## Routing

Problem: Given more than one path from source to destination, which one to take?


## Features:

- Architecture
- Algorithms
- Implementation
- Performance


## Architecture

Hierarchical routing:
$\longrightarrow$ Internet: intra-domain vs. inter-domain routing $\longrightarrow$ separate decision making


## Granularity

- Router
- Domain: autonomous system (AS)
$\rightarrow 16$ bit identifier

Network representation

- Router graph
- AS graph

Stub AS


Transit AS

Route or path: criteria of goodness

- Hop count
- Delay
$\rightarrow$ composed of three parts
- Bandwidth
$\rightarrow$ available bandwidth
- Loss rate

Composition of goodness metric:
$\longrightarrow$ quality of end-to-end path

- Additive: hop count, delay
- Min: bandwidth
- Multiplicative: loss rate


## Goodness of routing:

$\longrightarrow$ assume $N$ users or sessions
$\longrightarrow$ suppose path metric is delay

- System optimal routing
$\rightarrow$ choose paths to minimize $\sum_{i=1}^{N} D_{i}$
- User optimal routing
$\rightarrow$ each user $i$ chooses path to minimize $D_{i}$
$\rightarrow$ selfish actions

Pros/cons:

- System optimal routing:
- Good: minimizes delay for the system as a whole
- Bad: complex and difficult to scale
- User optimal routing:
- Good: simple
- Bad: may not make efficient use of resources
$\rightarrow$ utilization

Some pitfalls of user optimal routing:
$\longrightarrow$ stemming from selfishness

- Fluttering or ping pong effect
- Braess paradox

Braess paradox example:

- 6 users sending 1 Mbps traffic
- Delay on shared link increases with traffic volume $x$
- Users make routing decisions one after the other

- 3 users will take $A \rightarrow B \rightarrow D$
- 3 users will take $A \rightarrow C \rightarrow D$
- total delay per user: $(5 \cdot 3+1)+(3+25)=44$

Resource provisioning:
$\longrightarrow$ high bandwidth link is added between $B$ and $C$


- User 1: $A \rightarrow B \rightarrow C \rightarrow D$ (13)
- User 2: $A \rightarrow B \rightarrow C \rightarrow D$ (23)
- User 3: $A \rightarrow B \rightarrow C \rightarrow D$ (33)
- User 4: $A \rightarrow B \rightarrow C \rightarrow D$ (43)
- User 5: $A \rightarrow B \rightarrow D$ (52)
- User 6: $A \rightarrow C \rightarrow D$ (52)

Adding extra link should improve things, but has the opposite effect
$\longrightarrow$ paradox possible due to selfishness
$\longrightarrow$ D. Braess (1969)
$\longrightarrow$ cannot arise in system optimal routing
$\longrightarrow$ i.e., cooperative routing

Adam Smith: let the "invisible hand" do its work $\longrightarrow$ doesn't always lead to best outcome $\longrightarrow$ capitalism vs. communism

Modus operandi of the Internet: user optimal routing $\longrightarrow$ simplicity wins the day

## Algorithms

Find short, in particular, shortest paths from source to destination.

Key observation on shortest paths:

- Assume $p$ is a shortest path from $S$ to $D$
$\rightarrow S \xrightarrow{p} D$
- Pick any intermediate node $X$ on the path
- Consider the two segments $p_{1}$ and $p_{2}$
$\rightarrow S \xrightarrow{p_{1}} X \xrightarrow{p_{2}} D$
- The path $p_{1}$ from $S$ to $X$ is a shortest path, and so is the path $p_{2}$ from $X$ to $D$


## Illustration:


shortest path shortest path
$\longrightarrow$ reverse implication need not hold

Procedure: Grow a routing tree $\mathcal{T}$ rooted at source $S$
$\longrightarrow$ initially $\mathcal{T}$ only contains $S$

1. Find a node $X$ with shortest path from $S$
$\rightarrow$ there may be more than one such node
$\rightarrow$ add $X$ (and path $S \stackrel{p}{\sim} X$ ) to routing tree $\mathcal{T}$
2. Find node $Y$ with shortest path from $\mathcal{T}$
$\rightarrow$ update existing paths if going through $Y$ is shorter
$\rightarrow$ uses shortest path decomposition property
3. Repeat step two until no more nodes left to add

Observations:
$\longrightarrow$ once node is added, it's final (no backtracking)
$\longrightarrow$ builds minimum spanning tree routed at $S$
$\longrightarrow$ Dijkstra's algorithm

Remarks:

- Running time: $O\left(n^{2}\right)$ time complexity $\rightarrow n$ : number of nodes
- Can also be run "backwards"
$\rightarrow$ start from destination $D$ and go to all sources
$\rightarrow$ single-destination/all-source shortest path
- Source $S$ requires global link distance knowledge
$\rightarrow$ centralized algorithm (center: source $S$ )
$\rightarrow$ every router runs Dijkstra with itself as source
- Internet protocol implementation
$\rightarrow$ OSPF (Open Shortest Path First)
$\rightarrow$ link state algorithm
- Minimum spanning tree routed at $S$ :
$\rightarrow$ multicasting: multicast tree
$\rightarrow$ standardized but not implemented on Internet


# Distributed/decentralized shortest path algorithm: 

$\longrightarrow$ Bellman-Ford algorithm
$\longrightarrow$ based on shortest path decomposition property

Key procedure:

- Each node $X$ maintains current shortest distance to all other nodes
$\rightarrow$ a distance vector
- Each node advertises to neighbors its current best distance estimates
- A node $X$, upon receiving an update from neighbor $Y$, performs update: for all $Z$

$$
d(X, Z) \leftarrow \min \{d(X, Z), d(Y, Z)+\ell(X, Y)\}
$$

... same criterion as Dijkstra's algorithm

Remarks:

- Running time: $O\left(n^{3}\right)$
- Each source or router only talks to neighbors
$\rightarrow$ local interaction
$\rightarrow$ no need to send update if no change
$\rightarrow$ if change, entire distance vector must be sent
- Knows shortest distance, but not path
$\rightarrow$ just the next hop is known
- Elegant but additional issues compared to Dijkstra's algorithm
$\rightarrow$ e.g., stability
- Internet protocol implementation
$\rightarrow$ RIP (Routing Information Protocol)

