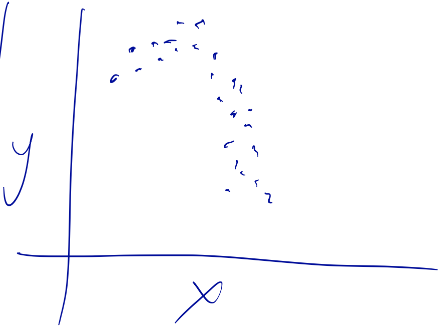
$c_1 = d_1 - d_2$

$$\underline{c} = \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}_{T} \underline{d}$$

So we'd get

$$\|y - A T \underline{d}\|_2^2$$

if we minimized over $\underline{d}$.



$$y = c_3 x^2 + c_2 x + c_1$$

is a better type of fit.

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ & \vdots & \\ 1 & x_N & x_N^2 \end{bmatrix} \quad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

find $\underline{c}$ to minimize

$$f(\underline{c}) = \|y - A\underline{c}\|_2^2$$

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2 instances of the general problem

find $\underline{c}$ to make

$$\|y - Ac\|_2^2$$

as small as possible.

3 general problems

1. Linear systems.

↳ find $\underline{x}$ s.t.

$$A\underline{x} = \underline{b}$$

2. Least sq.

↳ find $\underline{x}$ s.t.

$$\|A\underline{x} - b\|_2^2$$

is as small as possible.

3. Eigenvalu

find $\underline{x}$,

$$A\underline{x} =$$

Notation : Sca
gree

There are

i.e.

$$A^T A \underline{x} =$$

will conver
problem into a

λ s.t.

$\lambda \underline{x}$

las are
k letters :)

many relationship

$A^T \underline{b}$

a least sq
linear system..

$$(A - \lambda I)\underline{x} = 0$$