Fundamentals of information transmission and coding (a.k.a. communication theory)

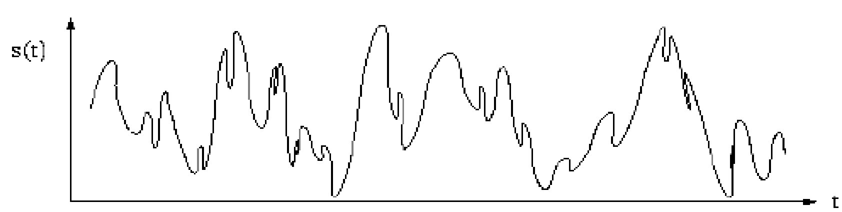
Signals and functions

Elementary operation of communication: send *signal* on medium from A to B.

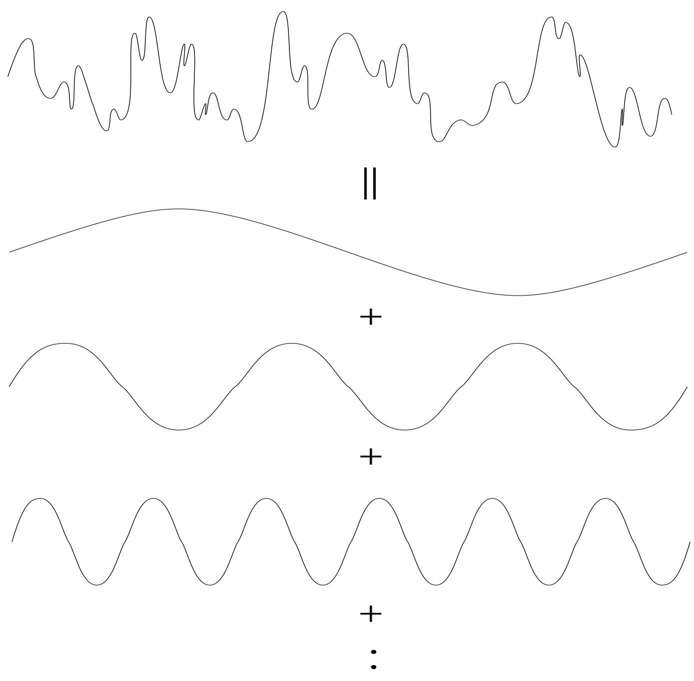
- media—copper wire, optical fiber, air/space, ...
- signals—voltage and currents, light pulses, radio waves, microwaves, ...

  $\rightarrow$ electromagnetic wave

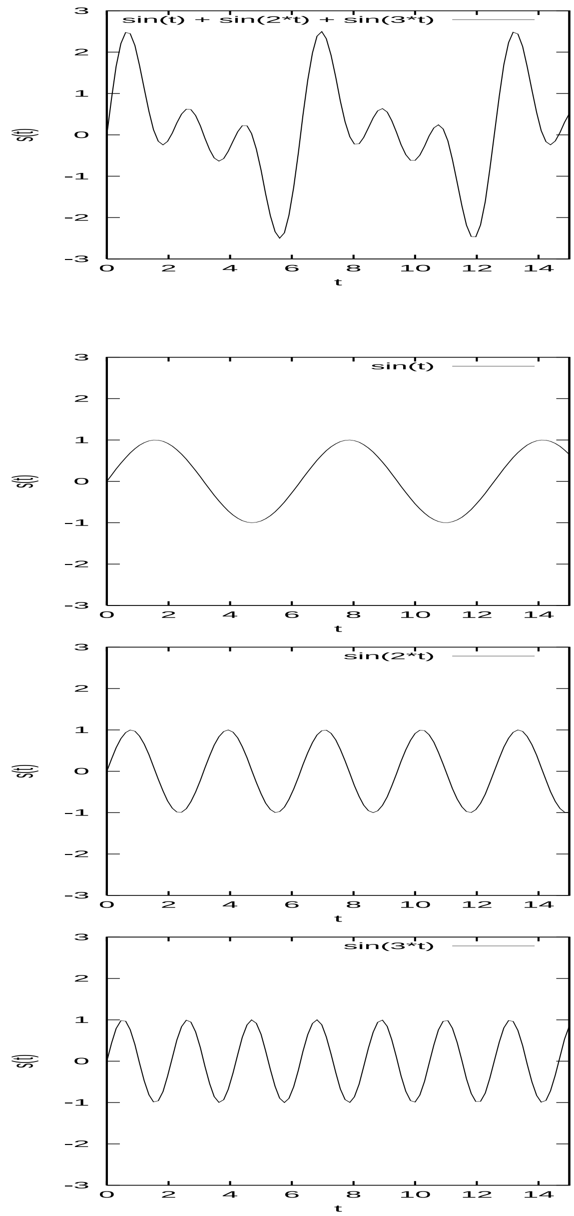
Signal can be viewed as a time-varying function $s(t)$.



