Computational methods in optimization

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CS 520

Course objectives

To understand why optimization is hard

To understand when we can optimize

To understand how to optimize

To be able to optimize a function

To understand optimization software

What is optimization?

- (Cheeky) I hope you know, you decided to take this class!
- Mathematical programming
- Operations research

What is optimization?

My attempt at a definition –

Optimization is the umbrella term for the

theory / software / methodology / models / process

of finding the

extreme / improved / better points

of a ***mathematical function*** representing a ***useful quantity*** while satisfying constraints on the feasible inputs to that function

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Syllabus

Course outline

Background

Software

Least Squares

Prerequisites

Calculus

Linear algebra (QR, SVD, Eigenvalues)

1d quadratic functions

Numerical linear algebra (CS515):

- Cholesky / iterative methods / etc.

Unconstrained optimization

- Non-linear equations
- Newton methods
- Line search
- Trust region
- Quasi-newton

$$\text{minimize} \quad f(x)$$

Constrained optimization

- Linear programming
- Quadratic programming
- Large-scale

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & l \leq \begin{bmatrix} x \\ Ax \\ c(x) \end{bmatrix} \leq u \end{aligned}$$

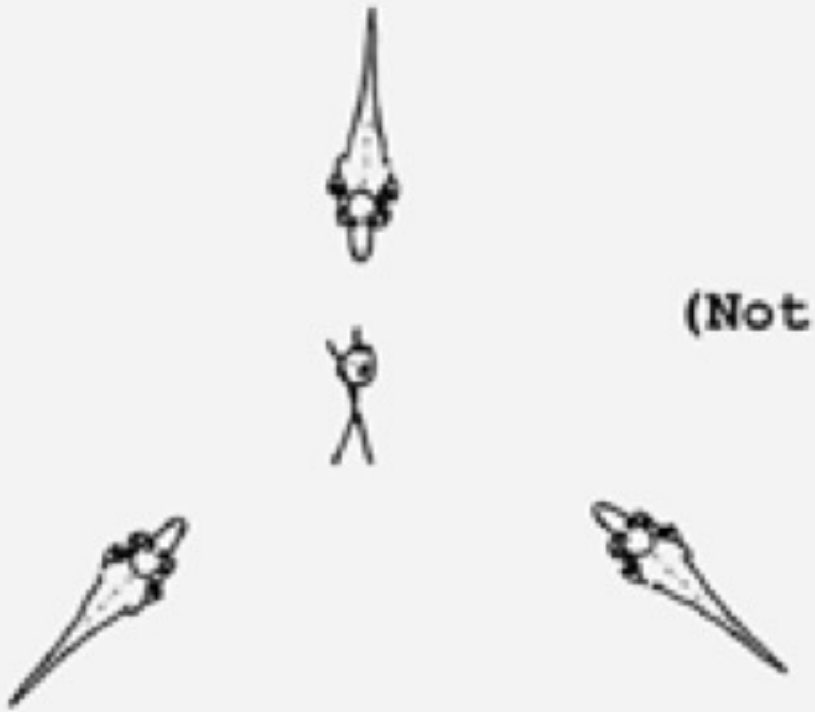
Modern topics

- Convex
- Integer
- Stochastic

Questions about topics?

Your first quiz!

2. You are at the center of a 20m equilateral triangle with a raptor at each corner. The top raptor has a wounded leg and is limited to a top speed of 10 m/s.



(Not to scale)

The raptors will run toward you. At what angle should you run to maximize the time you stay alive?

Raptors move at 15 m/s You move at 6 m/s

solution in Julia

Who cares?

The new model

choose direction to run $\mathbf{v}_p[j]$ for $j = \{1, \ldots, N\}$

to minimize "likelihood" of being eaten

$$\sum_{j=1}^{N} \sum_{i=1}^{3} \frac{1}{\|\mathbf{p}[j] - \mathbf{r}_i[j]\|^2} dt$$

