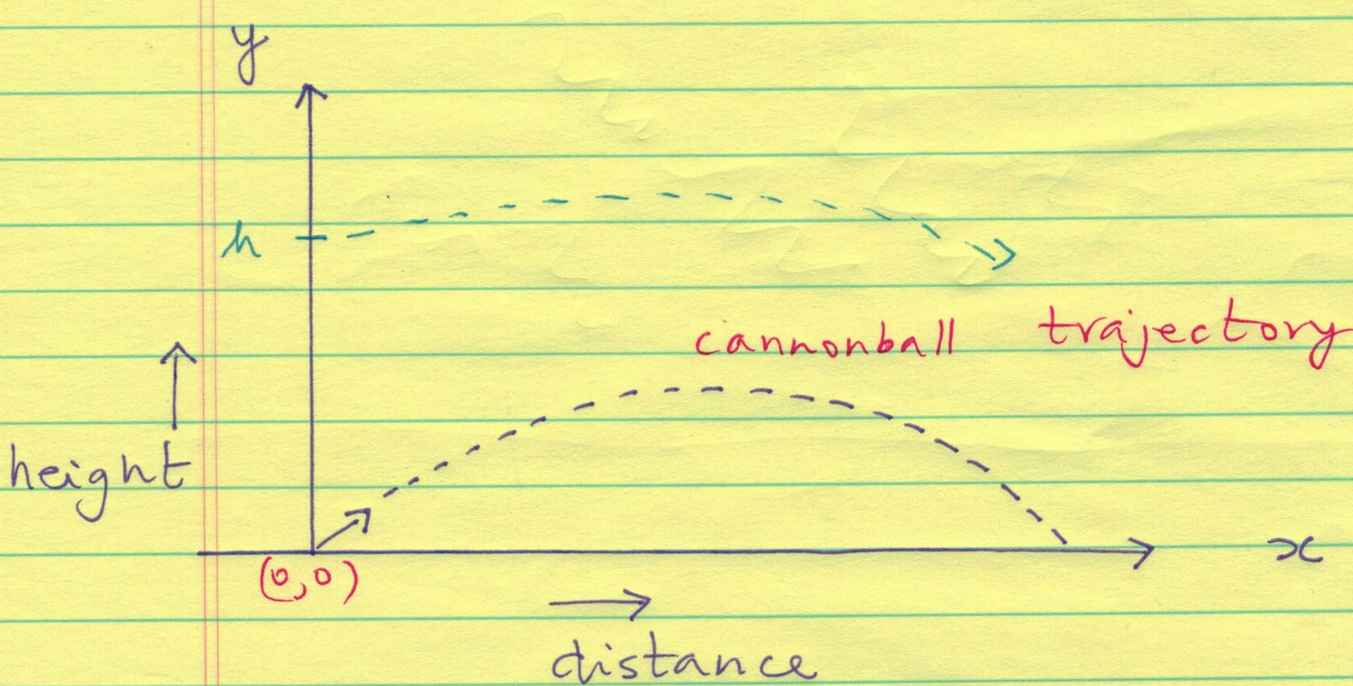
CHAPTER 10



problem : how to get path of cannonball?