If $s(t)$ is "sufficiently nice" (Dirichlet conditions) then $s(t)$ can be represented as a linear combination of complex sinusoids:



Simple example:



$\longrightarrow$ sinusoids form basis for other signals

Analogous to basis in linear algebra:

other elements can be expressed as linear combinations of elements in the basis set

Ex.: in 3-D, $\{(1, 0, 0),\ (0, 1, 0),\ (0, 0, 1)\}$ form a basis.

$\longrightarrow$ $(7, 2, 4) = 7 \cdot (1, 0, 0) + 2 \cdot (0, 1, 0) + 4 \cdot (0, 0, 1)$

$\longrightarrow$ coefficients: 7, 2, 4

$\longrightarrow$ spectrum

How many elements are there in a basis?

Vector spaces:

- finite dimensional
- infinite dimensional: signals

Given an arbitrary element in the vector space, how to find the coefficient of basis elements?

$\longrightarrow$ e.g., given $(7, 2, 4)$, coefficient of $(0, 1, 0)$?

Fourier expansion and transform:

$$s(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} d\omega,$$

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt.$$

$\longrightarrow$ recall: $e^{i\omega t} = \cos \omega t + i \sin \omega t$

$\longrightarrow$ $S(\omega)$: coefficient of basis elements

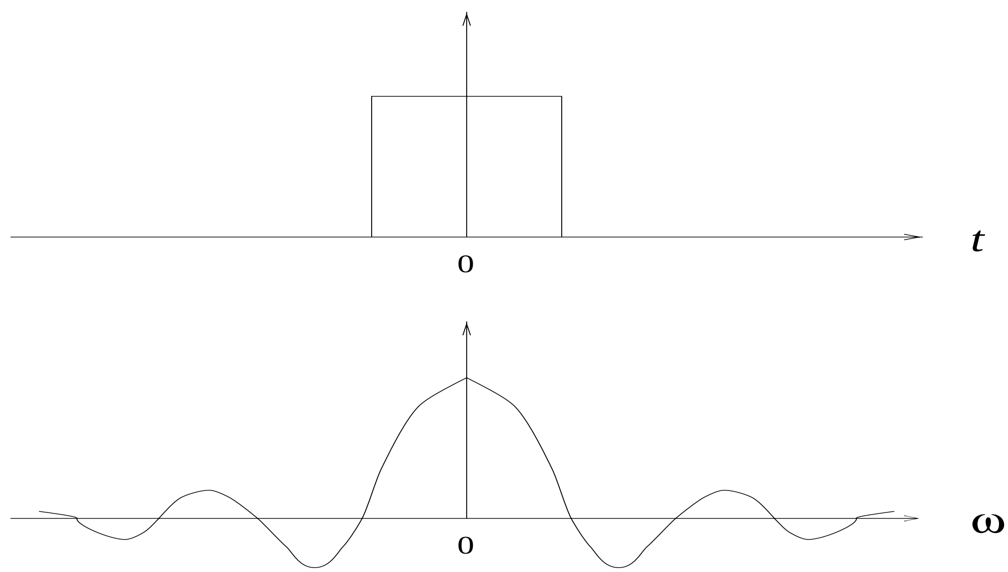
$\longrightarrow$ time domain vs. frequency domain

Frequency ω: cycles per second (Hz)

