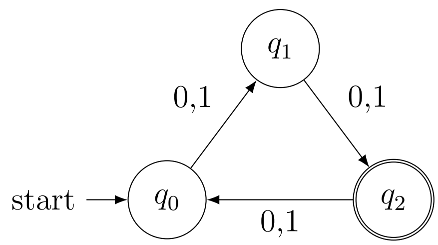
CS 483 - Introduction to the Theory of Computation Fall 2026

Assignment 1 (Total: 100 + 20 pts) *Due: Sep 10*

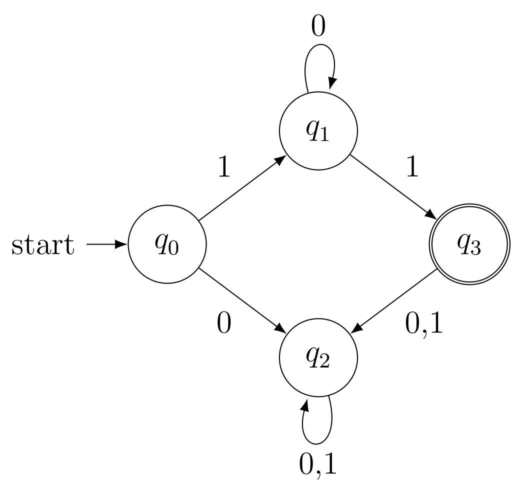
Note: Use LaTeX or Word to typeset your solutions, or write them legibly by hand and submit the scan. You can use the source code of this file as a template. To draw the finite automaton, you can use TikZ or `https://madebyevan.com/fsm/`.

Problem 1 (Identify DFAs). Consider the following state diagrams of DFAs over the alphabet $\{0, 1\}$. Describe the language accepted by each of them in one simple plain English sentence.

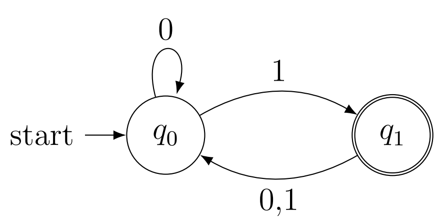
1. **(5 pts)**.



2. **(5 pts)**.



3. **(10 pts)**.



Problem 2 (Construct DFAs). Draw state diagrams of DFAs for the following languages over the alphabet $\{0, 1\}$. Try to use as few states as you can.

1. **(5 pts)**. $\{w \mid w \text{ starts with } 11\}$.
2. **(10 pts)**. $\{w \mid w \text{ is a string in } 0^* \cup 1^*\}$.
3. **(10 pts)**. $\{w \mid w \text{ contains the consecutive substring } 110\}$.

Problem 3 (Design regular expressions). Write regular expressions for the following languages over the alphabet $\Sigma = \{a, b, c\}$.

1. **(5 pts)**. $\{w \mid w \text{ only contains symbols } b, c\}$.
2. **(10 pts)**. $\{w \mid w \text{ starts and ends with the same symbol}\}$.
3. **(10 pts)**. $\{w \mid w \text{ does not contain any (not necessarily consecutive) substring } ab\}$.
4. **(Bonus, 10 pts)**. $\{w \mid w \text{ does not contain any consecutive substring } ab\}$.

Problem 3 (Prove regularity). Prove the following statements that certain languages are regular.

1. **(5 pts)**. If L is a regular language, show that the complement of L (i.e., $\Sigma^* \setminus L$) is also regular.
2. **(5 pts)**. If L_1, L_2 are both regular languages on the same alphabet, show that $L_1 \cap L_2$ is also regular.
3. **(10 pts)**. If L is a regular language on the alphabet Σ, then for every larger alphabet $\Sigma' \supset \Sigma$, show that L is also regular as a language on Σ'.
4. **(10 pts)**. Prove that

   $$L = \{w \mid w \text{ represents a non-negative binary number that is divisible by } k\}$$

   is regular. Here $k \in \mathbb{N}_+$ is a constant, and the binary number starts from the most significant bit with possible leading zeros.

   Hint. The DFA that accepts L could have k states.
5. **(Bonus, 10 pts)**. For languages A and B, let

   $$A/B = \{w \mid wx \in A \text{ for some } x \in B\}.$$

   If A is regular and B is any language, show that A/B is regular.