Attention Is All You Need:

Deriving the Seminal Transformer Architecture from First Principles

J. Setpal

September 3, 2024

MACHINE LEARNING @ PURDUE

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h. **Embedding:** A look-up table that translates categorical values (words) to vectors.

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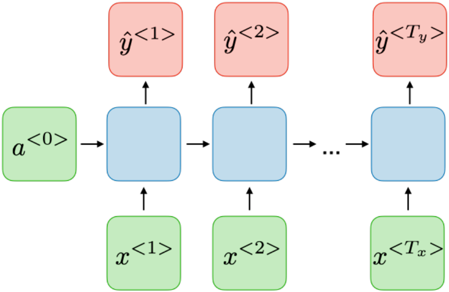

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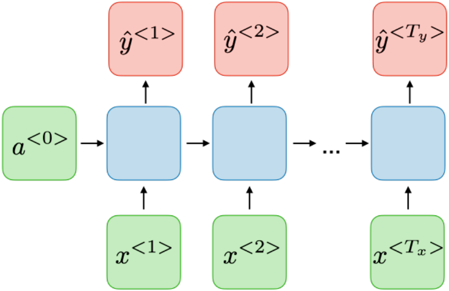

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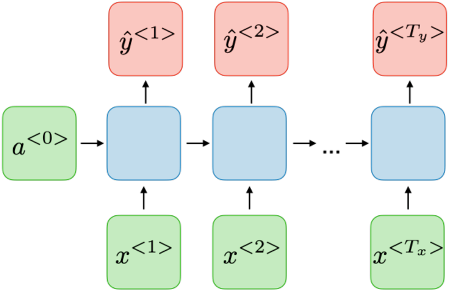

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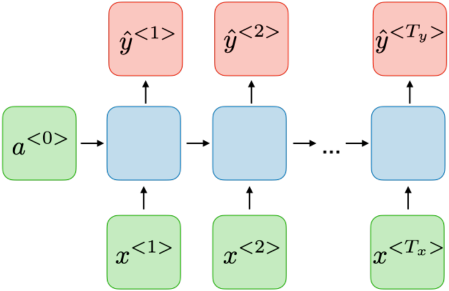

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One way to improve this is by incorporating *attention* into RNNs.[1]

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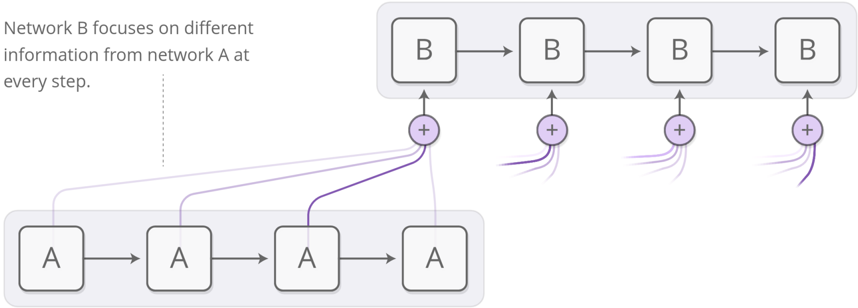

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We query this database to extract important information from our input sequence. For $\{x_i\}_{i=1}^{t}$,

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Where q_i, k_i, v_i are each independently computed latent matrices.

$$\text{Self-Attention}(Q, K, V) = \left(\frac{QK^T}{\sqrt{d_{out}}}\right) V \tag{6}$$

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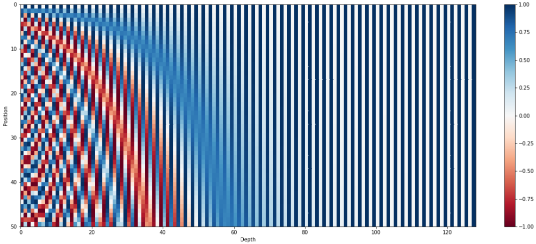
To resolve this, we add positional encodings to each word embedding:

$$PE_{(pos,2i)} = \sin\left({}^{pos}\!/_{1E4^{2i/d_{model}}}\right) \tag{7}$$

$$PE_{(pos,2i+1)} = \sin\left({}^{pos}\!/_{1E4^{2i/d_{model}}}\right) \tag{8}$$



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Therefore, we instantiate *multiple* heads within each layer, and concatenate to construct a final output represetnation.

$$MHA = \text{Concat}(\{h_i\}_{i=1}^{H})W_O \tag{9}$$

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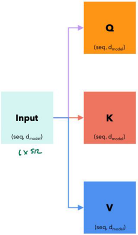

