Lecture 19

- Overview of Next Part of Class
- Project ②
- Interior Pt. Methods ②
- Midterm Notes.

· Lecture 19/20

Interior Pt. Methods for LPs.

· Lecture 21

- Autodiff, Matrix Calc, Finite differences.

- Lecture 22

- Practical Newton Method

Lecture 23:

Optimization w/o Derivatives.

① Non-linear Constraints!

↳ Penalty & AL methods

Lecture 24

methods

Lecture 25:

Lecture 26:

Lecture 27/28

Opt.

Lecture 29:

Lecture 30:

↳ Sampling meth

↳ Global opt

↳ Integer Opt.

: Quasi-Newton

Nonlinear Eqns

Trust Region Methods

: Large Scale

Quad Prog.

Misc Topics.

PROJECT NOTES

Interior Pt. Method for LPs

$$\min \ \underline{c}^T \underline{x}$$
$$\text{s.t.} \ A\underline{x} = \underline{b}$$
$$\underline{x} \geq 0$$

SIMPLEX

- Vertex Based.
- always on External polytope
- Worst Case Exponential time

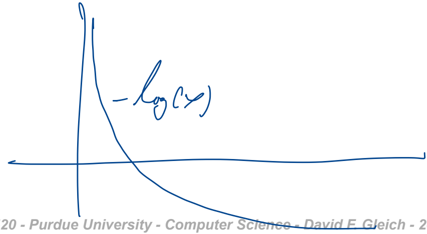
Interior Pt Method

- look at points strictly interior to feasible polytope
- Worst Case poly time.
- roughly $O(n^3)$

How to think about IP methods (not what they actually do!)

- Central path / log-barrier method
- We study

$$\min \; c^T x - \tau \sum_{i=1}^{n} \log(x_i)$$
$$\text{s.t. } Ax = b \qquad \textcircled{*}$$



So at a soln of $\textcircled{*}$, $x > 0$ (Because of log-barrier term).

Idea:

$$\min \; c^T x + \tau\,[\text{Something}]$$
$$\text{s.t. } Ax = b$$
$$\text{“ } x > 0 \text{ ”} \longrightarrow$$

look @ $\tau \to 0$

I'll

Min

st

is st

uniq.

$\Rightarrow$ c

look

State w/o Proof:

$c^T x - \tau \sum \log(x_i)$

$\underline{A}\underline{x} = \underline{b}$

ongly Convex w/a

minimizer.

unique $x(\tau)$.

at $\lim_{\tau \to 0} x(\tau)$.

Strongly Convex $(\Leftarrow)$

Use KKT to solve.

$$\mathcal{L}(x,\lambda) = \underline{c}^T \underline{x} - \tau \sum \log(x_i) - \lambda^T(\underline{A}\underline{x} - \underline{b})$$

$$\partial \mathcal{L} / \partial x = c - A^T \lambda - \text{``}\tau / x\text{''} = 0$$

Notation Alert! $X = \text{diag}(\underline{x})$ is a diag. matrix w/ $\underline{x}$ on the diag.

$$\partial \mathcal{L} / \partial x = \boxed{\begin{array}{l} \underline{c} - A^T \lambda - \tau X^{-1} \underline{e} = 0 \quad \text{vec of all 1s} \\ A\underline{x} = b, \quad \underline{x} > 0 \end{array}}$$

KKT for LP

$$\underline{c} - A^T\underline{\lambda} - \underline{s} = 0$$

$$\underline{x}^T\underline{s} = 0$$

$$A\underline{x} = \underline{b}$$

$$\underline{s} \ge 0$$

$$\underline{x} \ge 0$$

What if $\underline{s} = \tau X^{-1}\underline{e}$?

$c - A^T\lambda - s = 0$ ✓

$Ax = \underline{b}$ ✓

$s \ge 0$ ✓

$x \ge 0$ ✓

$x^Ts = x^T(\tau X^{-1}\underline{e}) = \tau n$

X_{-}

$x(\tau), \lambda(\tau), s(\tau)$ are all unique.

$$\lim_{\tau \to 0} x(\tau), \lambda(\tau), s(\tau)$$

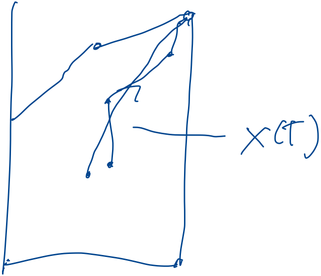
guess a point where

$$x^{*T}s^{*} = 0$$

$$x \ge 0$$

? .

$\Rightarrow$ Valid Soln for LP!

Called the Central Path!