Lecture 22: Message Authentication Codes from PRF

Outline

- In the previous lecture, we defined MACs and their security and constructed them using random functions
- In today's lecture, we shall construct MACs using pseudo-random functions

MAC using Pseudorandom Functions I

Scheme.

- Secret-key Generation. Sample sk uniformly at random from $\{0,1\}^{n/100}$ and provide sk to both the sender and the verifier
- Tagging a message $m \in \{0,1\}^n$. The sender computes tag $\tau = g_{\mathsf{sk}}(m)$ (evaluate using the GGM construction, where we consider functions $\{0,1\}^n \to \{0,1\}^{n/100}$ and id in $\{0,1\}^{n/100}$)
- Verifying a message-tag pair $(\widetilde{m}, \widetilde{\tau})$. Check whether $\widetilde{\tau}$ is same as $g_{\mathsf{sk}}(\widetilde{m})$ or not

MAC using Pseudorandom Functions II

Security

- An adversary cannot forge if it sees t message-tag pairs, where $t = \mathrm{poly}(n)$ and the adversary is computationally bounded
  If an adversary can forge a signature in this case, we can distinguish the random functions from pseudo-random functions. Because, in the former case, forgeability was impossible for any adversary. However, in the latter case, this adversary makes forgeability possible. s

Subtlety

The scheme mentioned above is secure ONLY for messages in $\{0,1\}^n$ and NOT $\{0,1\}^*$

What does it mean?

- The set $\{0,1\}^n$ represents n-bit messages, and $\{0,1\}^*$ represents arbitrary-length messages. This scheme is secure only when an adversary sees message-tag pairs for messages $m_1, m_2, \ldots, m_t$ such that all of them have identical length n. Moreover, the adversary has to forge by producing (m', τ') pair such that the length of the message m' is exactly n.
- The scheme is not secure if the adversary can produce a message of a different length. The attack is explained in the next slide

Subtlety

Adversarial strategy to forge a message-tag pair of different lengths.

- Suppose the adversary has seen a message-tag pair (m, τ) such that $\tau = F_{\mathsf{sk}}(m)$
- The adversary creates $m' = m0$ (i.e., the message m concatenated at the end with 0). The adversary computes τ' as the first half of $G(\tau)$.
- Verify that $F_{\mathsf{sk}}(m') = \tau'$
- In fact, the adversary can successfully tag any m' such that m is the prefix of m'

Lesson Learned (Very Important)

- The sender and the verifier should establish one secret key sk for EACH length of the message they want to sign. For example
    - They establish a secret-key sk $\in \{0,1\}^k$ for 1024-bit messages and use $F_{\mathsf{sk}}(m)$ as the tag for 1024-bit messages m
    - If they want to tag 2048-bit messages, then they establish a new secret-key sk$'$ $\in \{0,1\}^k$ and use $F_{sk'}(m)$ as the tag for 2048-bit messages m
    - The verifier should only check the validity of the tags corresponding to 2048-bit messages using the secret key associated with message length 2048 (in our case, it is the secret key sk$'$)

Food for thought

- Suppose we want to construct a MAC so that if t-parties among a set of n-parties decide to endorse a message m, they can add a tag that the verifier can verify. How to construct such a scheme?