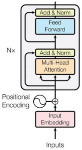

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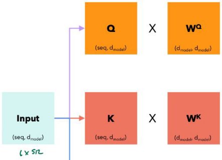

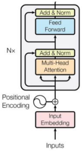

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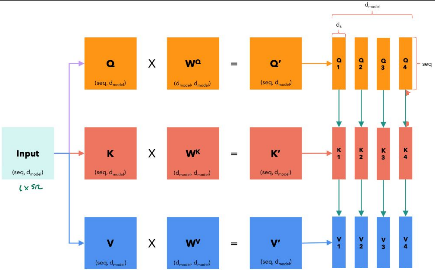

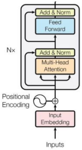

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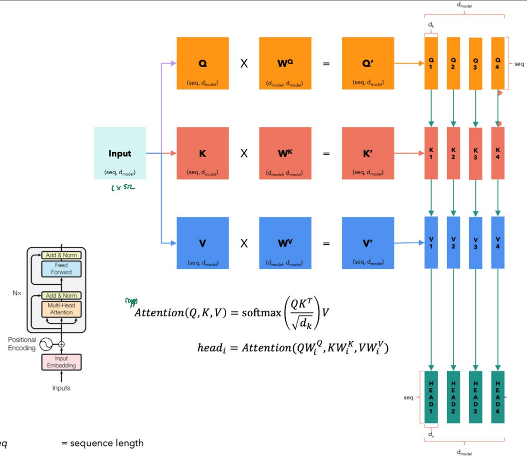

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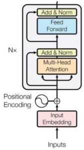

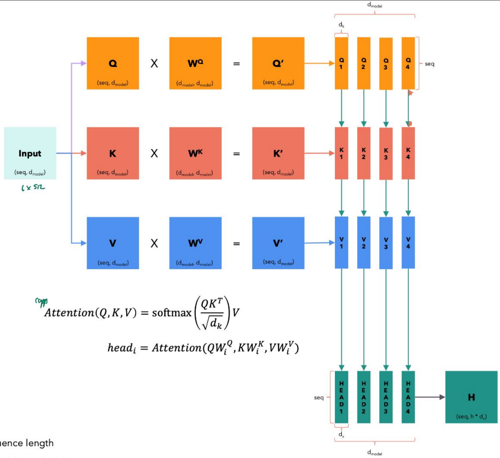

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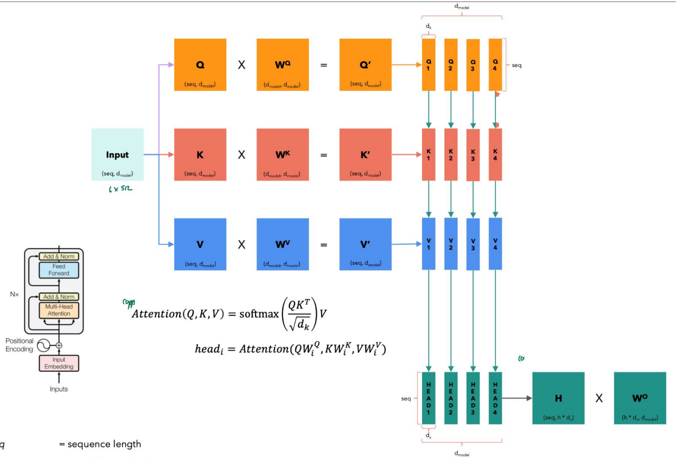

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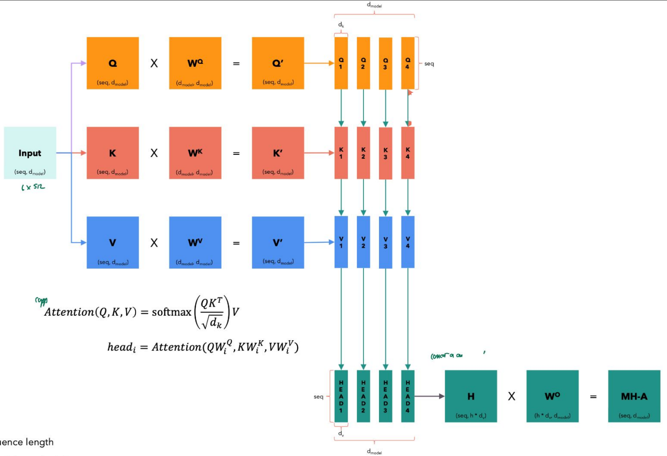