subject to raptor motion

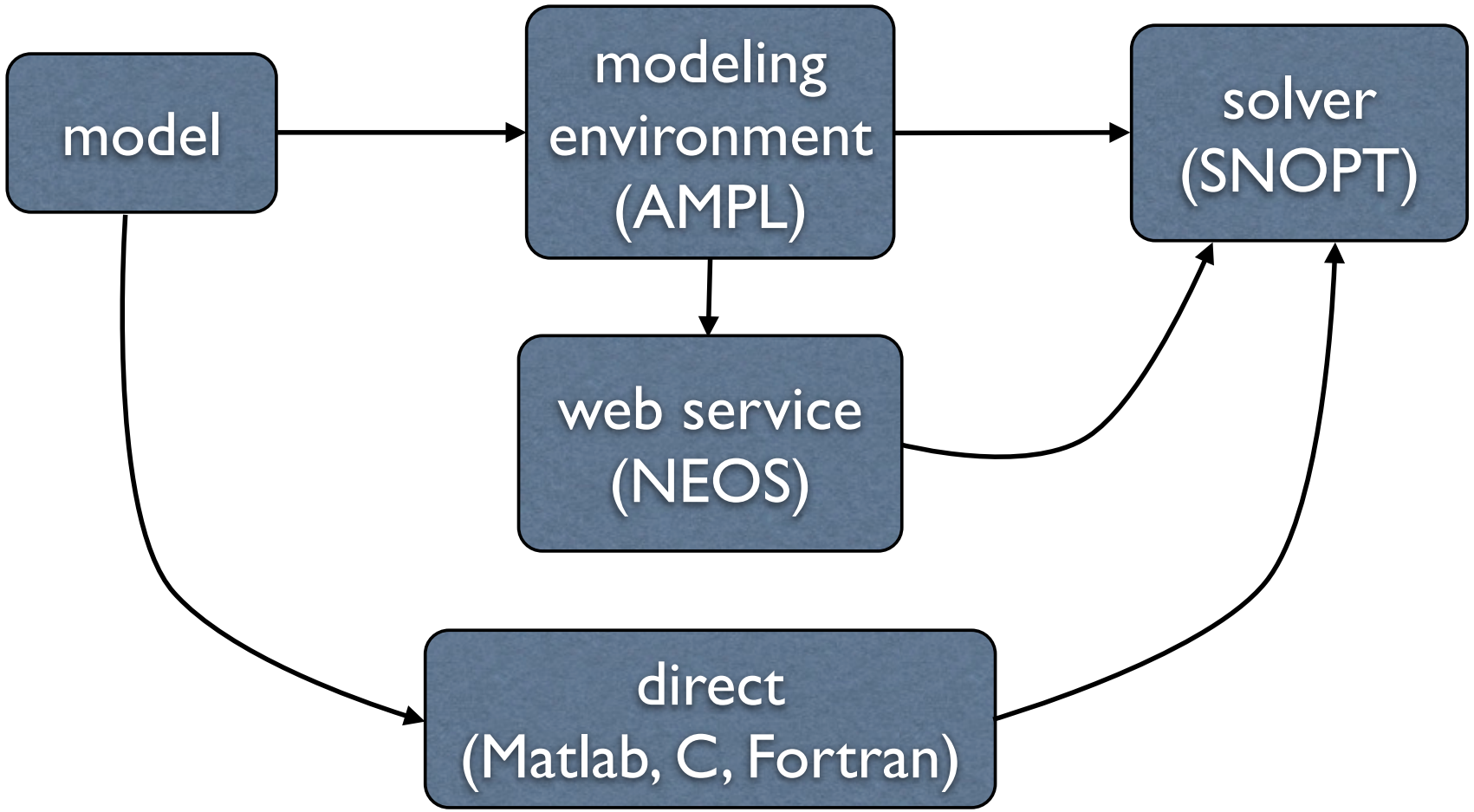
$$\mathbf{r}_i[j+1] = \mathbf{r}_i[j] + h v_i \frac{\mathbf{p}[j] - \mathbf{r}_i[j]}{\|\mathbf{p}[j] - \mathbf{r}_i[j]\|}$$

