CS 483 - Introduction to the Theory of Computation Fall 2026

Assignment 5 (Total: 100 + 20 pts) *Due: Nov 5*

Note: Use LaTeX or Word to typeset your solutions, or write them legibly by hand and submit the scan. You can use the source code of this file as a template.

Problem 1 (Completeness without context). Define the symmetric difference between two sets as

$$A \oplus B = \{x \mid x \in A \setminus B \text{ or } x \in B \setminus A\}.$$

1. **(10 pts)**. Let L be an arbitrary non-trivial language ($L \neq \varnothing, L \neq \Sigma^*$) and let D be a finite language. Prove that $L \oplus D \leq_{\mathrm{m}} L$.
2. **(10 pts)**. Let L_1 and L_2 be two languages such that $L_1 \oplus L_2$ is finite. Prove that L_1 is RE-complete if and only if L_2 is RE-complete.

Problem 2 (Double Halting). Consider the language

$$\textsc{Halt}_2 = \{(M, w_1, w_2) \mid M \text{ is a Turing machine, } M(w_1) \neq \bot, M(w_2) \neq \bot\}.$$

1. **(10 pts)**. Prove that $\textsc{Halt} \leq_{\mathrm{m}} \textsc{Halt}_2$.
2. **(10 pts)**. Prove that $\textsc{Halt}_2 \leq_{\mathrm{m}} \textsc{Halt}$.

Problem 2 (Determine decidability and recognizablity). Determine whether each of the following languages are decidable, undecidable but recognizable, or unrecognizable. Justify your answers.

1. **(5 pts)**. $\textsc{HaltTime} = \{(M, w, t) \mid M \text{ is a TM, } M(w) \text{ halts in } t \text{ steps}\}$.
2. **(5 pts)**. $\textsc{Empty}_{\mathrm{DFA}} = \{M \mid M \text{ is a DFA, } L(M) = \varnothing\}$.
3. **(10 pts)**. $\textsc{Empty}_{\mathrm{TM}} = \{M \mid M \text{ is a TM, } L(M) = \varnothing\}$.
4. **(10 pts)**. $\textsc{Fin} = \{M \mid M \text{ is a TM, } L(M) \text{ is finite}\}$.
5. **(10 pts)**. $\textsc{SelfHalt} = \{M \mid M \text{ is a TM, } M(M) \neq \bot\}$.
6. **(Bonus, 10 pts)**. $\overline{\textsc{Fin}} = \{M \mid M \text{ is a TM, } L(M) \text{ is infinite}\}$.
7. **(Bonus, 10 pts)**. $\textsc{Halt}_{\mathrm{rev}} = \{(M, w) \mid M \text{ is a TM, } M(w) \neq \bot \text{ and } M(w^{\mathrm{R}}) = \bot\}$.

Problem 4 (Busy beavers). A unary Turing machine is a Turing machine with unary alphabet $\Sigma = \{1\}$ and tape alphabet $\Gamma = \{1, \sqcup\}$.

1. **(10 pts)**. Let

$$\textsc{Halt}_{\text{unary}} = \{(M, w) \mid \text{M is a unary Turing machine}, w \in 1^*, M(w) \neq \perp\}.$$

   Prove that $\textsc{Halt}_{\text{unary}}$ is undecidable.

2. **(10 pts)**. Define the busy-beaver number $BB(n)$ as the maximum number of steps that a unary, single-tape Turing machine with n states halts on an empty input (among, of course, the ones that halts). Prove that BB is not a computable function.