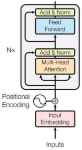

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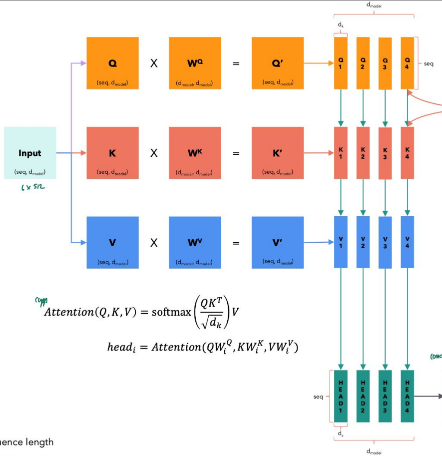
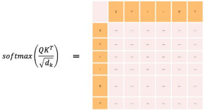
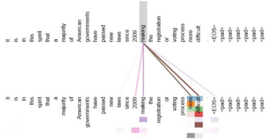
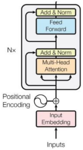




Attention Visualizations





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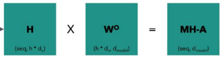

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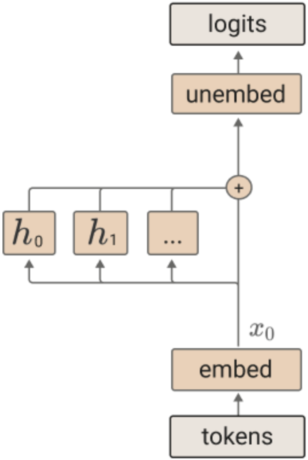
For this, a two-layer MLP is instantiated that expands and consequently contracts the input dimension.

$$FFN(x) = \sigma_{relu}(xW_1 + b_1)W_2 + b_2 \tag{10}$$

The original paper uses a factor of 4.

Scaling to Deeper Networks – A Better Interpretation



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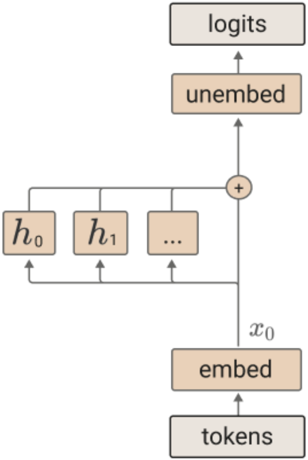

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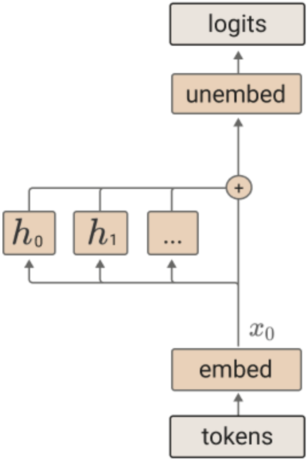

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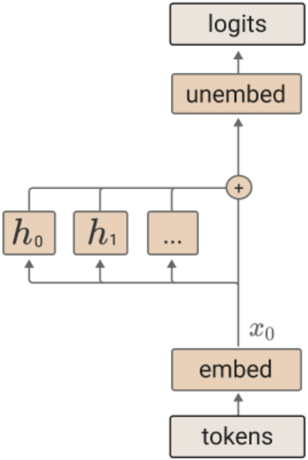
A result of this setup is that we can interpret each attention head and MLP as "reading from" and "writing to" a residual stream.

Scaling to Deeper Networks – A Better Interpretation



Deep neural architectures struggle with vanishing gradients, when trained at scale.

The approach used universally is to add "skip connections" to maintain a short gradient path, and mitigate this concern.

A result of this setup is that we can interpret each attention head and MLP as "reading from" and "writing to" a residual stream.

In addition, we also perform layer normalization over the latent vectors before MLP & self-attention.

Inference & Training

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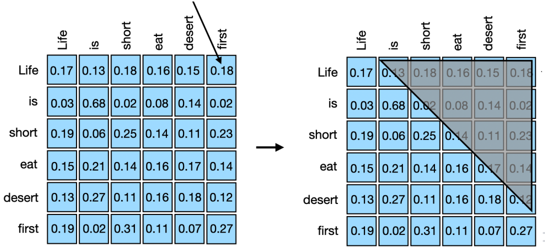
However, a consequence of this is that attention can *look into the future*. We prevent this by applying a causal mask:



Reviewing NanoGPT

If you can view this screen, I am making a mistake.

Thank you!

Have an awesome rest of your day!

Slides: `https://cs.purdue.edu/homes/jsetpal/slides/transformer.pdf`