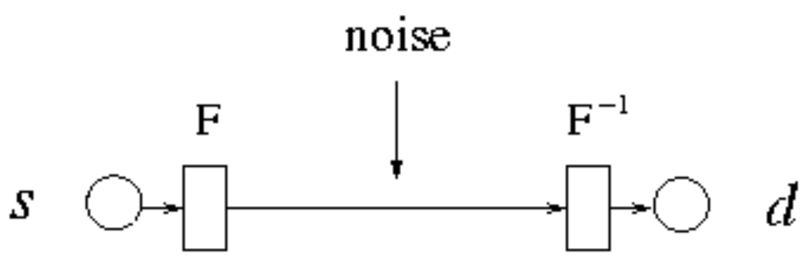
Information transmission under noise

How much throughput can we get in a network link that is subject to noise

$\rightarrow$ reliable throughput

$\rightarrow$ different models of noise

Set-up:



To communicate symbol $a \in \Sigma$ send code word w_a:

$$a \mapsto w_a \mapsto w \mapsto ?$$

$\rightarrow$ w_a gets corrupted and becomes w

$\rightarrow$ if $w = w_b$ for $b \neq a$, incorrectly conclude b was sent

Want F^{-1}:

- detect w has been corrupted

  $\rightarrow$ error detection

- correct w back to w_a

  $\rightarrow$ error correction

Coding theory

$\rightarrow$ error model

$\rightarrow$ e.g., how many bit flips, bit flip pattern

Examples:

- 1-bit flip detection: parity bit
- 1-bit flip correction: majority vote

  $\rightarrow$ 3-fold redundancy

Error detection:

- to communicate symbol $a \in \Sigma$, code word w_a is transmitted
- k bit flips change w_a into w
- to detect k-bit error, $w \neq w_b$ for any $b \in \Sigma$

  $\rightarrow$ i.e., w must not be a valid code word

Conceptually: code words live in higher dimensional space than symbols

$\rightarrow$ e.g., if a is n bits long, w_a is m bits long where $m > n$

Distance between code words $d(w_a, w_b) > k$

$\rightarrow$ Hamming distance (e.g., $d(0001, 0100) = 2$)

$\rightarrow$ detect up to k bit flips

$\rightarrow$ necessary and sufficient condition

Error correction: to correct k-bit error, $d(w, w_a) < d(w, w_b)$ for any $b \neq a$

$\rightarrow$ although w_a distorted into w, w most resembles w_a

$\rightarrow$ minimum distance matching

Geometrically: balls of radius k centered at code words must not intersect

$\rightarrow B_k(w_a) \cap B_k(w_b) = \emptyset$

$\rightarrow$ necessary and sufficient

Error detecting and correcting code constructed using algebra over finite fields

$\rightarrow$ e.g., CRC (cyclic redundancy check)

Shannon's result on reliable communication

$\rightarrow$ fundamental limit

$\rightarrow$ upper bound on bps

$\rightarrow$ depends only on bandwidth of physical link (Hz) and relative noise (dB)

Channel Coding Theorem (Shannon's 2nd Theorem): Given bandwidth W of physical link, signal power P_S, noise power P_N, link subject to white noise,

$$C \;=\; W \log\left(1 + \frac{P_S}{P_N}\right) \text{ bps}$$

$\rightarrow$ P_S/P_N: signal-to-noise ratio (SNR)

$\rightarrow$ increasing power yields logarithmic gain

Implications for networking:

- increase bandwidth W (Hz) to proportionally increase reliable throughput

  → e.g., FDM, OFDM

  → width not absolute frequency

  → what about AM (or PCM)?

- power control (e.g., handheld devices)

  → logarithmic gain

  → accelerates battery power depletion

  → multi-user interference: doesn't work if everyone increases power

  → signal-to-interference ratio (SIR)

  → in general: SINR

Signal-to-noise ratio (SNR) expressed as

$$\text{dB} = 10 \log_{10}(P_S/P_N)$$

Example: assuming a decibel level of 30, what is the channel capacity of a telephone line?

First, $W = 3000$ Hz, $P_S/P_N = 1000$. Using Channel Coding Theorem,

$$C = 3000 \log 1001 \approx 30 \text{ Kbps.}$$

→ compare against 28.8 Kbps modems

→ what about 56 Kbps modems?

→ incorrect assumptions

Last but not least: bandwidth (Hz) of signal $s(t)$

→ e.g., audio

Application: digitize analog signal

→ discrete time: sampling

→ discrete amplitude: quantization

Focus: digitize time so that fidelity is preserved

→ continuous time signal to discrete time samples

→ from discrete time samples back to continuous time signal

→ original replica

Sampling Theorem (Nyquist): Given continuous bandlimited signal $s(t)$ with bandwidth W (Hz), $s(t)$ can be reconstructed from its samples if

$$\nu > 2W$$

where ν is the sampling rate (unit: samples per second).

Human auditory system:

→ sensitivity: 20 Hz–20 KHz range (roughly 20 KHz)

→ voice: 300 Hz–3.3 KHz (roughly 4 KHz)

→ 8000 samples per second

→ note T1 line

CD quality audio: 44100 samples per second

→ also denoted Hz (44.1 KHz)