human motion

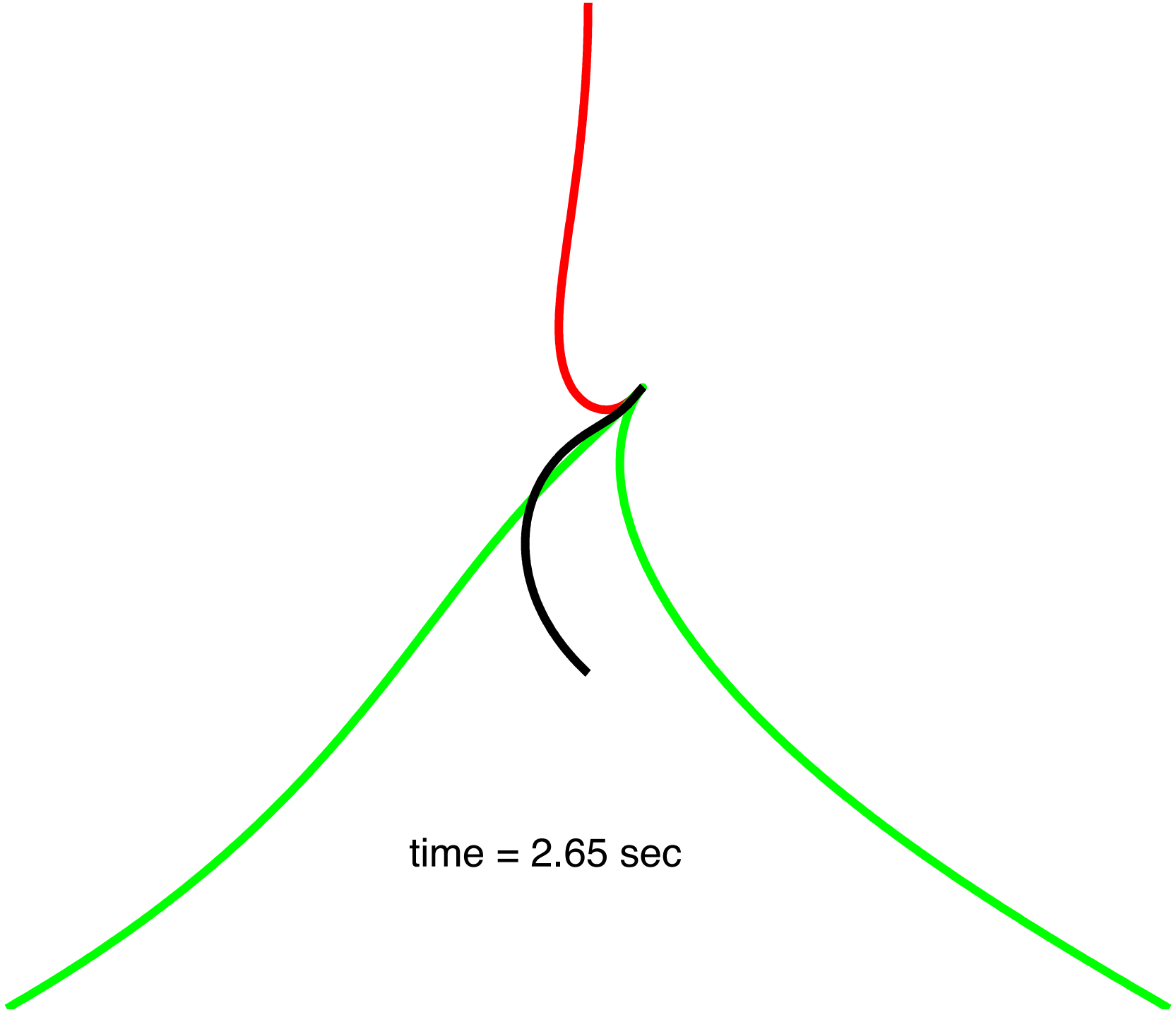
$$\mathbf{p}[j+1] = \mathbf{p}[j] + h\mathbf{v}_p[j]$$