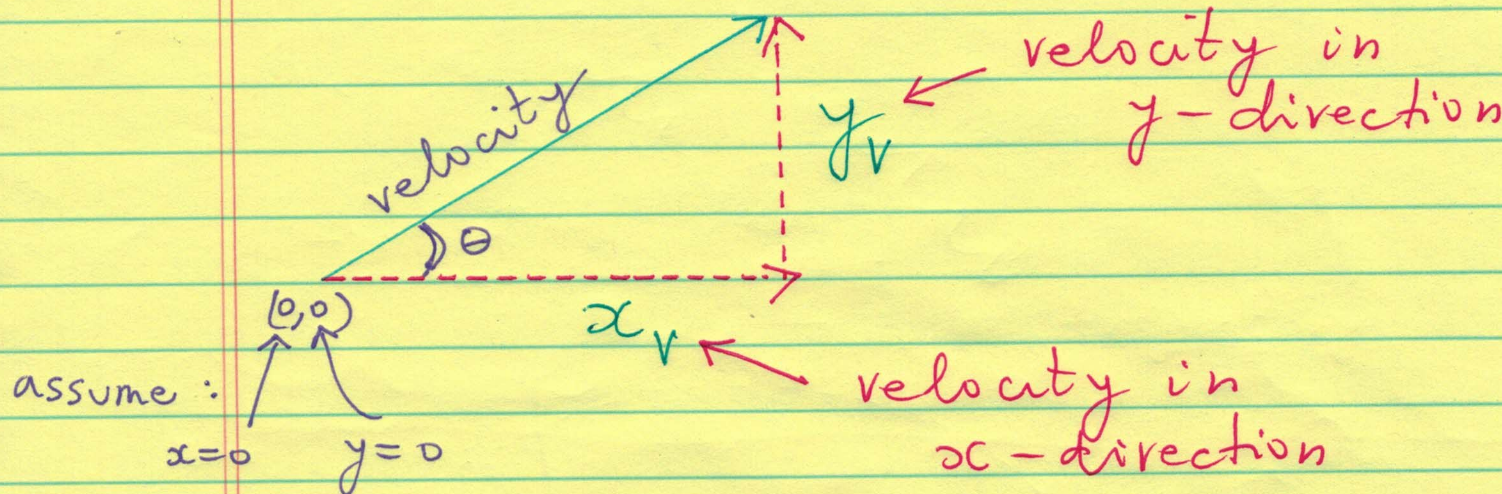
inputs : 1) angle of fire θ

2) (starting) velocity

3) height of cannon (h)

4) time-step (how often we update cannonball position)

assumption : ignore wind resistance



1. No wind resistance
   $\Rightarrow$ x_v is constant

2. y_v is positive at first,
   then becomes negative (gravity)

Suppose $y_v = 30$ m/sec at $t=0$

After 1 second, so at $t=1$,

$$y_v = 30 - 9.8 = 20.2 \text{ m/sec}$$

acceleration due to gravity is 9.8 m/sec

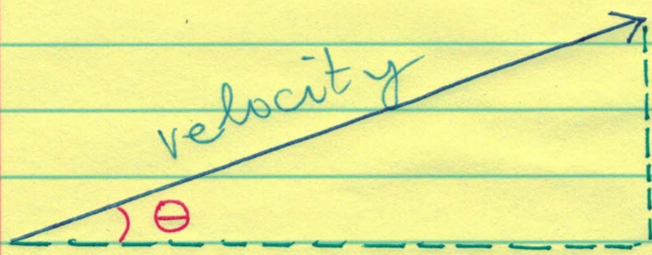
pulling the ball down toward earth.

At $t=2$, $y_v = 20.2 - 9.8 = 10.4$ m/sec

Soon y_v becomes negative

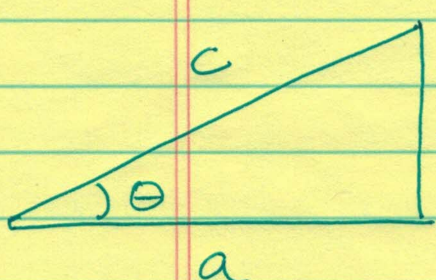
$\Rightarrow$ cannonball falls toward earth.

problem: How to compute x_v & y_v?



$y_v = velocity * \sin(\theta)$

$x_v = velocity * \cos(\theta)$



$\cos(\theta) = a/c$

$\sin(\theta) = b/c$

This enables you to split the initial firing velocity into its x-component (x_v) and y-component (y_v).

$180^\circ = \pi$ radians

$\Rightarrow$ $\underline{1}^\circ = \pi/180$ radians

$\Rightarrow$ $\theta^\circ = \theta * \pi/180$ radians

* Python needs the angle in radians to get $\sin(\theta)$ and $\cos(\theta)$

As cannonball travels, its position is given by (x, y)

x = horizontal distance from cannon

y = vertical " above ground

(or below ground if $y < 0$)

Let Δt = small time-step.

Suppose ball position is now (x, y)

After time Δt,

$$\underset{\leftarrow \text{new}}{x} = \underset{\leftarrow \text{old}}{x} + \underbrace{x_v}_{\text{x-velocity}} * \underbrace{\Delta t}_{\text{time}}$$

x-direction computation is easy because we assume no wind-resistance.

But y-direction is constantly acted upon by gravity.

yv decreases by 9.8 m/sec each passing second

yv decreases by 0.98 m/sec each passing 0.1 seconds.

Δt = time step

yv_1

velocity at start of time step

= y-velocity at end of previous time-step.

yv_2

velocity at end of time step.

so only need to compute this

old y-velocity

$$y = y + \Delta t * \underbrace{\frac{(y_{r_1} + y_{r_2})}{2}}$$

new y-velocity

time step

Approximation: using average velocity over Δt time step.

ALGORITHM:

input θ, v, h, t ← time step

velocity   height of cannon above ground

$r = (\theta * \pi)/180$ ← radians

$x = 0$

$y = h$

← $(0, h)$ is cannon position

$x_r = v * \cos(r)$

$y_r = v * \sin(r)$

] initial x and y velocities

While $(y \geq 0)$: (ball is above ground)

$$x = x + t * x_v$$

new x — old x — time — speed in x-direction

$$y_{v_1} = y_v$$ y-vel at start of interval

$$y_{v_2} = y_v - 9.8 * t$$ y-vel at end of interval

$$y = y + t * \underbrace{\left(\frac{y_{v_1} + y_{v_2}}{2}\right)}_{\text{average } y\text{-velocity in time step}}$$

new y — old y — time

———— x ————