# Week 3 (Wednesday) [No lecture on Monday Labor Day] 

Let's summarize what we have done so far to get to the programming stage:

## Variables

x, abcd, Abcd, ABCD, Joe ....
(Use variables that have meaning, i.e., total = sum1 + sum2 + sum3)

Algebraic Expressions and Functions
>> $3+8$
11
>> $3^{*} 2$
6
>> 3 *2.0
6.0
>>5/2
2.5
>> 4/2
2.0

5 has no decimal point, so it is type int (int means "integer")
5.0 has a decimal point, so it is type float (float means "floating point")

Why call it "floating point"? Because the decimal point can be moved anywhere
Convenient (say, to represent the number) without changing its value.
5.5 is same as $0.55 \times 10$

You can keep moving the decimal point right or left
In a computer word, 5.5 will be represented as 0.55 x 10

You already know how to represent 0.55 in binary. Use the 48 right-most bits (called "lower order
bits") to represent 0.55 in binary,
and use the 16 left-most bits (called "Higher order bits") to contain the number 1. The understanding is that 1 is the exponent of 10.

This is how floating point numbers are represented
.... by splitting a 64-bit word
Into two pieces, the low 48-bits holding the fractional part ("mantissa"), and the high 16-bits holding the exponent (with the understanding that it's the exponent of 10).

## Accuracy

This means the size of the exponent is constrained by 16 bits. So how large or how small can the exponent be?

Let's see. Suppose a computer word had only 3 bits. How many numbers could it hold?
$000=0$
$001 \quad 1$
$010 \quad 2$

| 011 | 3 |
| :--- | :--- |
| 100 | 4 |
| 101 | 5 |
| 110 | 6 |
| 111 | 7 |

So 3 bits can hold 8 numbers. Now these are all positive numbers. So a total of 8 positive numbers can be held in 3 bits.
$2^{* *} 3=8$
So 12 bits could hold 2**16 positive numbers.
$0,1,2,3, \ldots \ldots . .,\left(2^{* *} 16\right)-1$.
$\left(2^{* *} 12\right)-1 .=65535$
So the largest positive exponent we can have in 16 bits is 4095
I.e., $10 * * 65535$

What about negative exponents?
If there was a way we could signal that the number contained in the 16 bits is a negative number (and
yes, there is;
we'll see this way a bit later), then we could store the negative exponents
-65536, ........, -1
That means you can represent a number such as
$395 \times 10 * *(-100)$

## (Python will evaluate this as $3.9500000000000004 \mathrm{e}-98)$

If the number is too small for Python to print, it will print 0.0

Notice this "trade-off" with accuracy, depending on how we split the 64 bits into the two parts that contain the mantissa and the exponent. We don't
have a choice. The computer hardware/software decides.

Reduce the 16 bits to 12 bits or 8 bits and the exponent range gets smaller, but the accuracy of the number gets larger because we have more bits on the other side to represent the mantissa (the
number of digits after the decimal point).
Increase the number of bits for the exponent to 20 (say), and the accuracy of the mantissa
representation gets smaller, because fewer bits are left to hold it.

Final point:
To represent 5.5, we would put
0.55 in the mantissa bits and

1 in the exponent bits

So that we have the number (10**1) * 0.55
which is 5.5

Making sure the number after the decimal point is not 0 is called the "standard form".

In other words, we would not use a representation like
$\left(10^{* *} 2\right)^{*} 0.055$, or
$(10 * * 3) * 0.0055$ etc.
even though these are mathematically correct. We want to avoid losing valuable bits to the 0's after the decimal point, since we have only 48 bits to hold that fraction.

## Data Types

3,99, 17537531. Int
3.3.99.76 float
"Gosh! He talks a lot!" String (just a run of characters between quotes)

Algebraic expressions ALWAYS evaluate to a number:
>> $3 * 99$
297
What kind of number (int, float) you get as a result depends on what the inputs are. You saw this in the lecture on using ${ }^{*}$, /, //, \% operators.xpressions

## BOOLEAN EXPRESSIONS:

An important class of expressions do not evaluate to numbers. They evaluate to
True or False.
These are called Boolean expressions. We use these to make a program take one of two or three or four or umpteen different directions depending on what a particular expression evaluates to. We use the word "conditional" to describe that expression.

This is not Python code. Just demonstrating the idea.

If (temperature > 90F) then
print("Dress like Tarzan!")
Else
If (temperature < 42F) then print("Dress like an Eskimo!")
Else print("Dress normally!")

Expressions such as "temperature > 90f" have only one of two values: True or False.
They are Boolean expressions because a Boolean
variable can take on only one of two values: True or False.

Python understands Booleans.
>> $5<6$
True
>> $6<5$
False
>> $600+500>1200$
False
>>3 $==3$
True
>> $3=714$
False
$\gg 5+6+7==7+6+5$
True
When the expression evaluates to "True", the indented block under the "if" is executed.
When the expression evaluates to "False", the indented block under the "else"

Is executed.

So this is called an "if-else" statement, or an "if-then-else" statement. It makes your program go one way or another way depending on the conditions.