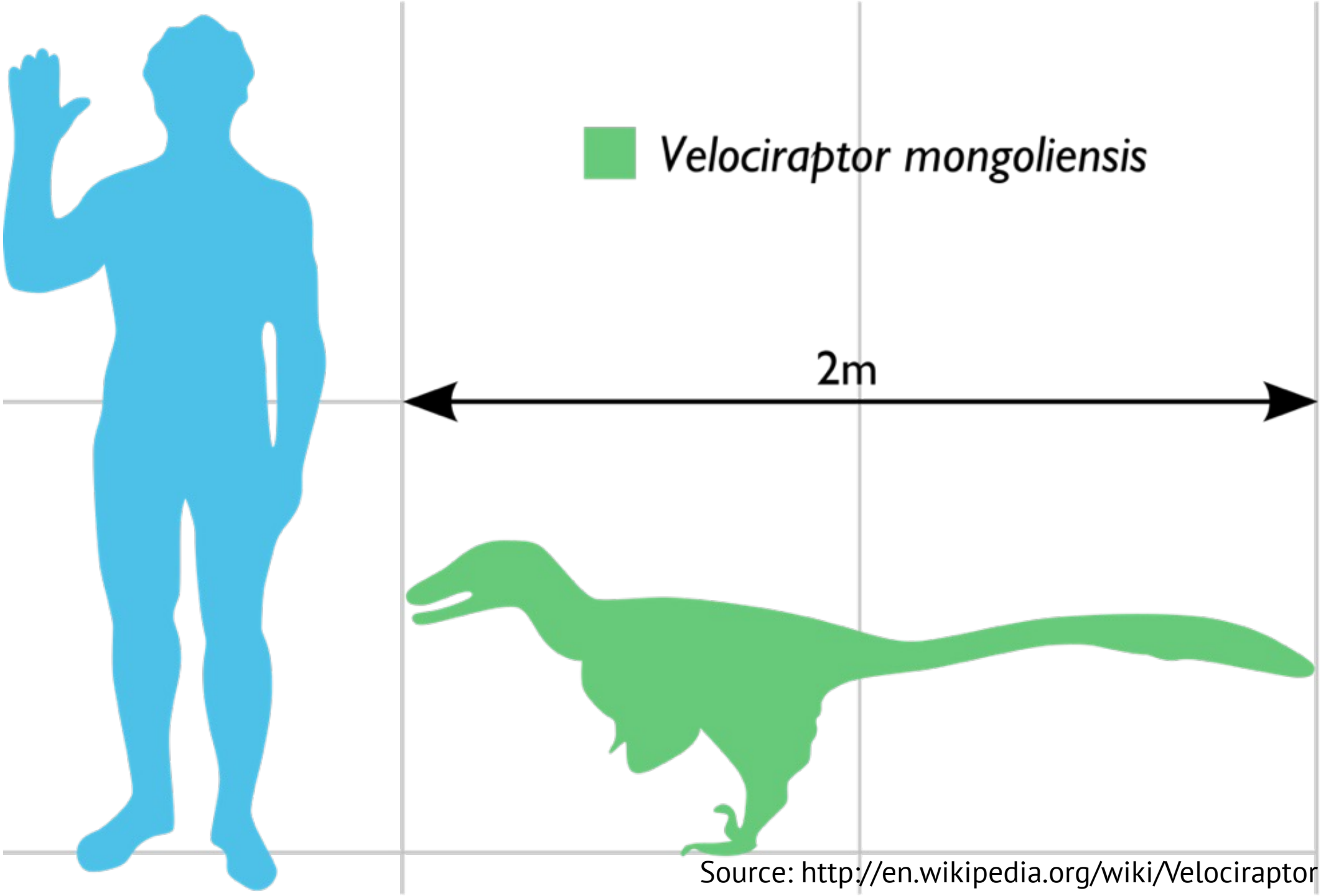
Thanks to Nick Henderson for many slides.

How it's done


model
modeling environment (AMPL)
solver (SNOPT)
web service (NEOS)
direct (Matlab, C, Fortran)

