

Lecture 20: Message Authentication Codes from PRF

- In the previous lecture, we defined MACs and their security and constructed them using random functions
- In today's lecture, we shall construct MACs using pseudo-random functions

Scheme.

- Secret-key Generation. Sample sk uniformly at random from $\{0, 1\}^{n/100}$ and provide sk to both the sender and the verifier
- Tagging a message $m \in \{0, 1\}^n$. The sender computes tag $\tau = g_{sk}(m)$ (evaluate using the GGM construction, where we consider functions $\{0, 1\}^n \rightarrow \{0, 1\}^{n/100}$ and id in $\{0, 1\}^{n/100}$)
- Verifying a message-tag pair $(\tilde{m}, \tilde{\tau})$. Check whether $\tilde{\tau}$ is same as $g_{sk}(\tilde{m})$ or not

Security

- An adversary cannot forge if it sees t message-tag pairs, where $t = \text{poly}(n)$ and the adversary is computationally bounded. If an adversary can forge a signature in this case, we can distinguish the random functions from pseudo-random functions. Because, in the former case, forgeability was impossible for any adversary. However, in the latter case, this adversary makes forgeability possible. s

The scheme mentioned above is secure **ONLY** for messages in $\{0, 1\}^n$ and **NOT** $\{0, 1\}^*$

What does it mean?

- The set $\{0, 1\}^n$ represents n -bit messages, and $\{0, 1\}^*$ represents arbitrary-length messages. This scheme is secure only when an adversary sees message-tag pairs for messages m_1, m_2, \dots, m_t such that all of them have identical length n . Moreover, the adversary has to forge by producing (m', τ') pair such that the length of the message m' is exactly n .
- The scheme is not secure if the adversary can produce a message of a different length. The attack is explained in the next slide

Adversarial strategy to forge a message-tag pair of different lengths.

- Suppose the adversary has seen a message-tag pair (m, τ) such that $\tau = F_{sk}(m)$
- The adversary creates $m' = m0$ (i.e., the message m concatenated at the end with 0). The adversary computes τ' as the first half of $G(\tau)$.
- Verify that $F_{sk}(m') = \tau'$
- In fact, the adversary can successfully tag any m' such that m is the prefix of m'

Lesson Learned (Very Important)

- The sender and the verifier should establish one secret key sk for EACH length of the message they want to sign. For example
 - They establish a secret-key $sk \in \{0, 1\}^k$ for 1024-bit messages and use $F_{sk}(m)$ as the tag for 1024-bit messages m
 - If they want to tag 2048-bit messages, then they establish a new secret-key $sk' \in \{0, 1\}^k$ and use $F_{sk'}(m)$ as the tag for 2048-bit messages m
 - The verifier should only check the validity of the tags corresponding to 2048-bit messages using the secret key associated with message length 2048 (in our case, it is the secret key sk')

- Suppose we want to construct a MAC so that if t -parties among a set of n -parties decide to endorse a message m , they can add a tag that the verifier can verify. How to construct such a scheme?