CS58400 - Theory of Computation and Computational Complexity Spring 2026

Assignment 2 *Due: Mar 12*

Note: Use LaTeX to typeset your solutions. You can use the source code of this file as a template or reference. Bonus questions are still capped by the total assignment grades, so only work on them if you want a challenge.

Problem 1 (Careless definition of NP) Define $\mathsf{NP}^{\times}$ as the class of languages L such that for some $L' \in \mathsf{P}$,

$$L = \{x \mid \exists y \in \{0,1\}^*, (x, y) \in L'\}$$

(in other words, it is "NP" but with arbitrarily long witnesses).

(5 pts). Prove that $\mathsf{NP}^{\times}$ is in fact RE.

Problem 2 (Applications of padding). Here are two statements we mentioned in class, and both can be proved using the padding technique.

1. **(5 pts)**. Show that for every language $L \in \mathsf{EXP}$, there exists $L' \in \mathsf{E}$ such that $L \leq_{\mathrm{m}}^{\mathrm{p}} L'$. Use it to conclude that $\mathsf{NP} \neq \mathsf{E}$.

2. **(5 pts)**. Let $\mathsf{EEXP} = \bigcup_{c>0} \mathsf{TIME}\big(2^{2^{n^c}}\big)$. Prove that $\mathsf{EXP}^{\mathsf{EXP}} = \mathsf{EEXP}$.

   Hint. In an oracle Turing machine with exponential running time, the content on the query tape can be exponentially longer than the original input.

Problem 3 (Properties of $\mathsf{NP} \cap \mathsf{coNP}$).

1. **(5 pts)**. Prove that $\mathsf{NP} \cap \mathsf{coNP}$ is closed under symmetric difference. That is, if $L_1, L_2 \in \mathsf{NP} \cap \mathsf{coNP}$ then

   $$L_1 \oplus L_2 = \{x \mid x \text{ is in exactly one of } L_1 \text{ or } L_2\}$$

   is also in $\mathsf{NP} \cap \mathsf{coNP}$.

2. **(5 pts)**. Prove that $\mathsf{P}^{\mathsf{NP} \cap \mathsf{coNP}} = \mathsf{NP} \cap \mathsf{coNP}$.

   Hint. What could be the witnesses for a language in $\mathsf{P}^{\mathsf{NP} \cap \mathsf{coNP}}$?

Problem 4 (More completeness practice). All completeness in this question refers to the completeness under polynomial-time reductions.

1. **(10 pts)**. Prove that the following language is NP-complete:

$$\textsc{SubsetSum} = \{(z_1, \ldots, z_n, s) \in \mathbb{Z}^{n+1} \mid \exists S \subseteq [n], \textstyle\sum_{i \in S} z_i = s\}.$$

   Hint. If each $z_i \leq \text{poly}(n)$, then the problem can be solved in polynomial time by dynamic programming. To make $\textsc{SubsetSum}$ hard, z_i's need to be large numbers.

2. **(5 pts)**. Prove that the following language is EXP-complete:

$$\textsc{HaltTime} = \{(\langle M \rangle, x, t) \mid \text{TM } M \text{ halts on input } x \text{ in } t \text{ steps.}\}$$

3. **(Bonus, 5 pts)**. Let DP be the class of languages L such that $L = L_1 \cap L_2$ for some $L_1 \in \mathsf{NP}$ and $L_2 \in \mathsf{coNP}$. Prove that the following language is DP-complete:

$$\textsc{ExactDomSet} = \{(G, k) \mid \gamma(G) = k\},$$

   where $\gamma(G)$ is the domination number of graph G, i.e. the minimum size of a dominating set of G.

   Hint. Try to modify the proof that $\textsc{DomSet}$ is NP-hard.