Quantum-Like Bits: Graph Constructions for Arbitrary Qubit States

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Abstract

Building on experimental observations of composite graphs that exhibit emergent eigenvectors in complex synchronized networks [1,3], we develop a rigorous graph theoretic framework for constructing quantum-like bits (QL-bits). Our approach builds a composite system from two k-regular subgraphs coupled via a bipartite connection matrix C, whose emergent eigenvectors form a natural qubit basis. Rigorous proofs establish that the composite matrix $R = [A\ C;\ C^T\ B]$ yields eigenvectors corresponding to eigenvalues $\lambda_- = k + l$ and $\lambda_+ = k - l$ under symmetric coupling. By introducing state tuning through detuning (varying subgraph regularity) and employing asymmetric coupling (via directed matrices C_A, C_B in replacement of C, C^T), we show how to generate an arbitrary state $\psi = a\psi_+ + b\psi_-$ (with $a^2 + b^2 = 1$). This work extends previous research on QL state representations and offers a flexible methodology for state manipulation with applications in quantum simulation and network synchronization.

Introduction and Motivation

Emergent Behavior: Complex synchronized networks give rise to emergent (satellite) eigenvectors that can be leveraged to mimic qubit states.

Graph Theory Meets Quantum Computing: By modeling qubit states through the structure of regular graphs, we can explore QL information processing using classical network constructs.

Objective: Develop a rigorous yet flexible method to construct and tune QL-bits by coupling two regular subgraphs. In particular, we show how to construct:

$$\psi = \begin{pmatrix} aV_A \\ bV_B \end{pmatrix} \text{ for } a^2 + b^2 = 1$$

Where ψ is the eigenvector associated with the top emergent eigenvalue of the adjacency matrix $R = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$.

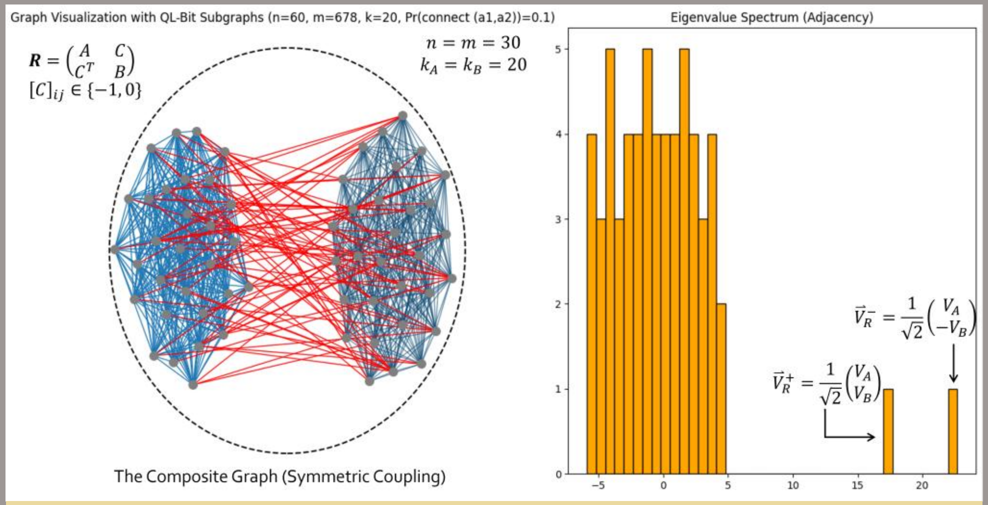

Fig 1. Regular graph and adjacency spectrum. Each subgraph has n/2 nodes. m_subgraph is set to None, indicating no edges deleted for a total of 678 edges ≈ 2*(#edges in a k-regular subgraph of size $\frac{n}{2}$)+Pr(connect)*(#possible (undirected) connecting edges) = $2(k(\frac{n}{2})/2)+0.1(\frac{n}{2})^2$ = 20*30+0.1*30^2=690. Note the two emergent eigenvectors are approximately $20 \pm 3 = k \pm l$ for k-regular subgraphs a_1, a_2 and approximately l-regular connecting graph G_C(randomly adding edges uniformly adds approximately l ≔ (#edges added)/v)*2 edges to each node).

Theoretical Framework

The composite (undirected) graph is formed by two subgraphs, G_A and G_B, of orders n and m, respectively, with adjacency matrices A and B. Both subgraphs are assumed to be k-regular, meaning each vertex has exactly k connections. They are coupled by a bipartite matrix C, whose entries are chosen from {-1, 0}. The composite adjacency matrix is then expressed as $R = [A\ C;\ C^T\ B]$.

Let V_A and V_B be the normalized Perron-Frobenius eigenvectors of A and B, given by $V_A = (\frac{1}{\sqrt{n}})[1, 1, ..., 1]^T$ and $V_B = (\frac{1}{\sqrt{m}})[1, 1, ..., 1]^T$ [2]. These eigenvectors allow us to define the qubit basis vectors as

$$\psi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} V_A \\ V_B \end{pmatrix} \quad \text{and} \quad \psi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} V_A \\ -V_B \end{pmatrix}$$

In our rigorous treatment (Lemma 4.1), we show that when C is l-regular and the subgraphs have equal order ($n = m$), these basis vectors become eigenvectors of R with eigenvalues $\lambda_- = k + l$ and $\lambda_+ = k - l$.

Tuning Mechanisms

To generate an arbitrary QL-bit state, we express the state as $\psi = a\psi_+ + b\psi_-$, where a and b satisfy the normalization condition $a^2 + b^2 = 1$. One tuning mechanism involves detuning the regularities of the subgraphs. By varying k_A and k_B, we define a detuning parameter $\Delta = (k_A - k_B)/2$, which controls the relative contributions of ψ_+ and ψ_- in the composite state. However, in regimes where detuning leads to divergence (for instance, when either a or b approaches zero) or becomes infeasible (e.g. $\Delta \geq n$), symmetric coupling becomes less practical. To remedy this, we propose the use of asymmetric coupling. In this alternative approach, the undirected coupling matrix C is replaced by two directed matrices, C_A and C_B, satisfying $C_A V_A = -l_A V_A$ and $C_B V_A = -l_B V_A$. This method allows independent adjustment of the coupling strengths and ensures a balanced emergent state.

Practical Considerations + Extensions

The present framework employs discrete coupling weights (-1 and 0), which may limit the precision of state tuning. Strategies such as using high-precision rational entries or employing ensemble averaging can address these limitations. Future work may extend this approach to multi-qubit systems and explore the effects of heterogeneous graph structures, thereby broadening the applicability of QL-bits in quantum simulation and complex network analysis.

References

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