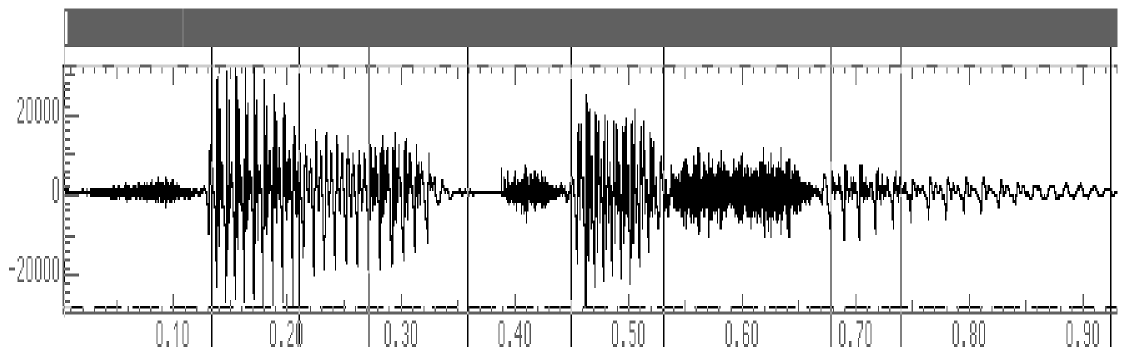
$\longrightarrow$ $\omega = 1/T$

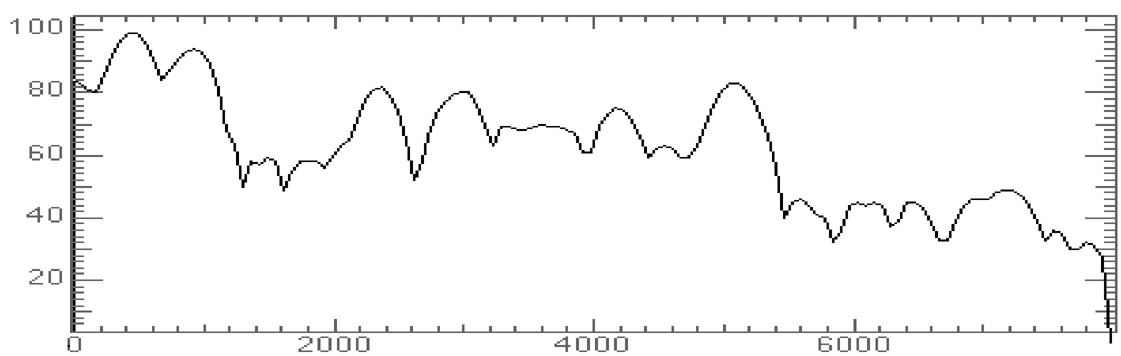
T: period of sinusoid

Example: square wave



Example: audio (e.g., speech) signal





Source: Dept. of Linguistics and Phonetics, Lund University

Random function (i.e., white noise) has "flat-looking" spectrum.

$\longrightarrow$ unbounded bandwidth

Why bother with frequency domain representation?

$\longrightarrow$ contains same information . . .

$\longrightarrow$ i.e., invertible

Luckily, most "interesting" functions arising in practice are "special":

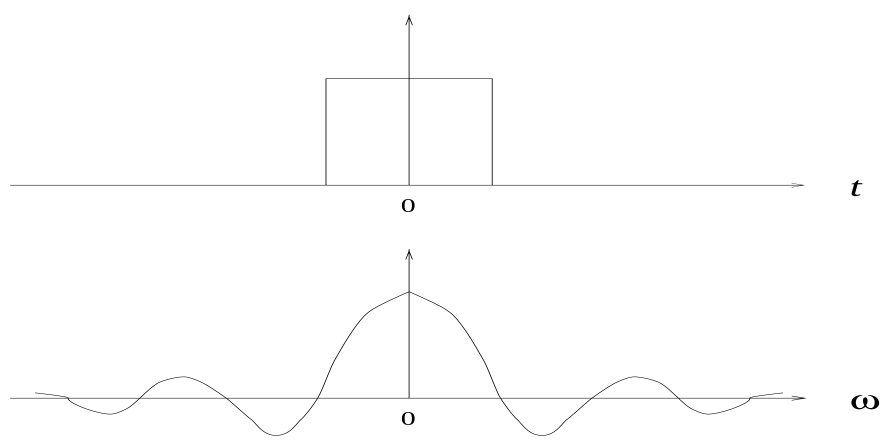
⟶ bandlimited

⟶ i.e., $S(\omega) = 0$ for ω sufficiently large

⟶ when $S(\omega) \approx 0$, can treat as $S(\omega) = 0$

⟶ let's approximate!