## Built-in functions:

>> abs (-5.5)
5.5
$\gg \min (5,6,7)$
5
>>max (-5, -6, -7)
-5

Functions that are not built-in must either come from existing libraries (e.g., like the sqrt() function that we import from the math module):
>>x = math.sqrt(256)
>> $x$
16
or functions you must write yourself. Say your teacher catches you giggling loudly in class and makes you stay after class and write some string (e.g., "I will not giggle in class!") 100 times on a sheet of paper. Being clever you learn Python and write a function to do this:
def me_smart( $\mathrm{s}, \mathrm{n}$ ): \#s is the string to be written (printed) $n$ times
for $k$ in range(n): \#using $k$ as index in a loop going from 0,1,2 .... to $n$ (stop at $n-1$ )
print(s)
return
>> me_smart("I will not giggle in class!", 100)
will print the string 100 times.

## Type Conversions

>> $5+6$ \#algebraic expression
>> ( $5<6$ ) \#logical expression .... evaluates to
True or False. (Boolean)
Important:
True is treated as the integer 1
False is treated as the integer 0
So, if in the shell, you type:
>> 10 + True
11 \#Python does IMPLICIT TYPE
CONVERSION because of the " + "
\#Converts True to the int 1 and does integer addition
>>10 + False \#False is converted to 0 because of the " + "
10
This means in the SET of integers (all the whole numbers), the Boolean values

True and
False
are a SUBSET because they are 1 and 0 .
The SET of integers are a subset of the FLOATS (because any integer becomes a float if you place a decimal point after it).
>> $4+.4545$. \# 4 is int, but Python converts it to float before addition
4.4545

The set of ints is contained in the set of floats. The latter is a larger set.

BUT, the range of values that int objects can have is much larger than the range of values that float objects can have.

That means you cannot always convert an int value to a float value (because the number of digits is too large and causes OVERFLOW)
>>> 2**10000 + 5
1995063116880758384883742162683585083

8234968318861924548520089498529438830 22194663191996168403619459789933112942 32091242715564913494137811175937859320 9632395785573004679379452676524655126 6059895520550086918193311542508608460 618104685509074866089624888090489894 8380092539416332578506215683094739025 5691238806522509664387444104675987162 6985453222868538161694315775629640762 8368807607322285350916414761839563814 5896946389941084096053626782106462142 733339403652556564953060314268023496 9400335934316651459297773279665775606 17258203140799419817960737824568376228 0037302885487251900834464581454650557 92960141483392161573458813925709537976 91192778008269577356744441230620187578 3632550272832378927071037380286639303 14281332414016241956716905740614196543 42324638801248856147305207431992259611 7962501309928602417083408076059323201 6126849228849625584131284406153673895 148711425631511108974551420331382020293 1640957596464756010405845841566072044 96286701651506192063100418642227590867 0900574606417856951911456055068251250 4060075198422618980592371180544447880

7290639524254833922198270740447316237 6760846613033778706039803413197133493 6546227005631699374555082417809728109 8329131440357187752476850985727693792 643322159939987688666080836883783802 76432827751722736575727447841122943897 33810861607423253291974813120197604178 2819656974758981645312584341359598627 8413012818540628347664908869052104758 0882615823961985770122407044330583075 8690393196046034049731565832086721059 13300903752823415539745394397715257455 29051021231094732161075347482574077527 3986348298498340756937955646638621874 56949927901657210370136443313581721431 1791398222983845847334440270964182851 0050729277483645505786345011008529878 1238947392869954083434615880704395911 89858151457791771436196987281314594837 83202081474982171858011389071228250905 82681743622057747592141765371568772561 4904582904992461028630081535583308130 1019876758562343435389554091756234008 4488752616264356864883351946372037729 324009445624692325435040067802727383 77553764067268986362410374914109667185 5705075909810024678988017827192595338

1282421954028302759408448955014676668 3896979968862416363133763939033734558 01407636741877711055384225739499110186 46821969658165148513049422236994771476 30691554682176828762003627772577237813 65331611196811280792669481887201298643 6607685516398605346022978715575179473 8524636944692308789426594821700805112 0322365496288169035739121368338393591 75641873385051097027161391543959099159 81546544173363116569360311222499379699 99226781732358023111862644575299135758 1750081998392362846152498810889602322 4436217377161808635701546848405862232 9792853875623486556440536962622018963 57102881236156751254333830327002909766 86505685571575055167275188991941297113 37690149916181315171544007728650573189 55745092033018530484711381831540732405 3319038462084036421763703911550639789 0007428536721962809034779745333204683 68795868580237952218629120080742819551 31794815762444829851846150970488802727 47215746881315947504097321150804981904 55803416826949787141316063210686391511 681774304792596709381

So no problem representing that integer. What if we want a float?
>>> 2**10000 + 5.0
Traceback (most recent call last):
File "<pyshell\#37>", line 1, in <module>
$2^{* *} 10000+5.0$
OverflowError: int too large to convert to float

EXPLICIT type-conversion is the conversion you do by yourself:
>> int (5.6)
5
>> int (-5.9)
-5
>> float(7)
7.0
>> int("5.9"). \#makes no sense, so Python will give you a ValueError

There is also a str() function (actually a "string constructor", just like int() is the "int constructor") that converts an int into a string
$\gg x=\operatorname{str}(5.9)$
>> x
'5.9'
\# x is the string " 5.9 "

