Lecture 4!

- Announcements
  ↳ My office Hours
  Fri: 1:15 - 2:15pm
- Questions on HW?
- Question from Last Class?

Today

- A few more types of matrix Structures
  ↳ S.P.S.D.
  ↳ Orthogonal
  ↳ M-Matrices.
- The game of Candyland.
- Sparse matrices and How to Store them in Julia.
  ↳ Indexed or Coord. Storage.
  ↳ Compressed Sparse row.

Structure:

Symmetric

This class generalizes

$A^T A$

A matrix A

$A = P^T F$

Some F.

Pos. Semi-definite (spsd)

of matrices

in the normal Eqns for least sq.

is spsd iff

for

If A is spsd.

Then

$$\underline{x}^T A \underline{x} \geq 0$$

for any vector $\underline{x}$

Pf: $A = F^T F$

So $\underline{x}^T A \underline{x} =$

$\underline{x}^T F^T F \underline{x} =$

$y^T y$ for $y = F\underline{x}$

$$\sum_{i=1}^{n} y_i^2 \geq 0.$$

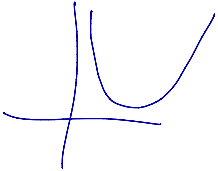
A matrix A is Symm. positive definite (spd) iff

$$\underline{x}^T A \underline{x} > 0$$

$\forall \underline{x} \neq 0.$

Why spsd/spd?

In 1d a quadratic is



Orthogonal Matrices

An orthogonal matrix consists of a set of orthonormal vectors.

x, y are orthonormal if

$$\underline{x}^T \underline{x} = 1$$

$$y^T y = 1$$

$$\underline{x}^T y = 0.$$

So, a set of vectors $\{\underline{v}_1, \ldots, \underline{v}_n\}$ are orthonorm ⟺

$$\underline{v}_i^T \underline{v}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else.} \end{cases}$$

So if

$$V = [\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_n]$$

Then

$$V^T V = I$$

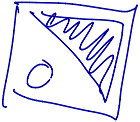
$$[V^T V]_{i,j} = \underline{v}_i^T \underline{v}_j.$$

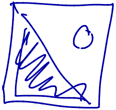
M-matrices

An M-matrix is a square matrix whose inverse is element-wise non-negative.

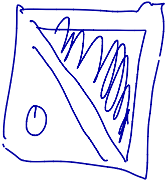
$$\Leftrightarrow \quad \{A^{-1}\}_{i,j} \geq 0.$$

Triangular





Hessenberg.



Candy Land as a Matrix.

There are 133 squares on the board, there are also, 2 special states: Start, destination.
(140) (134)

States 135 – 139 are used to handle the short-cuts / sticky states.

$\Leftrightarrow$

Candyland = 140 × 140 Matrix!

$T = \{\underline{t}_1, \ldots, \underline{t}_{140}\}$

$T(i,j)$ = prob of moving from state j to state i.

$\Longleftrightarrow$

$\underline{t}_{130}$ = prob of going somewhere given I'm in cell 130.

$\underline{t}_{140}$ = Prob of where I go on the first step.

What's the prob of being in any state after 2 steps?

Prob of state i in 2 steps =

$$\sum_K T(i,K)\, \underline{t}_{140}(K)$$

for all states i

$$\underline{p}_2 = T\, \underline{t}_{140}$$

Quiz:

What is an Expression for $\underline{p}_3$

where

$\underline{p}_3(i)$ = Prob of landing on state i in 3 steps of the game.

Soln

$$p_3(i) = \sum_k T(i,k) p_2(k)$$

$$\Leftrightarrow$$

$$= T \underline{p}_2$$

$$= T^2 \underline{e}_{1,100}$$

Generalization

$$\underline{p}_k = T \underline{p}_{k-1}$$

and

$$\underline{p}_k = T^{k-1} \underline{e}_{1,100}$$

How to store T?

Case 1:

A dense 140 x 140 matrix.

There are 2272 non-zero entries in T.

So we store

19600 - 2272

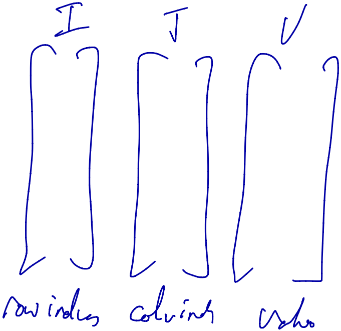
elements that are 0.

Case 2:

We just store a list of the non-zero entries.

$\Leftrightarrow$

Indexed or Coord format.



I	J	V
1	140	0.09375
2	140	0.09375
102	140	0.015625
50	37	0.046875

We want to use I, J, V to compute

$$y = T \underline{x}$$

$$\Longleftrightarrow$$

$$y_i = \sum_j T_{ij} x_j$$

$$y_i = \sum_{j \mid T_{ij} \neq 0} T_{ij} x_j$$

Alg

Allocate y w/ all zeros.

For each tuple $(i, j, v) \in I, J, V$

$$y_i \Leftarrow y_i + v x_j$$

That it!

What is

Compressed Sparse Column Storage?

I	J	V
3	1	37
37	1	1
81	1	$1/2$
2	2	-3
5	2	$-\pi$
	⋮	

→

rowval	nzval.	colptr
3	37	1
37	1	4
81	$1/2$	6
2	-3	6
5	$-\pi$	
		⋮

we group all elements of a col together, and we order columns seq.

□