⟶ e.g., square wave



Ex.: human auditory system

⟶ 20 Hz–20 kHz

⟶ speech is intelligible at 300 Hz–3300 Hz

⟶ broadcast quality audio; CD quality audio

Telephone systems: engineered to exploit this property

⟶ bandwidth 3000 Hz

⟶ copper medium: various grades

Digital data vs. analog data

Digital data: bits.

$\longrightarrow$ discrete signal

$\longrightarrow$ both in time and amplitude

Analog data: audio/voice, video/image

$\longrightarrow$ continuous signal

$\longrightarrow$ both in time and amplitude

$\longrightarrow$ analog data is often digitized

$\longrightarrow$ digital signal processing

How to digitize such that digital representation is faithful?

$\longrightarrow$ sampling

Sampling theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with $S(\omega) = 0$ for $|\omega| > W$, $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

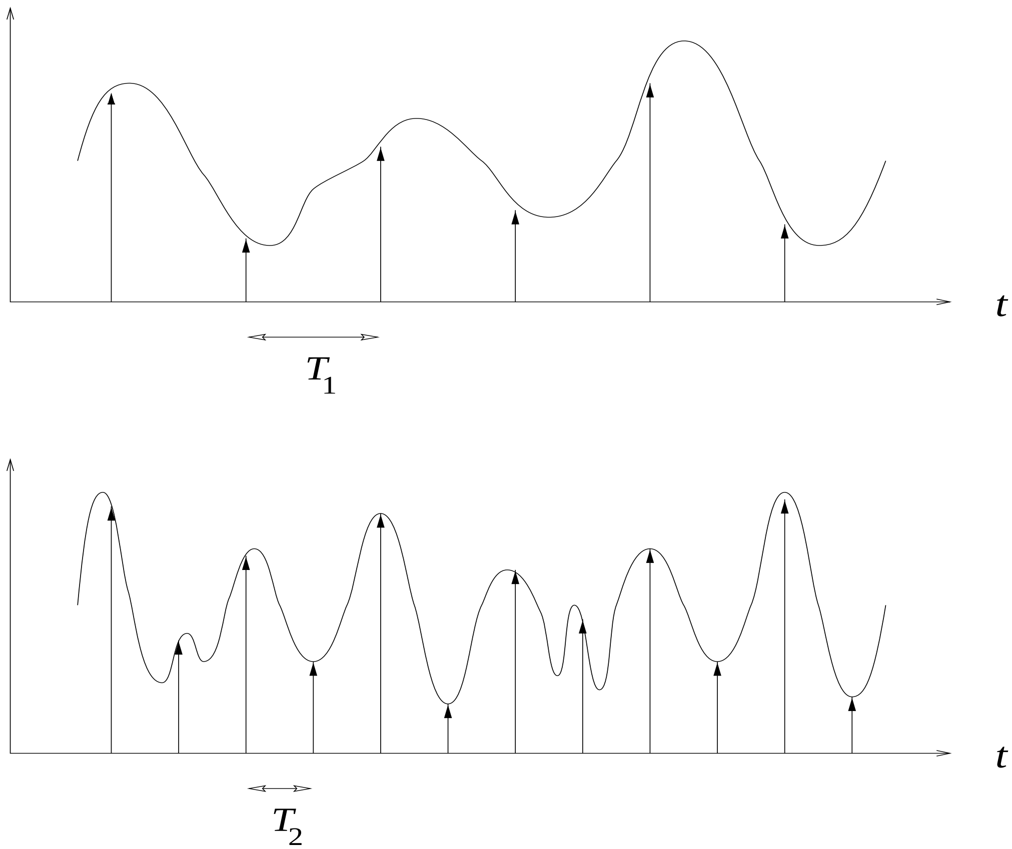
where ν is the sampling rate.

$\longrightarrow$ ν: samples per second

Quantization issue ignored

$\longrightarrow$ amplitude must also be digitized

Slowly vs. rapidly varying signal:



$$\nu_1 = \frac{1}{T_1} \;<\; \nu_2 = \frac{1}{T_2}$$