time = 0.01 sec

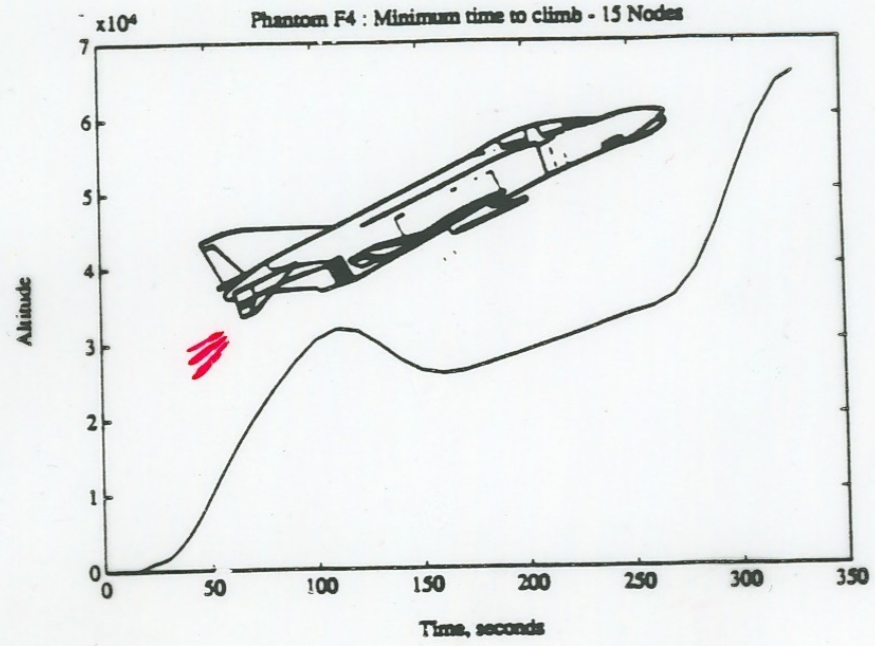

time = 2.65 sec



Source: http://en.wikipedia.org/wiki/Velociraptor

Aerospace Applications of NPSOL and SNOPT

OTIS #1

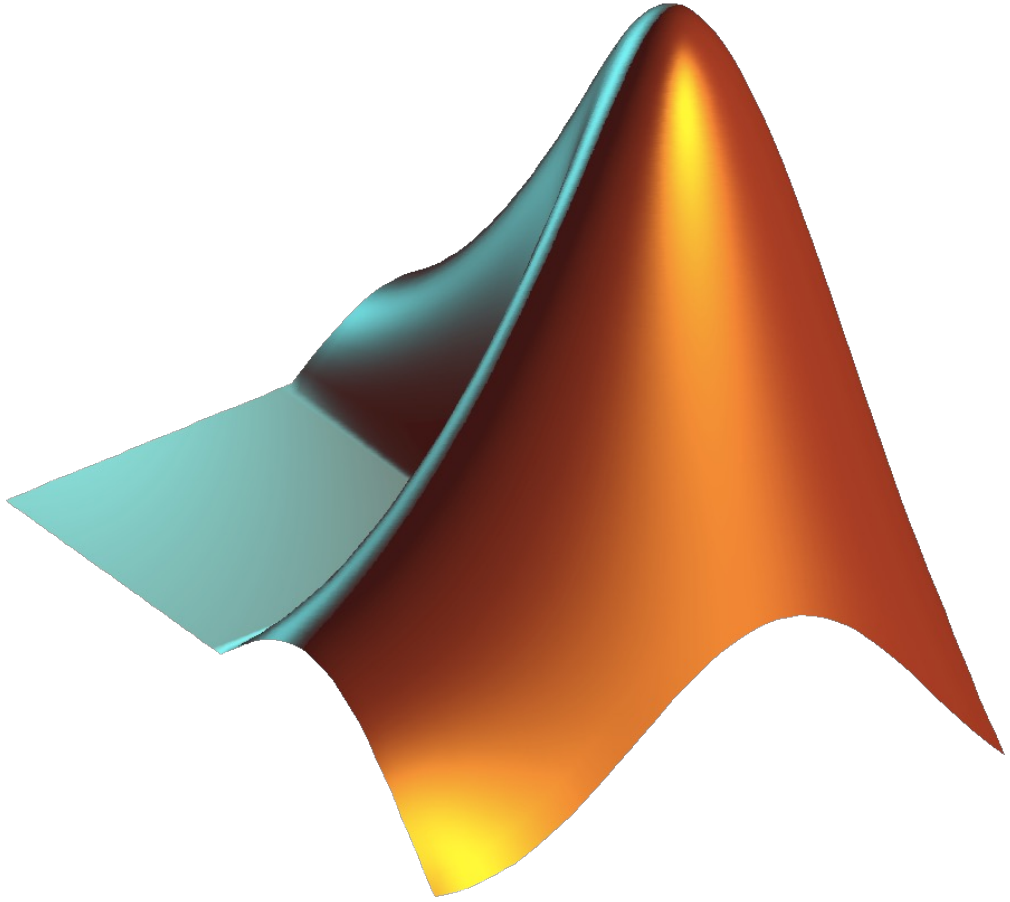


What are you applications?

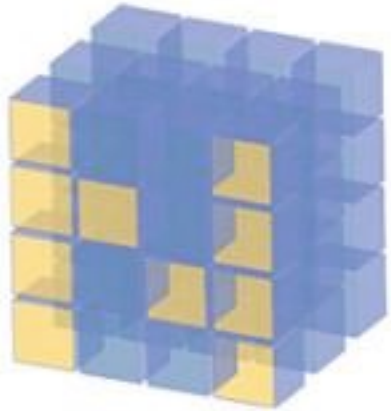
Course logistics!

Numerical computing software


julia



MATLAB



NumPy

Why I've moved everything to Julia

- Free, free, free! (Run it anywhere.)
- Close enough to Matlab
  - A(5,6) vs. A[5,6]
  - [V,D] = eig(A) vs. V,d = eigen(A)
  - "my string" vs. 'my string'
- For-loops are "more efficient" (used a lot here!)
- Many helpful tools for numerical analysis
  - BigFloat has enhanced precision
- Great optimization tools in Julia! (the de-facto standard now?)
  - World leading automatic differentiation!

What I'm worried about

- Julia is still under active development
- There are ~~many~~ rough edges
- Graphics / plotting are still ~~very (2017)~~ ~~rough~~ ~~(2020)~~ not always reliable (2023)
- Time-to-first-plot/function/etc is slow… ☹
  - Should be much better in the next 1-2 years

How to use Julia

On your own computer

- VSCode – what I use for normal development
  - (but our demos don't work because I used a bunch of interactive stuff … sigh)
- Pluto (what I would redo scripts with now…)
- Jupyter notebook (Julia-Python-R)
  - What you will see in class!

- Julia command line (not recommended)
- ~~or **nteract**~~
- ~~Atom / Juno~~